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Adequate Liquid Provision for a Run Preventing Contract

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Abstract

We extend Ross and Cooper (1998) and find an adequate liquidate provision as a function of liquidity cost in CRRA (Constant Relative Risk Aversion) environment. Our study shows that a RPC (run preventing contract) in a MMMF (money market mutual fund) requires a higher amount of liquidity provision when the capital loss becomes greater in a credit crunch period.

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1. Introduction

In the 2007 subprime mortgage crisis, obtaining adequate short-term liquidity was important to some financial intermediaries, particularly MMMFs (money market mutual fund), in order to prevent a possible fund run. When financial markets go through a credit crunch period, the liquidation cost of long-term assets increases as the market value of such assets decreases under mark-to-market accounting rules. As a result, some investors worry about capital loss of their investment and therefore may wish to withdraw earlier than they would have if the crisis had not occurred. For example, a recent report under the title "Shadow banking, financial markets and financial stability" by the Deputy Governor of the Bank of England stated the following: "Money market funds have become a gigantic part of the US and European financial systems, and just like banks, they are subject to runs…So their own maturity mismatches mask the true liquidity position of the banking sector, and inject extra fragility into the financial system as a whole."[1]

One way to give confidence back to the investors and to ease their concerns would be deposit insurance, as in banks, where there was no run concern in the US. Another way would be to reserve sufficient liquid assets; nevertheless, the returns on liquid assets may be lower. However, the literature on the fund industry has not yet considered such liquidity provision to prevent runs. In bank run literature, Cooper and Ross show the necessity of liquidity provision in order to prevent runs. Thus it would be worthwhile to extend Cooper and Ross (1998). By allowing varying liquidation costs, captured by τ, where $\tau \in [0,1]$,[2] we theoretically derive the adequate holding of liquidity in MMMF to prevent a run as a function of τ.

2. The model

The model is a modified version of Diamond-Dybvig (1983). There is a unit mass of an infinite number of identical agents within a range of [0, 1], each of whom lives over three periods, t=0, 1, and 2, and is born with a unit endowment, which they deposit with an intermediary in period 0. At the start of period 1, agents are informed about their consumption types. A fraction π learn that they obtain utility from period 1 consumption only (early consumers), while the others obtain utility from period 2 consumption (late consumers). We assume that π is non-stochastic and known to all agents and an intermediary; c_1 and c_2 denote the consumption levels for early and late consumers,

[1] "Money Market funds have become a gigantic part of the US financial system; at about \$3trn, being roughly the same size as the transaction deposits of commercial banks. They are pretty big in Europe too − around \$1.5trn. They offer a bank-like service: almost instant liquidity... And they lend it out, purchasing commercial paper of various types as well as treasury bills and providing repo financing....On both sides of the Atlantic, many are so-called Constant Net Asset Value (CNAV) funds. Stripping through the detail, this means that they promise to return to savers, on demand, at least as much as they invest. Just like a bank. And just like a bank, they are subject to runs… And, if a Constant-NAV fund's value goes just a few basis points below par (100p in the £1), they effectively have to close, fuelling the incentive to run... So their own maturity mismatches mask the true liquidity position of the banking sector, and inject extra fragility into the financial system as a whole."

[2] However, they did not examine an adequate level of liquidity provision across different liquidity costs, because the bank is free of a run concerns for the presence of either deposit insurance protection or other guarantees.

respectively. The agent's utility function of $u_i(c_i)$ is described as $\log c_i$, where $i \in \{1,2\}$, is strictly increasing and strictly concave.

2.1 With No Liquidation Cost

There are two investment technologies available for transferring resources over time. First, there is a productive technology that provides a means of shifting resources from period 0 to 2, with a return of R>1 over the two periods. However, early liquidation of the technology in period 1 yields only unit per unit of period 0 investment. Second, there is a liquid technology that yields one unit in period t+1 per unit of period t investment, t=0, 1.We denote the amounts of liquid and illiquid investments as i_1 and i_2, respectively.

