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Pitfall of simple permanent income hypothesis model

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Abstract

Permanent Income Hypothesis (hereafter, PIH) is one of the central concepts in macroeconomics. Single equation version of PIH is often appeared in textbooks and academic papers. But, even in single equation version of PIH Romer(2006) suggested, to get economic insights from estimation, we need to consider the additional income determination equation and then we can't ignore “Simultaneous equations bias.” In this note, we examine this “Simultaneous equations bias” effect theoretically and empirically. Our results suggest that ignoring this bias will lead to the wrong estimates and conclusion. More attention should be given on simultaneous equations approach.

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1. Introduction

As Friedman(1957) suggested, Permanent Income Hypothesis (hereafter, PIH) is one of the central concepts in macroeconomics. And, some textbooks introduce and explain the simple, single equation version of PIH, and many papers with this model are published in academic journals (see Romer[2006], DeJuan and Seater[2006], for example). But, are the results brought by single equation PIH consistent with formal econometric methodology? To be more precise, does this single equation version of PIH successfully bring the consistent estimator when we consider the simultaneous relation between necessary variables such as consumption and income in PIH?

In this note, we examine the effect of "Simultaneous equations bias" on single equation version of PIH theoretically and empirically. Simultaneous equations bias is the typically classical concept in econometrics, and is known to violate the classical assumption of OLS estimators (the independence of regressors from the disturbance term) as many econometrics textbook such as Johnston and DiNardo(1997) shows. In such a case, the application of OLS will give biased and inconsistent estimates.

This note is organized as follows. In section 2, after we introduce the single equation version of PIH, we examine its relation to "Simultaneous equations bias" and show the classical solution, Instrumental Variable method. In section 3, we show the empirical evidence with OLS and IV, and discuss how simultaneous equations bias is important practically. And in section 4, we summarize our conclusion.

2. PIH, Simultaneous Equations Bias and Its Classical Solution

(1) Textbook Model –Single Equation Version of PIH

This section is based on Romer(2006), and DeJuan and Seater(2006). Suppose the next simple PIH model. Consumption is equal to permanent income: $C = Y^P$ and income equation has 2 parts: permanent income part and transitory income part, $Y = Y^P + Y^T$. Transitory income reflects differences of current income from permanent income. In consumption equation, consumption is affected by current income: $C_t = \alpha + \beta Y_t + u_t$, and α, β are parameters and u_t follows Gaussian white noise with mean 0 and variance σ_u^2. Note that the assumed statistics of transitory income Y^T is a mean 0, uncorrelated with permanent income Y^P and $E(u_t | Y_t^T) = 0$ holds for any t since Y^T is assumed to be exogenous. So, income is determined by $Y_t = C_t + Y_t^T$ (Clearly, this model is simultaneous equations model, as we discuss soon after this paragraph).

Then, from the well-known result of OLS regression, in the special case of univariate regression such as our consumption function, as Romer(2006) shows, we have $\hat{\alpha}_{OLSQ} = (1-\hat{\beta}_{OLSQ})\overline{Y}^P$, $\hat{\beta}_{OLSQ} = \dfrac{Var(Y^P)}{Var(Y^P)+Var(Y^T)}$. $\hat{\beta}_{OLSQ}$ is 1 if there is no transitory income and Var(Y^T) is zero. This is the single equation version of Permanent Income Hypothesis.

But, as long as we use the above simultaneous equation model to derive the result of single equation version of PIH, this simple result is wrong because of the existence of "Simultaneous equations bias." Next, we show the evidence of "Simultaneous equations bias" theoretically.

(2) Effect of Simultaneous Equations Bias and Its Classical Solution

We follow Ban et al.(2006) to derive the next result.

At first, suppose that $\underset{T\to\infty}{\text{plimit}}\dfrac{1}{T}\sum_{t=1}^{T}(Y_t^T-\overline{Y}^T)^2 = Var(Y_T)$. Then, the setup of model in this section (1) leads to $\underset{T\to\infty}{\text{plimit}}\,\hat{\beta}_{OLSQ} = \beta + (1-\beta)\dfrac{Var(u)}{Var(Y_T)+Var(u)}$.

This result shows the existence of "Simultaneous equations bias" in the above model. In such a case, it is widely known that the instrumental variable is effective. In our model, instrumental variable Y^T resolves this bias and we get consistent $\hat{\beta}$. In short, the interpretation of coefficient β based on this section (1) does not hold. In the following section, we investigate how large this simultaneous equation bias is and how it will lead the wrong conclusion based on real data.

3. Empirical Evidence

(1) Data Description

Data are "Household consumption expenditure (including Non-profit institutions serving households)" and "Gross Domestic Product (GDP)" of Japan, UK and US at constant 1990 price in national currency from CY1970 to CY2008, downloaded from National Accounts Main Aggregates Database in the website of United Nations Statistics Division.

(2) Estimation Results

Since all of these data show unit root by augmented Dickey-Fuller test, for OLS and IV estimation, we take the first order difference of independent, dependent and instrumental variables. Before that, we need to separate the permanent income and transitory income from income data. To do this, we use Hodrick-Prescott filter with the smoothing parameter lambda 100. Hodrick-Prescott filter distinguishes smoothed and cycle series, and we use the first order difference of cycle series as instrumental variable. Note that cycle series of Hodrick-Prescott filter has mean zero and the problem is whether it is exogenous from error term in consumption equation or not. The estimation result is as follows.

