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Fees versus Royalties under Partial Cross Ownership

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Abstract

This paper studies an independent patent-holder's licensing of a process innovation to a Cournot duopoly characterized by partial cross ownership. We find that royalty licensing is preferred by the patentee when the degree of cross ownership is high, whereas fixed fee licensing is preferred when the degree of cross ownership is low. When the degree of cross ownership is in the intermediate range, the superiority of fixed fee and royalty licensing depends on the magnitude of the innovations.

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1. Introduction

The seminal works of Kamien and Tauman (1986) and Kamien *et al*. (1992) on patent licensing show that an independent patent holder of a cost-reducing innovation prefers fixed fee licensing to royalty licensing. Nevertheless, empirical studies such as Rostoker (1984), Macho-Stadler *et al*. (1996), and Jensen and Thursby (2001) consistently document the importance of royalties in licensing agreements. Subsequent theoretical models then try to justify the use of royalties from various directions, such as asymmetric information (Beggs 1992), product differentiation (Muto 1993, Wang and Yang 1999, and Wang 2002), insider innovator (Wang 1998), strategic delegation (Saracho 2002), and cost asymmetry (Wang and Yang 2004).

The purpose of this paper is to provide a new explanation for the prevalence of royalty licensing by considering partial cross ownership among firms in the same industry. In the real world it is commonly observed that firms (or their majority shareholders) own rival firms' stock as passive investments ("silent financial interests" according to Bresnahan and Salop 1986), which give them a share in the rival firms' profits but not in the rival firms' decision making (see, for example, Alley 1997, Dietzenbacher *et al*. 2000, Gilo 2000, and Gilo *et al*. 2006). In light of this, we consider an independent patent holder's licensing of a cost-reducing innovation to a Cournot duopoly characterized by partial cross ownership. We analyze how the degree of cross ownership affects the patent holder's licensing behavior and her preference between fixed fee and royalty licensing. We find that royalty licensing is preferred by the patentee when the degree of cross ownership is sufficiently high, whereas fixed fee licensing is preferred when the degree of cross ownership is low. When the degree of cross ownership is in the intermediate range, the superiority of fixed fee and royalty licensing depends on the magnitude of the innovations.

The remainder of the paper is organized as follows. We introduce our model in Section 2. Fixed fee and royalty licensing are analyzed in Sections 3 and 4, respectively, and are compared in Section 5. Section 6 concludes. All proofs are provided in the Appendix.

2. The Model

Consider an industry with two firms, 1 and 2, producing a homogeneous product. The inverse market demand is given by $p = 1 - q_1 - q_2$, where p is the market price and q_i represents firm i's output level, $i = 1, 2$. Both firms have the same constant marginal cost, c, where $0 < c < 1$. Inspired by Macho-Stadler and Verdier (1991), we consider the simplest possible cross ownership structure between the duopoly: firm 1 is completely owned by shareholder 1, while firm 2 is jointly owned by shareholders 1 and 2. Shareholder 2 is assumed to possess a $\beta > 1/2$ fraction of firm 2's shares (and retain the full decision power of firm 2), while shareholder 1 possesses the remaining $1 - \beta$ fraction. Following the literature on cross-shareholdings with silent interests (e.g., Gilo *et al*. 2006, Macho-Stadler and Verdier

1991, and Reynolds and Snapp 1986), the objective functions of firms 1 and 2, Π_1 and Π_2, are:

$$\Pi_1 \equiv \pi_1 + (1-\beta)\pi_2 \quad \text{and} \quad \Pi_2 \equiv \beta\pi_2, \tag{1}$$

where π_i is the profit derived from firm i's production of q_i.

There is an independent patent holder (e.g., a research lab) owning a cost-reducing technology which, once licensed, can reduce the licensee's marginal cost from c to $c - \Delta c$, where $0 < \Delta c < c$. The interactions between the patentee and the duopoly are described by the following three-stage game. In the first stage, the patentee either announces a fixed fee, f, or a per-unit royalty, r, to maximize her licensing revenue. In the second stage, the two firms simultaneously and independently decide whether or not to purchase the license. In the third stage, both firms simultaneously choose outputs to maximize their respective payoffs, with the production costs being determined by the licensing outcome of the previous stages. The appropriate equilibrium concept for this game is that of the subgame perfect equilibrium.