We assume that there is an intermediary operating in a competitive environment, which compels it to offer contracts that maximize a representative agent(consumer)'s ex-ante expected utility, subject to its break-even constraint. The intermediary's goal then is to choose optimal levels of c_1, c_2, i_1, and i_2 with two associated Lagrange multipliers as follows:

$$\max_{c_1,c_2,i_2} \pi u_1(c_1) + (1-\pi)u_2(c_2)$$

s.t. $\pi c_1 \le 1 - i_2$, $(1-\pi)c_2 \le R i_2$ as i_1 is dominated.

Here we ignore liquid technology, because it is completely dominated by illiquid technology from the choice set in period 0, as the productive technology provides a higher return in two periods of investment as well as yielding the same return as the liquid technology in one period of investment. Taking the two resource feasibility conditions as binding based on the property of $u_i'(c_i) > 0$, we can solve for the values of i_2 and c_2 as a function of c_1. The intermediary's optimization problem is then simplified for maximizing the representative consumer's utility with respect to c_1 as follows:

$$\max_{c_1} \pi u_1(c_1) + (1-\pi)u_2\left(\frac{1-\pi c_1}{1-\pi}\right)$$

First Order Condition (FOC) with respect to c_1 gives the solution as $c_1^* = 1$. Now, replacing this value into the above two resource constraints generates the associated optimal values of i_2 and c_2 as $i_2^* = 1-\pi, c_2^* = R$. The above solution becomes the first best allocation because, if the ex-post agent consumption types were costlessly verifiable to the intermediary in advance, he (or she) would offer the same contract as above. Also, the above solution is not vulnerable to a run, because as c_2^* turns out to be greater than c_1^*, the late consumer's motive for misrepresentation of being an early consumer through a fund run disappears.

2.2 With Liquidation Cost

Thus far, early liquidation cost of the productive technology was assumed to be zero. Hereafter, however, we introduce its liquidation cost. The cost is denoted by $\tau \in [0,1]$, where $\tau \neq 0$.[3] Thus, early liquidation of an illiquid investment in period 1 yields only $1-\tau$ per unit of period 0 investment when the financial market is in a credit crunch. The liquid investment still provides the same one unit in period t+1 per unit of period t investment as before. Thus, the one-period return of the liquid technology becomes higher than that of the illiquid investment. Hence, for the intermediary, an incentive to hold the liquid asset now arises[4] The resource feasibility conditions then change as follows when the intermediary decides to hold a nonnegative i_1:

$$\pi c_1 + i_1 \leq 1 - i_2, \quad (1)$$

$$(1-\pi)c_2 \leq Ri_2 + i_1 \quad (2)$$

With $\tau \neq 0$, holding too much illiquid asset may bring a run against the intermediary when late consumers misrepresent themselves as early ones by withdrawing i_2 earlier, and therefore $c_1 > 1 - i_2\tau$ occurs. In particular, if the level of i_2 is one, a fund run is likely. With this run possibility, it is worthwhile to construct a run preventing contract (hereafter RPC).[5] No run condition is characterized by $c_1 \leq 1 - i_2\tau$, as suggested in Cooper and Ross (1988). Also, holding a positive i_1 may be of value for either preparing for early consumption or preventing a run. The RPC contract of the intermediary involves levels of c_1, c_2, i_1, and i_2, which is rewritten as follows:

$$\max_{c_1, c_2, i_1, i_2, \lambda} \pi u_1(c_1) + (1-\pi)u_2(c_2) + \lambda[1 - i_2\tau - c_1]$$

s.t. both equation (1) and (2), which are assumed to be binding hereafter

By substituting c_1 and c_2 from the two equations (1) and (2) respectively into the above object function, we can rewrite the optimization only for i_1, i_2, and the multiplier λ as follows.

$$\max_{i_1, i_2, \lambda} \pi u_1(c_1(i_1, i_2; \pi, \tau, R)) + (1-\pi)u_2(c_2(i_1, i_2; \pi, \tau, R)) + \lambda[1 - i_2\tau - c_1(i_1, i_2; \pi, \tau, R)] \quad (3)$$

Taking derivatives on the object function with respect to i_1, i_2, and the multiplier λ yields the subsequent three FOC (First Order Condition)s, which can be written as in (4) through (6) below.

