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Network externalities and differentiation in an entry model

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Abstract

In a market characterized by network externalities, we consider a situation in which an established incumbent faces a new entrant: the differentiation degree chosen by the entrant increases with the network externalities, while the price set by the incumbent initially increases with the network externalities but eventually it decreases with the network externalities.

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Introduction

Price and product differentiation equilibria in entrance models have been investigated in many studies, including, for example, Pepall (1997), Anderson and Engers (2001), Lambertini (2002) and Davis *et al*. (2004). These papers are developed in markets characterized by no network externalities, where the consumer utility depends only on the products' attributes. However, network externalities are relevant in many markets (thinks for instance to computer, telephone and Internet services). In this case, the surplus each consumer receives from buying the good depends also on the user base.

In a market characterized by network externalities, we consider a situation in which an established incumbent faces a new entrant, and we posit the following questions: 1) how do network externalities affect the price decision of the incumbent? 2) how do network externalities affect the product differentiation degree chosen by the entrant? This note attempts to answer these questions.

We show that the equilibrium incumbent's price initially increases with the network externalities but eventually it decreases. This is due to the fact that the network externalities affect the pricing decision of the incumbent in two ways. First, higher network externalities increase the competitive advantage of the incumbent by increasing the importance of the user base owned by the incumbent at the entrance moment. Second, higher network externalities induce the incumbent to increase the current user base in order to further increase future demand and obtain higher profits. When network externalities are sufficiently low, the first effect dominates and the incumbent sets a higher price when externalities increase, while when externalities are sufficiently high the second effect dominates and the incumbent sets a lower price when externalities increase. Moreover, we show that network externalities positively affect the differentiation degree chosen by the entrant. In fact, the entrant chooses the differentiation from the incumbent by optimally balancing the benefit it obtains from higher demand with the costs originating from more aggressive pricing attitude of the incumbent. *Ceteris paribus*, higher network externalities lower the entrant's demand and the aggressive attitude of the incumbent, and the entrant replies by increasing equilibrium differentiation.

The rest of the note is organized as follows. Section 2 presents the model. Section 3 contains the analysis and the results. Section 4 concludes.

The model

Suppose that there are two firms: firm *I* (incumbent) and firm *E* (entrant). Suppose that there are two periods: period 1 (*current period*) and period 2 (*future*). At time 1 firm *E* enters and decides how much to be differentiated from the incumbent. Firm *I* observes the differentiation degree and sets the price. Following Armstrong and Vickers (1993), the entrant is *price taker*[1]. Following Casadesus-Masanell and Ghemawat (2006) and Jing (2007), network externalities imply that current demand of each firm is positively affected by its past demand. Therefore, demand at time 2 is positively affected by demand at time 1, and the demand at time 1 is positively affected by the past demand (i.e. the installed base). Let us denote by p_i^I the price set by firm *I* in period $i = 1, 2$, by Q_i^I the demand of firm *I* in period $i = 1, 2$ and by Q_i^E the demand of firm *E* in period $i = 1, 2$. The demand of firm *I* and firm *E* at time 1 are given respectively by:

$$Q_1^I = Q_1^I(e, h, p_1^I)$$

$$Q_1^E = Q_1^E(Q_1^I)$$

where $e > 0$ indicates the externality and $h \geq 0$ is the degree of homogeneity between the two firms. Assume: $\frac{\partial Q_1^I}{\partial p_1^I} < 0$, $\frac{\partial Q_1^I}{\partial e} > 0$, $\frac{\partial Q_1^I}{\partial h} > 0$ and $\frac{\partial Q_1^E}{\partial Q_1^I} < 0$. The first inequality is obvious and needs no explanation. The second inequality states that higher network externalities imply higher current demand for firm *I*. This is due to the fact that the incumbent (but not the entrant) has an installed base (since it is already in the market when firm *E* enters, some consumers have bought from it in the past, and this increases, *via* externalities, the demand in the current period). The third inequality follows from

[1] "*This assumption requires that the entrant be small relative to the incumbent* (Armstrong and Vickers, 1993, p. 340)".

the second inequality: if firms sell homogenous goods, all consumers naturally buy from firm *I* due to the existence of the installed base and positive network externalities. In other words, the more the firms are similar in the products they offer, the more the installed base determines the choice of the consumers in the current period. The fourth inequality follows Armstrong and Vickers (1993), and characterizes firm *E*' demand as a residual demand.