In the spirit of backward induction, we first characterize the Cournot-Nash equilibrium in the third stage. The objective functions of firms 1 and 2, respectively, are:

$$\max_{q_1} \Pi_1(q_1, q_2) = (1 - q_1 - q_2 - c_1)q_1 + (1-\beta)(1 - q_1 - q_2 - c_2)q_2 \quad \text{and} \tag{2}$$

$$\max_{q_2} \Pi_2(q_1, q_2) = \beta(1 - q_1 - q_2 - c_2)q_2, \tag{3}$$

where $c_1, c_2 \in \{c, c - \Delta c\}$ and $\{c, c - \Delta c + r\}$ under fixed fee licensing and royalty licensing, respectively. The reaction functions are:

$$\hat{q}_1(q_2) = \begin{cases} \dfrac{1 - c_1 - (2-\beta)q_2}{2}, & \text{if } q_2 \le \dfrac{1-c_1}{2-\beta} \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \hat{q}_2(q_1) = \begin{cases} \dfrac{1 - c_2 - q_1}{2}, & \text{if } q_1 \le 1 - c_2 \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

For simplicity, we assume that both firms produce positive outputs under all possible licensing outcomes. That is, we only consider non-drastic innovations as in Wang and Yang (2004). Simultaneously solving the first lines of the reaction functions in (4) yields the following equilibrium outputs:

$$q_1(c_1, c_2, \beta) = \frac{\beta - 2c_1 + (2-\beta)c_2}{2+\beta} \quad \text{and} \quad q_2(c_1, c_2, \beta) = \frac{1 + c_1 - 2c_2}{2+\beta}. \tag{5}$$

It is readily verifiable that if $c_1 = c_2$, then $q_1 > 0$ and $q_2 > 0$. However, if $c_1 = c - \Delta c$ and $c_2 = c$, then $q_1 > 0$ but $q_2 > 0$ iff $\Delta c < 1 - c$; and if $c_1 = c$ and $c_2 = c - \Delta c$, then $q_2 > 0$ but $q_1 > 0$ iff $\Delta c < \beta(1 - c)/(2 - \beta)$. Because $\beta(1 - c)/(2 - \beta) < 1 - c$, we then assume that

$$\Delta c < \frac{\beta(1-c)}{2-\beta}. \tag{6}$$

That is, the assumption of non-drastic innovations entails that firm 1 produces a positive output even when firm 2 is the sole licensee of the innovation.[1]

To see the effect of partial cross ownership on output choice, envision the reaction curves in (4) in a two-dimensional figure with q_1 and q_2 drawn on the horizontal and vertical axes, respectively. Compared to the conventional case without cross ownership (i.e., $\beta = 1$), firm 1's reaction curve here with $\beta < 1$ *pivots inwards* around the horizontal axis, while firm 2's reaction curve remains unchanged. Accordingly, firm 1 produces *less* and firm 2 produces *more* under cross ownership than in the conventional case with $\beta = 1$. Note that firm 1's shareholder owns a fraction of firm 2's profits and thus has incentives to compete less vigorously.[2] From (5), we have the total industry output:

$$Q(c_1, c_2, \beta) = \frac{(1+\beta) - c_1 - \beta c_2}{2+\beta}. \tag{7}$$

It can be readily verified that the total industry output, $Q(c_1, c_2, \beta)$, is *smaller* and the resulting market price, $p(c_1, c_2, \beta) = (1 + c_1 + \beta c_2)/(2 + \beta)$, is *higher* under cross ownership than under $\beta = 1$. Finally, the associated profits from production are