$$-\frac{1}{c_1} + R\frac{1}{c_2} + \lambda\left(\frac{1}{\pi} - \tau\right) = 0 \quad (4)$$

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3 Diamond and Dybvig assume that $\tau = 0$, ignoring the liquidation costs. However, in reality, liquidating illiquid projects earlier than before the maturity is associated with some costs.
4 Holding i_1 over two periods is not desirable when consumer types are observable, because holding i_1 brings a lower rate of return than that of i_2 over the same period.
5 Another candidate for RPC could be deposit insurance by the government. However, the insurance provision by the government sometimes becomes the very cause of an intermediary moral hazard, and hence we ignore the insurance provision here. Instead, we assure RPC by holding sufficient liquidity provision in advance.

$$-\frac{1}{c_1}+\frac{1}{c_2}+\frac{\lambda}{\pi}=0 \qquad (5)$$

$$1-i_2\tau-c_1=0 \qquad (6)$$

The above three First Order Conditions jointly combined with the two resources constraints, equations (1) and (2), give the following optimal values.

$c_1^{**}=\frac{\pi(\varphi+1)}{\pi+\varphi}$, $c_2^{**}=\varphi+1$, where $\varphi=\frac{R-1}{\tau}$[6]

$i_1^{**}=\frac{(1-\pi)}{(\pi+\varphi)}(\varphi(1+\pi)+\pi-\frac{\varphi}{\tau})$, $i_2^{**}=\frac{\varphi(1-\pi)}{\tau(\pi+\varphi)}$, and $\lambda=\frac{\varphi}{\varphi+1}$

under the condition of $\varphi(1+\pi)+\pi\geq\frac{\varphi}{\tau}$ to yield a non-negative value of i_1^{**}.

Sketch of proof: First, when we substitute $-\frac{1}{c_1}$ from equation (4) into equation (5), it solves for the value of c_2 as $\frac{R-1}{\tau}\lambda$. Second, substituting this into equation (2)(i.e., $(1-\pi)c_2=Ri_2+i_1$) yields $\frac{(1-\pi)\varphi}{\lambda}=i_2R+i_1$, which is denoted as equation (7). Third, replacing c_1 from equation (1) by using equation (6) yields $1-\pi=i_1+(1-\pi\tau)i_2$, which is denoted as equation (8). Fourth, using both equations (7) and (8) to solve for optimal values of i_1 and i_2 as a function of λ yields the following equations, given the parameter values π, τ, and R :

$$i_2=\frac{(1-\pi)(\frac{\varphi}{\lambda}-1)}{R-1+\pi\tau},\quad i_1=\frac{(1-\pi)(R-\frac{\varphi}{\lambda}+\frac{\varphi}{\lambda}\pi)}{R-1+\pi\tau}$$

Fifth, substituting the previous two expressions of i_1 and i_2 into the resource constraints will provide optimal values of c_1 and c_2 as a function of λ. Sixth, plugging these two values into both c_1 and c_2 from equation (5) will solve for a unique optimal value of λ, which is $\frac{\varphi}{\varphi+1}$. Lastly, plugging this value of λ back into the previously driven values of i_1, i_2, c_1, and c_2 yields the optimal values of c_1^{**}, c_2^{**}, i_1^{**} and i_2^{**}. QED

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[6] For the sake of notation simplicity we introduce φ, which represents the ratio of net return (R-1) to liquidation cost (τ): φ being greater than one means that the net return on illiquid technology is greater than the cost of liquidation. On the other hand, φ being less than one denotes that the net return is less than the liquidation cost.

Now, based on the derived optimal values of i_1^{**} and i_2^{**}, we can derive the adequate amount of liquidity holding in MMF as a function of τ. Our main findings are summarized as Proposition 1:

Proposition 1. ***As*** τ ***(liquidation cost) increases,*** i_1^{**} ***(liquid investment) increases while*** i_2^{**} ***(illiquid investment) decreases.***