At time 1, firm *I* maximizes the following discounted profit function:

$$\Pi^I = p_1^I Q_1^I(e,h,p_1^I) + \delta p_2^I * (e,h) Q_2^I * (e,h,Q_1^I(e,h,p_1^I))$$

where the asterisk indicates equilibrium values and $\delta \in (0,1)$ is the discount factor. Note that the equilibrium demand in period 2 depends on the quantity sold at time 1 thanks to the existence of network externalities. Define: $\partial Q_2^I * / \partial Q_1^I \equiv f(e)$, where $f(e)$ is a strictly-increasing function of *e*. Finally, in order to keep tractability, the following assumptions on the parameters of the model are introduced:

a) $\delta < \min\left\{ \left| -2 \frac{\partial Q^I(p_1^I *)}{\partial p_1^I} \Bigg/ \left[f(e) p_2^I * \frac{\partial^2 Q_1^I(p_1^I *)}{\partial {p_1^I}^2} \right] - \frac{p_1^I *}{f(e) p_2^I *} \right| ; \left| - \left[Q_1^E * \frac{\partial p_1^I *}{\partial h} \Bigg/ Q_2^E * \frac{\partial p_2^I *}{\partial h} \right] \right| \right\}$

b) $\frac{\partial Q_1^I *}{\partial e} \geq - \frac{\partial Q_1^I *}{\partial p_1^I *} \frac{\partial p_1^I *}{\partial e}$

c) $\frac{\partial Q_1^I *}{\partial h} \geq - \frac{\partial Q_1^I *}{\partial p_1^I *} \frac{\partial p_1^I *}{\partial h}$

Analysis

Firm *I* maximizes the discounted profits at time 1 by choosing the optimal price $p_1^I *$. The first order condition is:

$$\frac{\partial \Pi^I}{\partial p_1^I} = Q_1^I(p_1^I *) + p_1^I * \frac{\partial Q_1^I(p_1^I *)}{\partial p_1^I} + f(e) \delta p_2^I * \frac{\partial Q_1^I(p_1^I *)}{\partial p_1^I} = 0 \qquad (1)$$

Assumption (a) guarantees that the second order condition is satisfied. Consider now the derivative of (1) with respect to *h*:

$$\frac{\partial^2 \Pi^I}{\partial p_1^I \partial h} = \frac{\partial Q_1^I(p_1^I*)}{\partial h} + p_1^I * \frac{\partial^2 Q_1^I(p_1^I*)}{\partial p_1^I \partial h} + f(e)\delta[\frac{\partial p_2^I *}{\partial h}\frac{\partial Q_1^I(p_1^I*)}{\partial p_1^I} + p_2^I * \frac{\partial^2 Q_1^I(p_1^I*)}{\partial p_1^I \partial h}]$$

Conjecture for the moment that $\partial p_2^I * / \partial h > 0$ (we check later that this conjecture is indeed correct) and assume that the second derivatives are negligible in dimension[2]. It follows that when externalities are sufficiently high, the term in the square bracket dominates and $\frac{\partial^2 \Pi^I}{\partial p_1^I \partial h}$ is negative, while the reverse is true when externalities are low. Therefore, we get $\frac{\partial p_1^I *}{\partial h} > 0$ when $e < e*$ and $\frac{\partial p_1^I *}{\partial h} < 0$ when $e > e*$, where *e** is defined by $\frac{\partial^2 \Pi^I}{\partial p_1^I \partial h}(e*) = 0$. Hence, we can state the following result:

Result 1: The incumbent's first-period equilibrium price increases with the homogeneity when network externalities are low, while it decreases when network externalities are high.

The intuition of result 1 is the following. When homogeneity increases there are two effects at work: a *direct effect* and an *indirect effect*. Higher *h* implies that firm *I* can set a higher price today and in the future (*direct effect*) because it has a higher competitive advantage with respect to the entrant. However, a higher price in the current period reduces the current demand, and, through the externalities, it reduces also the future demand (*indirect effect*). When externalities are particularly high, the *indirect effect* dominates, and the impact of *h* on the equilibrium price at time 1 is negative. Turn now to the conjecture $\partial p_2^I * / \partial h > 0$. Since in period 2 there is no future left, firm *I* has no incentive to decrease its price when *h* increases in order to exploit higher future demand. That is, no *indirect effect* arises. It follows that, in equilibrium, the price in

[2] Note that this is always true with linear demand functions.

period 2 must increase with homogeneity. Therefore, conjecture $\partial p_2^I * / \partial h > 0$ is correct.[3]