$$\pi_1(c_1, c_2, \beta) = \frac{[1-(1+\beta)c_1 + \beta c_2][\beta - 2c_1 + (2-\beta)c_2]}{(2+\beta)^2} \quad \text{and} \quad \pi_2(c_1, c_2, \beta) = \frac{(1-2c_2+c_1)^2}{(2+\beta)^2}, \tag{8}$$

and the associated equilibrium payoffs of shareholders 1 and 2 are $\Pi_1(c_1, c_2, \beta) = \pi_1(c_1, c_2, \beta) + (1 - \beta)\pi_2(c_1, c_2, \beta)$ and $\Pi_2(c_1, c_2, \beta) = \beta\pi_2(c_1, c_2, \beta)$, respectively.

3. Fixed Fee Licensing

We first find the firms' equilibrium outputs and profits under all possible licensing outcomes when fixed fee licensing is adopted. If firm 1 is the sole licensee, then $c_1 = c - \Delta c$ and $c_2 = c$. Substituting these into (5) and (8) yields:

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[1] The assumption in (6) precludes the potential corner solution problem, which may arise in a Cournot model with asymmetric costs. We are grateful to the referee for drawing our attention to this.

[2] If $\beta = 1$, then (5) yields the conventional solution $q_1(c_1, c_2) = (1 - 2c_1 + c_2)/3$ and $q_2(c_1, c_2) = (1 - 2c_2 + c_1)/3$.

$$q_1(c-\Delta c, c, \beta) = \frac{\beta(1-c)+2\Delta c}{2+\beta}, \quad q_2(c-\Delta c, c, \beta) = \frac{1-c-\Delta c}{2+\beta}, \tag{9}$$

$$\pi_1(c-\Delta c, c, \beta) = \frac{[(1-c)+(1+\beta)\Delta c][\beta(1-c)+2\Delta c]}{(2+\beta)^2}, \quad \pi_2(c-\Delta c, c, \beta) = \frac{(1-c-\Delta c)^2}{(2+\beta)^2}. \tag{10}$$

If firm 2 is the sole licensee, then $c_1 = c$ and $c_2 = c - \Delta c$, such that from (5) and (8) we have:

$$q_1(c, c-\Delta c, \beta) = \frac{\beta(1-c)-(2-\beta)\Delta c}{2+\beta}, \quad q_2(c, c-\Delta c, \beta) = \frac{(1-c+2\Delta c)}{2+\beta}, \tag{11}$$

$$\pi_1(c, c-\Delta c, \beta) = \frac{[(1-c)-\beta\Delta c][\beta(1-c)-(2-\beta)\Delta c]}{(2+\beta)^2}, \quad \pi_2(c, c-\Delta c, \beta) = \frac{(1-c+2\Delta c)^2}{(2+\beta)^2}. \tag{12}$$

When both firms license, substituting $c_1 = c_2 = c - \Delta c$ into (5) and (8) yields:

$$q_1(c-\Delta c, c-\Delta c, \beta) = \frac{\beta(1-c+\Delta c)}{2+\beta}, \quad q_2(c-\Delta c, c-\Delta c, \beta) \equiv \frac{1-c+\Delta c}{2+\beta}, \tag{13}$$

$$\pi_1(c-\Delta c, c-\Delta c, \beta) = \frac{\beta(1-c+\Delta c)^2}{(2+\beta)^2}, \quad \pi_2(c-\Delta c, c-\Delta c, \beta) = \frac{(1-c+\Delta c)^2}{(2+\beta)^2}. \tag{14}$$

Finally, if neither firm licenses, then with $c_1 = c_2 = c$, we have:

$$q_1(c, c, \beta) = \frac{\beta(1-c)}{2+\beta}, \quad q_2(c, c, \beta) = \frac{1-c}{2+\beta}, \tag{15}$$

$$\pi_1(c, c, \beta) = \frac{\beta(1-c)^2}{(2+\beta)^2}, \quad \pi_2(c, c, \beta) = \frac{(1-c)^2}{(2+\beta)^2}. \tag{16}$$