Sketch of proof: After switching the expression of φ in i_1^{**} and i_2^{**} as an explicit function of τ (i.e., $\varphi = \frac{R-1}{\tau}$), we can compute the values of $\frac{di_1^{**}}{d\tau}$, $\frac{di_2^{**}}{d\tau}$. In the re-expressed $i_2^{**} = \frac{(R-1)(1-\pi)}{\tau(\tau\pi + R - 1)}$, τ term only appears in the denominator. Therefore, it is clear that $\frac{di_2^{**}}{d\tau} < 0$. Now, for $\frac{di_1^{*}}{d\tau}$, differentiating $i_1^{**} = (1-\pi)\frac{(R-1)(1+\pi) + \tau\pi - \frac{R-1}{\tau}}{(\tau\pi + R - 1)}$ with respect to τ gives $\frac{di_1^{**}}{d\tau} = (1-\pi)\frac{(R-1)\pi(\frac{2}{\tau} - \pi) + \frac{(R-1)^2}{\tau^2}}{(\tau\pi + R - 1)^2}$, which is positive as $\frac{2}{\tau} > 2 > \pi$. QED

Proposition 1 implies that when the liquidation cost of assets increases in a credit crunch period, reserving sufficient liquid assets is necessary to give confidence back to the MMF investor, even though the returns on liquid assets may be lower. Our analysis proves the prediction made by Cooper and Ross(1998).[7] Owing to the advantage of adopting a specific utility function of $\log c_i$, we succeed in deriving the optimal amounts of c_1^{**}, c_2^{**}, i_1^{**}, and i_2^{**} as explicit functions of π, τ, and R.

We simulate the optimal level of illiquid and liquid investments depending on the liquidation cost. For the simulation, we assume R as 1.15 with net returns of 15%; the fraction of early consumer π is 1/3 and the endowment is 1.

[7] Cooper and Ross (1998) analyze two extreme cases where τ values are either 1 or 0. Based upon those two cases, then, they predict that the amount of excess liquidity to be held for the RPC will depend upon the liquidation costs.

Figure 1

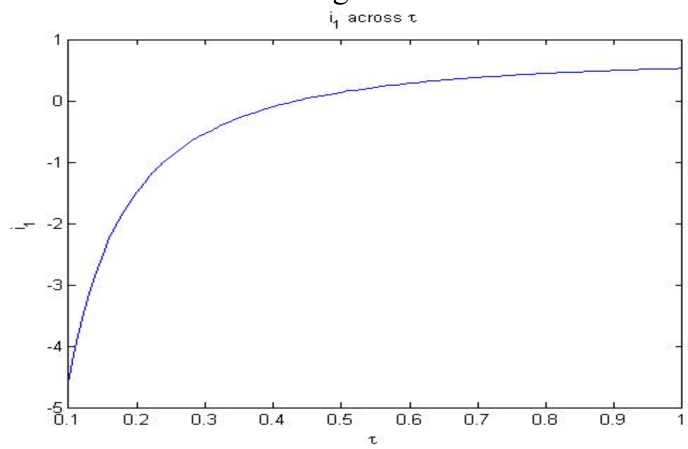
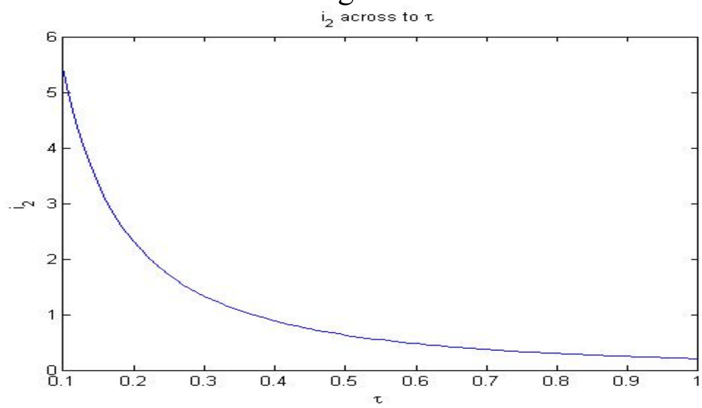


Figure 2



As shown in both figures, the optimal level of liquid investment, i_1^*, increases as the liquidation cost τ increases while the optimal level of illiquid investment, i_2^*, decreases.