Moreover, note that the impact of *e* on the equilibrium price at time 1 is similar to the impact of *h*, since both *e* and *h* affect positively the current and the future demand. By taking the derivative of (1) with respect to *e*, we get:

$$\frac{\partial^2 \Pi^I}{\partial p_1^I \partial e} = \frac{\partial Q_1^I (p_1^I *)}{\partial e} + p_1^I * \frac{\partial^2 Q_1^I (p_1^I *)}{\partial p_1^I \partial e} + f(e)\delta[\frac{\partial p_2^I *}{\partial e}\frac{\partial Q_1^I (p_1^I *)}{\partial p_1^I} + p_2^I * \frac{\partial^2 Q_1^I (p_1^I *)}{\partial p_1^I \partial e}] + \delta p_2^I * \frac{\partial f}{\partial e}\frac{\partial Q_1^I (p_1^I *)}{\partial p_1^I}$$

Take again the derivative with respect to *e*. Disregarding the second and third derivatives and conjecturing that $\partial p_2^I * / \partial e > 0$, we get:

$$\frac{\partial^3 \Pi^I}{\partial p_1^I \partial e \partial e} = 2\delta \frac{\partial f}{\partial e}\frac{\partial p_2^I *}{\partial e}\frac{\partial Q_1^I (p_1^I *)}{\partial p_1^I} < 0$$

Therefore, when network externalities are sufficiently high, $\frac{\partial^2 \Pi^I}{\partial p_1^I \partial e}$ is negative, while the reverse is true when network externalities are low. Define $\hat{e}$ by: $\frac{\partial^2 \Pi^I}{\partial p^I \partial e}(\hat{e}) = 0$. We get the following result:

Result 2: The incumbent's first-period equilibrium price initially increases with the network externalities, but eventually it decreases with the network externalities.

The intuition for Result 2 is analogous to the intuition for Result 1, and therefore it is not repeated. Moreover, conjecture $\partial p_2^I * / \partial e > 0$ is correct, because at time 2 only the

[3] This analysis can be easily extended to a *T*-period model. Suppose that there are *T* periods, indexed with $t = 1, ..., T$. For low values of *t*, the *direct effect* is less important than the *indirect effect*, because there is scarce installed base, while there is a great incentive to exploit future demand. Therefore, we expect that the critical value of *e* is lower when *t* is low, that is: $\partial e * / \partial t > 0$. Moreover, $e*(t = 1) = 0$, because when the market starts the incumbent has no installed base: very low externalities are sufficient for higher homogeneity to induce the incumbent to reduce current price in order to increase future demand and exploit future profits. At the opposite, $e*(t = T) = \infty$, because when there is no future to exploit, even high externalities have only the effect to increase the competitive advantage of the incumbent, inducing it to set an higher price when *h* increases.

direct effect works (see discussion above). Note that $\hat{e}$ does not necessarily coincides with e*. Assume for the moment that $\hat{e} = e^*$: we will turn later to the case in which $\hat{e} \neq e^*$.

The equilibrium demand of firm I at time 1 is:

$$Q_1^I * = Q_1^I * (e, h, p_1^I * (e, h))$$

Assumptions (b) - (c) allow us to write:

$$Q_1^I * = Q_1^I * (e, h)$$

At time 1, firm E faces the following discounted profit function:

$$\Pi^E = Q_1^E (Q_1^I *) p_1^I * + \delta Q_2^E * (Q_2^I * (Q_1^I *)) p_2^I *,$$

Firm E maximizes Π^E with respect to h. The first order condition is:

$$\frac{\partial \Pi^E}{\partial h} = [\frac{\partial Q_1^E}{\partial Q_1^I *} \cdot \frac{\partial Q_1^I *}{\partial h} \cdot p_1^I *] + [\frac{\partial p_1^I *}{\partial h} \cdot Q_1^E + \delta \frac{\partial p_2^I}{\partial h} \cdot Q_2^E *] + \delta [\frac{\partial Q_2^E *}{\partial Q_2^I *} \cdot \frac{\partial Q_2^I *}{\partial Q_1^I *} \cdot \frac{\partial Q_1^I *}{\partial h} \cdot p_2^I *] = 0 \quad (2)$$

Consider first $e > e^*$. Due to assumption (a), equation (3) is always negative[4]. Therefore, firm E chooses minimal homogeneity from the incumbent firm.

Consider now $e < e^*$. Assume that an h* such that: $\frac{\partial \Pi^E}{\partial h}(h^*) = 0$ exists. The second order condition is:

[4] If assumption (a) – which amounts to require that the discount factor is not too high – does not hold, equation (2) may be positive or negative. More importantly, the satisfaction of the second order condition would require further assumptions on the parameters of the model, and no clear results could be obtained.