The maximum fee each firm is willing to pay is the amount that makes it indifferent between licensing and not licensing. If firm 1 is the sole licensee, it is willing to pay:

$$L_1 \equiv \Pi_1(c-\Delta c, c, \beta) - \Pi_1(c, c, \beta) = \frac{(3+\beta)\Delta c}{(2+\beta)^2}[\beta(1-c)+\Delta c]. \tag{17}$$

Firm 1's maximum willingness to pay when firm 2 also licenses is:

$$LL_1 \equiv \Pi_1(c-\Delta c, c-\Delta c, \beta) - \Pi_1(c, c-\Delta c, \beta) = \frac{(3+\beta)\Delta c}{(2+\beta)^2}[\beta(1-c)-(1-\beta)\Delta c]\,. \tag{18}$$

Similarly, firm 2's maximum willingness to pay when it is the sole licensee is:

$$L_2 \equiv \Pi_2(c, c-\Delta c, \beta) - \Pi_2(c, c, \beta) = \frac{4\beta\,\Delta c}{(2+\beta)^2}[(1-c)+\Delta c]\,, \tag{19}$$

while firm 2 is willing to pay LL_2 when firm 1 also licenses, where LL_2 is given by:

$$LL_2 \equiv \Pi_2(c-\Delta c, c-\Delta c, \beta) - \Pi_2(c-\Delta c, c, \beta) = \frac{4\beta\,\Delta c(1-c)}{(2+\beta)^2}\,. \tag{20}$$

It is apparent that $L_1 > LL_1$ and $L_2 > LL_2$, implying that each firm is willing to pay more when it is the sole licensee than when the rival also licenses. Next, it can be shown that $LL_1 < LL_2$, such that the patent holder can only charge a fee equal to LL_1 if she licenses to both firms. As for the comparison between L_1 and L_2, it is straightforward to show that L_1 > (=, <) L_2 iff Δc > (=, <) $\beta(1-c)/3$. By comparing the patent holder's potential licensing revenues, L_1, L_2, and $2LL_1$, we have the following proposition:

Proposition 1: Under fixed-fee licensing, the patent holder will license to both firms if $\Delta c \le \dfrac{\beta(1-c)}{3-2\beta}$ and to firm 1 only if $\Delta c > \dfrac{\beta(1-c)}{3-2\beta}$, with licensing revenues, F^*, equal to $2LL_1$ and L_1, respectively.

It can be easily verified that the critical value of Δc in Proposition 1, $\beta(1-c)/(3-2\beta)$, is smaller than the upper bound of Δc defined in (6). Proposition 1 shows that for all degrees of cross ownership $1/2 < \beta < 1$, if the patent holder licenses her innovations via fixed fees, she will license larger innovations only to firm 1 and smaller innovations to both firms.

4. Royalty Licensing

We now consider licensing by means of a royalty. Under this licensing arrangement, a licensee pays r dollars to the patent holder for each unit of output it produces with the new technology. Thus, a licensee's unit cost of production is $c - \Delta c + r$. It is obvious that no firms will purchase the license if $r > \Delta c$, while both firms will purchase if $r \le \Delta c$. Using (7), we have the patent holder's licensing revenue, R, for $r \le \Delta c$ as follows:

$$R = rQ(c-\Delta c + r, c-\Delta c + r, \beta) = \frac{r(1+\beta)(1-c+\Delta c - r)}{2+\beta}. \quad (21)$$

Maximizing the expression in (21) subject to the constraint that $r \leq \Delta c$, we have:

Proposition 2: Under royalty licensing, the patent holder's optimal royalty rate is $r^* = \Delta c$, which yields a licensing revenue of $R^* = \frac{(1+\beta)(1-c)\Delta c}{2+\beta}$.

5. Fees versus Royalties

By comparing the patent holder's licensing revenues, F^* and R^*, we can identify the superiority of fixed fee and royalty licensing as follows:

Proposition 3:

(a) For $\Delta c \leq \frac{\beta(1-c)}{3-2\beta}$, we have:

(a-i) If $0.5 < \beta \leq 0.562$, the patent holder prefers royalty licensing.