2.3 Extension beyond the log functional form

In this subsection, we extend the results driven in section 2.2 into a general CRRA (Constant Relative Risk Aversion) utility function, which is $u\,(c_i) = \dfrac{c_i^{\sigma}}{\sigma}$, where $\sigma < 1$.[8] The purpose of this exercise is to confirm that the finding in section 2.2 is applicable to a more general environment. Now the problem we face is to solve the previous constraint optimization once again with respect to c_1, c_2, i_1 and i_2, not in log utility but in a more general utility function:

$$\max_{i_1, i_2, \lambda} \pi u_1(c_1(i_1, i_2; \pi, \tau, R)) + (1-\pi) u_2(c_2(i_1, i_2; \pi, \tau, R)) + \lambda[1 - i_2\tau - c_1(i_1, i_2; \pi, \tau, R)] \qquad (3)$$

The subsequent three FOC(First Order Condition)s in the CRRA environment are written below as (8) through (10).

[8] If σ becomes zero, then the utility function returns to the previous log utility. In this line, this section tries to extend the previous result to a more general case.

$$-c_1^{\sigma-1} + Rc_2^{\sigma-1} + \lambda\left(\frac{1}{\pi} - \tau\right) = 0 \tag{8}$$

$$-c_1^{\sigma-1} + c_2^{\sigma-1} + \frac{\lambda}{\pi} = 0 \tag{9}$$

$$1 - i_2\tau - c_1 = 0 \tag{10}$$

When we combine the above three First Order Conditions with the two binding resources constraints, equations (1) and (2), we can derive the following optimal values.

$$c_1^{***} = \frac{\varphi + 1 - (1-\pi)c_2^{***}}{\pi + \varphi} = \frac{\varphi + 1 - (1-\pi)\left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}}}{\pi + \varphi}, \quad c_2^{***} = \left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}} = \frac{\varphi + 1}{\left[\frac{(\varphi+\pi)^{\sigma}}{\pi}\right]^{\frac{1}{\sigma-1}} + 1 - \pi},$$

where $\varphi = \dfrac{R-1}{\tau}$, $\lambda = \left[\dfrac{\varphi^{\frac{1}{\sigma-1}}(\varphi+1)}{\left[\frac{(\varphi+\pi)^{\sigma}}{\pi}\right]^{\frac{1}{\sigma-1}} + 1 - \pi}\right]^{\sigma-1}$

$$i_2^{***} = \frac{(1-\pi)(\left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}} - 1)}{R - 1 + \pi\tau}, \quad i_1^{***} = \frac{(1-\pi)(R - (1-\pi\tau)\left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}})}{R - 1 + \pi\tau}$$

Sketch of proof: First, when we substitute $-c_1^{\sigma-1}$ from equation (8) into equation (9), it solves for the value of c_2 as $\left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}}$. Second, substituting this into equation (2) yields $(1-\pi)\left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}} = i_2R + i_1$, which is denoted as equation (11). Third, replacing c_1 from equation (1) by using equation (10) yields $1 - \pi = i_1 + (1-\pi\tau)i_2$, which is denoted as equation (12). Fourth, combining both equations (11) and (12) to solve for optimal values of i_1 and i_2 as a function of λ gives the following equations, given the parameter values π, τ, and R :

$$i_2 = \frac{(1-\pi)(\left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}} - 1)}{R - 1 + \pi\tau}, \quad i_1 = \frac{(1-\pi)(R - (1-\pi\tau)\left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}})}{R - 1 + \pi\tau}$$

Fifth, substituting the previous two expressions of i_1 and i_2 into the resource constraints will provide optimal values of c_1 and c_2 as a function of λ. Sixth, plugging these two values into both c_1 and c_2 from equation (9) will give a unique optimal value of λ, which is $\left[\dfrac{\varphi^{\frac{1}{\sigma-1}}(\varphi+1)}{\left[\dfrac{(\varphi+\pi)^{\sigma}}{\pi}\right]^{\frac{1}{\sigma-1}}+1-\pi}\right]^{\sigma-1}$. Lastly, plugging this of λ back into the previously driven values of i_1, i_2, c_1, and c_2 yields the optimal values of c_1^{***}, c_2^{***}, i_1^{***}, and i_2^{***}. QED

To check the consistency between sections 2.2 and 2.3, we put the value of 0 into σ, which is the power term of CRRA utility, and then derive λ and c_1^{***}. Now we can see that these values are exactly the same values as driven in section 2.2 (i.e., $c_1^{**}=\dfrac{\pi(\varphi+1)}{\pi+\varphi}$, $\lambda=\dfrac{\varphi}{\varphi+1}$). The remaining c_2^{***}, i_1^{***}, and i_2^{***} values become the same c_2^{**}, i_1^{**}, and i_2^{**} in section 2.2. Now, using i_1^{***}, and i_2^{***}, we can derive the optimal amount of liquidity holding in MMMF as a function of τ in Proposition 2 once again.