$$\frac{\partial^2 \Pi^E}{\partial h^2} = [\frac{\partial Q_1^E}{\partial Q_1^I *} \cdot \frac{\partial Q_1^I *}{\partial h} \cdot \frac{\partial p_1^I}{\partial h} *] + [\frac{\partial p_1^I *}{\partial h} \cdot \frac{\partial Q_1^E}{\partial h} + \delta \frac{\partial p_2^I}{\partial h} \cdot \frac{\partial Q_2^E *}{\partial h}] + \delta [\frac{\partial Q_2^E *}{\partial Q_2^I *} \cdot \frac{\partial Q_2^I *}{\partial Q_1^I *} \cdot \frac{\partial Q_1^I *}{\partial h} \cdot \frac{\partial p_2^I}{\partial h}] \quad (3)$$

Since (3) is always negative *h** is a maximum. Let us consider the derivative of (2) with respect to *e* (disregarding the second derivatives). We get:

$$\frac{\partial^2 \Pi^E}{\partial h \partial e} = [\frac{\partial Q_1^E}{\partial Q_1^I *} \cdot \frac{\partial Q_1^I *}{\partial h} \cdot \frac{\partial p_1^I *}{\partial e} + \frac{\partial p_1^I *}{\partial h} \cdot \frac{\partial Q_1^E}{\partial e}] + \delta [\frac{\partial Q_2^E *}{\partial Q_2^I *} \cdot \frac{\partial Q_2^I *}{\partial Q_1^I *} \cdot \frac{\partial Q_1^I *}{\partial h} \cdot \frac{\partial p_2^I}{\partial e} + \frac{\partial p_2^I}{\partial h} \cdot \frac{\partial Q_2^E *}{\partial e}] \quad (4)$$

Note that all terms are negative. Therefore, $\partial h^* / \partial e < 0$: that is, higher externalities are associated with lower equilibrium homogeneity.

We can write the following result:

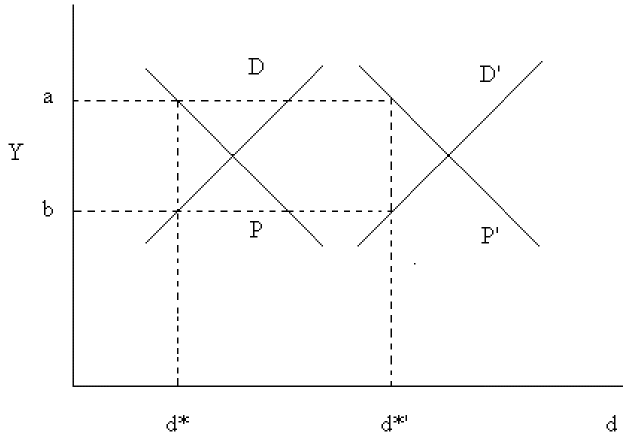
Result 3. When $e > e^*$, the entrant chooses to maximally differentiate. When $e < e^*$, the equilibrium differentiation degree increases with the network externalities.

The intuition of Result 3 can be obtained by looking at figure 1. Define with *d* the differentiation degree. When $e < e^*$, higher differentiation increases the demand of firm *E* but lowers the price set by firm *I*. Define with *D* the "demand" curve, i.e. the curve describing the effect of *d* on firm *E*' demand, and with *P* the "price" curve, i.e. the curve describing the effect of *d* on firm *I*' price. Suppose that optimal balancing between demand and price implies that the demand is equal to *a* and the price is equal to *b*, i.e. the distance between the *D*-curve and the *P*-curve is $Y = a - b$. Optimal differentiation degree is therefore given by *d**. Suppose now that network externalities increase. *Ceteris paribus*, the higher is *e* the lower is the demand of firm *E* and the higher is the price set by firm *I*. The *D*-curve shifts downward and the *P*-curve shifts upward. In order to maintain the same *Y*-distance between *D* and *P*, the new equilibrium differentiation degree, *d**', must be located at the right of *d**.

Instead, when $e > e^*$, firm *E* sets the highest possible *d*, since *d* increases both the demand of firm *E* and the price set by firm *I*.

Consider now the case $\hat{e} \neq e^*$. Equation (4) has an ambiguous sign for intermediate values of *e*: i.e., when *e* is such that $\max[\hat{e}, e^*] > e > \min[\hat{e}, e^*]$. Instead, when $e < \min[\hat{e}, e^*]$ equation (4) is unambiguously negative. Therefore, *for sufficiently low externalities*, equilibrium differentiation increases with externalities. Finally, when $e > \max[\hat{e}, e^*]$ firm *E* maximally differentiates.

Figure 1



Conclusion

Using an entrance model in which a *price-taker* entrant chooses the differentiation degree from the incumbent, we have shown that for low levels of network externalities equilibrium differentiation increases with externalities, while when network externalities are sufficiently high the entrant maximally differentiates from the incumbent. The equilibrium price set by the incumbent initially increases with externalities but eventually it decreases with externalities.

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