(a-ii) If $0.562 < \beta < 0.772$, the patent holder prefers royalty and fixed fee licensing for $\Delta c > \Delta\hat{c}$ and $\Delta c < \Delta\hat{c}$, respectively, where $\Delta\hat{c} \equiv \frac{(\beta^2+3\beta-2)(1-c)}{2(1-\beta)(3+\beta)}$ and $0 < \Delta\hat{c} < \frac{\beta(1-c)}{3-2\beta}$.

(a-iii) If $0.772 \leq \beta < 1$, the patent holder prefers fixed fee licensing.

(b) For $\Delta c > \frac{\beta(1-c)}{3-2\beta}$, we have:

(b-i) If $0.5 < \beta \leq 0.702$, the patent holder prefers royalty licensing.

(b-ii) If $0.702 < \beta < 0.772$, the patent holder prefers royalty and fixed fee licensing for $\Delta c < \Delta\tilde{c}$ and $\Delta c > \Delta\tilde{c}$, respectively, where $\Delta\tilde{c} \equiv \frac{2(1-c)}{3+\beta}$ and $\frac{\beta(1-c)}{3-2\beta} < \Delta\tilde{c} < \frac{\beta(1-c)}{2-\beta}$.

(b-iii) If $0.772 \leq \beta < 1$, the patent holder prefers fixed fee licensing.

Proposition 3 shows that, regardless of the innovation sizes, royalty licensing is preferred by the patentee when the degree of cross ownership is high (i.e., a small β), whereas fixed fee licensing is preferred when the degree of cross ownership is low (i.e., a large β). For intermediate degrees of cross ownership, royalty licensing is preferred for intermediate innovations. Note that unlike Wang and Yang (2004) in which royalty licensing is superior to fixed fee licensing only for smaller values of non-drastic innovations, in our model with cross ownership, we can find conditions under which royalty licensing is preferred for *all* values of non-drastic innovations. We discuss our results as follows.

When the value of β is large enough (e.g., close to 1) as in Parts (a-iii) and (b-iii), our model resembles the basic model of Kamien and Tauman (1986), such that their well-known result that fixed fee licensing is preferred to royalty licensing still prevails. By contrast, when the value of β is small as in Parts (a-i) and (b-i), we find a new result that royalty licensing is preferred. The intuition for this new finding is as follows. As explained by Kamien and Tauman (1986) and Wang and Yang (2004), the superiority of fixed fee licensing lies in the efficiency gain a licensee can enjoy. Under royalty licensing with $r^* = \Delta c$, a licensee does not vary its output post licensing, whereas under fixed fee licensing a licensee can benefit from cost reduction by adjusting its output. A fixed fee allows the patentee to extract the licensee's efficiency gains so that it is preferred. However, the presence of cross ownership mitigates the power of fixed fees as a means to extract the licensee's surplus. The mitigation occurs in one of the two forms. When the patentee licenses to both firms, she has to license at a fee firm 1 is willing to pay, LL_1, which moves in the same direction as β. The smaller the β, the more firm 1 can benefit from firm 2's license (since shareholder 1 owns a $1-\beta$ fraction of π_2) and the less incentive firm 1 has to become a licensee itself (so as to avoid fierce competition with firm 2), which then undermines the patentee's revenues under fixed fees, $2LL_1$. When the patentee licenses only to one firm, she licenses to firm 1 at L_1, which again, moves in the same direction as β. The smaller the β, the more firm 1 cares about firm 2's profits and the less firm 1 will pay to become a sole licensee with a cost advantage over firm 2. In both cases, small values of β render fixed fee licensing unattractive so that royalty licensing is preferred.