Proposition 2. ***As τ (liquidation cost) increases, i_1^{***} (liquid investment) increases while i_2^{***} (illiquid investment) decreases.***

Sketch of proof: Considering $c_2^{***}=\left[\dfrac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}}$, we can rewrite i_1^{***} and i_2^{***} as follows:

$$i_2^{***}=\frac{(1-\pi)(c_2^{***}-1)}{R-1+\pi\tau},\quad i_1^{***}=\frac{(1-\pi)(R-(1-\pi\tau)c_2^{***})}{R-1+\pi\tau}$$

Now, Regarding computation of the values of $\dfrac{di_1^{***}}{d\tau}$, $\dfrac{di_2^{***}}{d\tau}$, from the above expression, we can see that the sign of $\dfrac{dc_2^{***}}{d\tau}$ is crucial and that the signs of $\dfrac{di_1^{***}}{d\tau}$, $\dfrac{di_2^{***}}{d\tau}$ are subsequently determined depending on the sign of $\dfrac{dc_2^{***}}{d\tau}$ or $\dfrac{dc_2^{***}}{d\varphi}$, where $\varphi=\dfrac{R-1}{\tau}$.

Note that the sign of $\dfrac{dc_2^{***}}{d\tau}$ is exactly the opposite of the sign of $\dfrac{dc_2^{***}}{d\varphi}$.

Now, for $\frac{dc_2^{***}}{d\varphi}$, differentiating $c_2^{***} = \left[\frac{\lambda}{\varphi}\right]^{\frac{1}{\sigma-1}} = \frac{\varphi+1}{\left[\frac{(\varphi+\pi)^\sigma}{\pi}\right]^{\frac{1}{\sigma-1}} + 1 - \pi}$ with respect to φ

gives $$\frac{dc_2^{***}}{d\varphi} = \frac{\left[1 - \frac{\sigma}{\sigma-1}\frac{\varphi+1}{\varphi+\pi}\right]\left[\frac{(\varphi+\pi)^\sigma}{\pi}\right]^{\frac{1}{\sigma-1}} + 1 - \pi}{\left\{\left[\frac{(\varphi+\pi)^\sigma}{\pi}\right]^{\frac{1}{\sigma-1}} + 1 - \pi\right\}^2}$$, which is positive, as both the

denominator and numerator are positive. In particular, the numerator is positive as $\sigma < 1$.

$\frac{dc_2^{***}}{d\tau} < 0$ because $\frac{dc_2^{***}}{d\varphi} > 0$, which means that

$\frac{di_1^{***}}{d\tau} = \frac{-(1-\pi)(1-\pi\tau)}{R-1+\pi\tau}\frac{dc_2^{***}}{d\tau} > 0$, $\frac{di_2^{***}}{d\tau} = \frac{(1-\pi)}{R-1+\pi\tau}\frac{dc_2^{***}}{d\tau} < 0$. QED

Thus, in this section, we extend the result of log utility into CRRA utility and find that the necessity of short-term liquidity holding becomes stronger to prevent runs when the market crashes.

3. Conclusion

We derive the intermediary's optimal holdings of both liquid and illiquid investments as a function of the liquidation cost. We show that when the level of liquidation cost becomes higher as the loss of asset value accumulates, the associated level of liquid holding should increase rapidly in order to prevent a run. Further, we extend the finding in simple log utility into CRRA utility. Finally, for future research, it would be interesting to consider a randomized liquidation value, as the outlook on future financial market variables becomes more volatile with a certain extent of uncertainty rather than being deterministic. This will be a good complement to the current study.

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