Estimated Beta with OLS and IV

	OLS	IV	Wald Test(Null Hyp.:IV estimates is not different from OLS estimates)
Japan	0.336	0.292	Reject
	0.046	0.051	0.000
UK	0.618	0.565	Accept
	0.060	0.071	0.066
US	0.575	0.497	Reject
	0.060	0.069	0.006

Note: 1. Estimates of OLS and IV are in 1st row, and their standard errors are in 2nd row of each country.
2. 2nd row of Wald Test column in each country shows P-value. Accept/Reject decision is judged at 5% significance level.

The Wald test result in the above table, in which null hypothesis is that β_{INST} is equal to β_{OLSQ}, shows that we get wrong results in Japan and US cases if we use OLS. See the next table. This table shows the weak exogeneity test (Wu-Hausman Test) result of our cycle series.

Weak Exogeneity Test

	Wu-Hausman Test Statistics	5% Crit. Value of $\chi(1)$
Japan	4.306	3.841
UK	1.753	3.841
US	5.020	3.841

The null hypothesis of weak exogeneity test is plimit $\frac{1}{n}Yu = 0$. Then, this "Weak Exogeneity Test" table shows, at least in Japan and US, the null hypothesis is rejected. In other words, the assumption $E(u_t | Y_t^T) = 0$ of our IV estimates of Japan and US is successfully satisfied. And, our IV estimates of Japan and US cases is valid. On the other hand, regarding for UK case, we can accept OLS result. Then, how should we interpret these results?

First, substituting $Y_t = C_t + Y_t^T$ for Y_t in consumption equation, we have $C_t = \alpha + \beta_{INST}(C_t + Y_t^T) + u_t$. And solving for C_t, then we have the following equality: $C_t = \frac{\alpha}{1-\beta_{INST}} + \frac{\beta_{INST}}{1-\beta_{INST}} Y_t^T + \frac{1}{1-\beta_{INST}} u_t$. Therefore, in our model, the esitmated β_{INST} is used to calculate the sensitivity of consumption to transitory income, $\frac{\beta_{INST}}{1-\beta_{INST}}$. This sensitivity is calculated in the following table, "Estimated Sensitivity with OLS and IV."

Estimated Sensitivity with OLS and IV

	OLS	IV
Japan	0.336	0.412
UK	0.618	-
US	0.575	0.989

The result in "Estimated Sensitivity with OLS and IV" means that the sensitivity of consumption to transitory income by IV estimate is higher than that by OLS, in Japan and US. In short, the true sensitivity is underestimated by OLS. And, Japan is relatively less sensitive to transitory income, while US is relatively more sensitive to transitory income. If we only use the OLS results, we have the wrong conclusion that UK is most sensitive to transitory income, but its sensitivity is slightly higher than US. However, if we use the IV estimator, we reach the true result that US is most sensitive, and its sensitivity is apparently higher than UK. Actually, our IV estimates result is consistent with Campbell and Mankiw(1991). Also with IV, they estimate the share of "Rule of thumb" consumers in some countries who do not follow Permanent Income Hypothesis and decides their consumption based on their current income. They report that this type's share is 0.351, 0.203 and 0.035 in US, UK and Japan. Implication of their result is that consumption depends on transitory income in some degree and its magnitude is ranked with the following order: US, UK and Japan. This conclusion is consistent with our result considering "Simultaneous equations bias."

Clearly, these results show the existence of "Simultaneous equations bias" and it should not be ignored. One important problem of "Simultaneous equations bias" is that it will affect estimated β in both positive and negative directions and we cannot predict this direction in advance (see section 2[2]). For example, DeJuan and Seater(2006) tries to measure different β s by IV assuming measurement errors in variables which is a different setting from us[1]. Without such careful considerations, we may get wrong estimates because of simultaneous equations bias as we show in this note. Therefore, we cannot ignore this bias theoretically and empirically.

Note that, in our result, however, IV method is not important in UK case since our estimated transitory income, cycle series of Hodrick-Prescott filter, does not show weak exogeneity in UK case. This fact may suggest that we should construct "transitory income" more rigorously by superior statistical method to Hodrick-Prescott filter or larger scale simultaneous equations model to include the relationship between errors in consumption function and transitory income in UK. However, in this note, we stop exploring better methods since our result is sufficiently robust and exploring better methods surely leads to other different topics and obscure our aim.

4. Conclusion

Our results suggest that "Simultaneous equations bias" is important both theoretically and empirically. We should take much care of treating single equation version of Permanent Income Hypothesis. Ignoring this bias will lead to the wrong conclusion, particularly when measuring β is the main purpose as we see in this note.

Furthermore, we should not limit our attention only to Permanent Income Hypothesis. Our conclusion about single equation version of PIH is only one suggestion, and there may be many other single equation examples. More attention should be given on simultaneous equations approach.

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[1] In our case, to be more precise, we consider "TSLS" and simultaneous equations model. But, since theoretically IV includes TSLS, we mainly use the word IV in this note.