When the degree of cross ownership β is in the intermediate range (i.e., Parts (a-ii) and (b-ii)), the above effects of β on L_1 and LL_1 are still at work but to a lesser extent, so that the effect of the innovation size Δc is non-negligible. When the patent holder licenses to both firms at the fee LL_1 (i.e., Part (a-ii)), larger values of Δc lead to a *lower* LL_1, because firm 1 can benefit more from firm 2's adoption (of larger innovations) and the two firms will compete too vigorously if firm 1 also adopts. Thus, royalty licensing is preferred to fixed fee licensing for a larger Δc in this parameter range (i.e., $\Delta\hat{c} < \Delta c \leq \beta(1-c)/(3-2\beta)$). When the patent holder licenses to firm 1 only at L_1 (i.e., Part (b-ii)), firm 1 is willing to pay a *greater* L_1 for larger values of Δc. This is because in firm 1's calculation of L_1, the benefit from becoming a sole licensee is to enjoy the cost reduction, while the concern is not to compete too aggressively with firm 2. The latter concern is relatively minor here in (b-ii) (for the value of β is not as small and the degree of cross ownership is not as high as in (b-i)), while the former benefit increases in the innovation size Δc. Thus, fixed fee licensing is still preferred for larger values of Δc, while royalty licensing is preferred for smaller values of Δc in this parameter range (i.e., $\beta(1-c)/(3-2\beta) < \Delta c < \Delta\tilde{c}$).

6. Conclusion

We compare fixed fee and royalty licensing in a simple game, in which an independent patent holder seeks to license her cost-reducing technology to a Cournot duopoly with partial cross ownership. It is found that the degree of cross ownership is an important factor in determining the superiority of fixed fee and royalty licensing. We show that, for each non-drastic innovation size, the patent holder prefers royalty licensing when the degree of cross ownership is sufficiently high. If the degree of cross ownership is low, then fixed fee licensing is still preferred. For intermediate degrees of cross ownership, the superiority of both licensing means is determined by the innovation size. Our results provide a new explanation for the prevalence of royalty licensing in practice.

Appendix

Proof of Proposition 1: It is straightforward to show that $LL_1 < LL_2$ for all $0 < \beta < 1$ and that L_1 > (=, <) L_2 iff Δc > (=, <) $\beta(1-c)/3$. We thus have two cases to consider. First, for $\Delta c \leq \beta(1-c)/3$, licensing to both firms yields a revenue of $2LL_1$, while licensing to one firm only yields L_2. Direct computations yield $2LL_1 > L_2$ iff $\Delta c < \beta(1+\beta)(1-c)/(3-\beta^2)$. Note that $\beta(1+\beta)(1-c)/(3-\beta^2) > \beta(1-c)/3$ such that $2LL_1 > L_2$ holds for all $\Delta c \leq \beta(1-c)/3$. Next, for $\Delta c > \beta(1-c)/3$, licensing to both firms yields $2LL_1$, while licensing to one firm only yields L_1. Direct computations yield $2LL_1$ > (=, <) L_1 iff Δc < (=, >) $\beta(1-c)/(3-2\beta)$. Note that $\beta(1-c)/(3-2\beta)$ lies between $\beta(1-c)/3$ and $\beta(1-c)/(2-\beta)$, such that we have two sub-cases. For $\beta(1-c)/3 < \Delta c < \beta(1-c)/(3-2\beta)$, we have $2LL_1 > L_1$, while for $\Delta c > \beta(1-c)/(3-2\beta)$, we have $2LL_1 < L_1$.

Proof of Proposition 2: Without considering the constraint of $r \leq \Delta c$, the first-order condition for an interior solution to (21) is given by $\frac{\partial R}{\partial r} = \frac{(1+\beta)(1-c+\Delta c-2r)}{2+\beta} = 0$ with the second-order condition satisfied, $\frac{\partial^2 R}{\partial r^2} = \frac{-2(1+\beta)}{2+\beta} < 0$. Thus, the unconstrained solution is given by $\hat{r} = \frac{1-c+\Delta c}{2}$, which, however, does not satisfy the constraint of $r \leq \Delta c$. This is because $\hat{r} \leq \Delta c$ iff $\Delta c \geq 1 - c$, which violates our assumption for Δc specified in equation (6). Thus, the solution to (21) is given by $r^* = \Delta c$. Given that each firm's post-licensing cost is $c - \Delta c + r^* = c$, we have $Q(c,c,\beta) = \frac{(1+\beta)(1-c)}{2+\beta}$ from (7), such that $R^* = \frac{(1+\beta)(1-c)\Delta c}{2+\beta}$.

Proof of Proposition 3: There are two cases to consider, depending on the size of Δc.

(a) For $\Delta c \leq \beta(1-c)/(3-2\beta)$, we have $F^* = 2LL_1$, such that $R^* > F^*$ iff

$$\frac{(1+\beta)(1-c)\Delta c}{2+\beta} > \frac{2(3+\beta)\Delta c[\beta(1-c)-(1-\beta)\Delta c]}{(2+\beta)^2}, \tag{A.1}$$

which after rearranging becomes:

$$\Delta c > \frac{(\beta^2+3\beta-2)(1-c)}{2(1-\beta)(3+\beta)} \equiv \Delta\hat{c}. \tag{A.2}$$

The denominator of $\Delta\hat{c}$ is positive while the numerator is positive only for $\beta > 0.562$. Thus, for $\beta \le 0.562$, we have $\Delta\hat{c} \le 0$ such that (A.2) holds for sure. This is our result in Part (a-i). We next check the consistency between the condition for Δc in (A.2) and our parameter condition for Case (a). It is straightforward to show that $\Delta\hat{c} < \beta(1-c)/(3-2\beta)$ only for $\beta <$ 0.772. Thus, for $0.562 < \beta < 0.772$, we have $R^* > F^*$ for $\Delta c \in (\Delta\hat{c}, \beta(1-c)/(3-2\beta)]$ and $R^* < F^*$ for $\Delta c < \Delta\hat{c}$. These are our results in Part (a-ii). As for $\beta \ge 0.772$, we have $\Delta\hat{c} \ge \beta(1-c)/(3-2\beta)$, such that it is impossible for (A.2) to hold for any Δc in Case (a). This completes our proof for Part (a-iii).

(b) For $\beta(1-c)/(3-2\beta) < \Delta c < \beta(1-c)/(2-\beta)$, we have $F^* = L_1$, such that $R^* > F^*$ iff

$$\frac{(1+\beta)(1-c)\Delta c}{2+\beta} > \frac{(3+\beta)\Delta c[\beta(1-c)+\Delta c]}{(2+\beta)^2}, \tag{A.3}$$

which after rearranging becomes:

$$\Delta c < \frac{2(1-c)}{3+\beta} \equiv \Delta\tilde{c}. \tag{A.4}$$

We first check the consistency between the condition for Δc specified in (A.4) and our parameter condition for Case (b). It is straightforward to show that $\Delta\tilde{c} < \beta(1-c)/(2-\beta)$ for $\beta > 0.702$ and that $\Delta\tilde{c} > \beta(1-c)/(3-2\beta)$ for $\beta < 0.772$. Thus, we have three sub-cases depending on the value of β. First, for $\beta \le 0.702$, we have $\Delta\tilde{c}$ lying above $\beta(1-c)/(2-\beta)$, such that (A.4) holds for all Δc in Case (b). This is our result in Part (b-i). Next, for $0.702 < \beta < 0.772$, we have $\Delta\tilde{c}$ lying between $\beta(1-c)/(3-2\beta)$ and $\beta(1-c)/(2-\beta)$. Thus, we have $R^* > F^*$ for $\Delta c \in (\beta(1-c)/(3-2\beta), \Delta\tilde{c})$ and $R^* < F^*$ for $\Delta c \in (\Delta\tilde{c}, \beta(1-c)/(2-\beta))$. These are our results in Part (b-ii). Lastly, for $\beta \ge 0.772$, we have $\Delta\tilde{c}$ lying below $\beta(1-c)/(3-2\beta)$, such that for all Δc in Case (b), we have $\Delta c > \Delta\tilde{c}$ holding true such that $R^* < F^*$. This completes our proof for Part (b-iii).

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