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# Boundary and interior equilibria: what drives convergence in a ‘beauty contest'?

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## Abstract

We present an experimental game in the p-beauty framework. Building on the definitions of boundary and interior equilibria, we distinguish between ‘speed of convergence towards the game-theoretic equilibrium' and ‘deviations of the guesses from the game-theoretic equilibrium'. In contrast to earlier findings (Güth et al., 2002), we show that (i) interior equilibria initially produce smaller deviation of the guesses from the game-theoretic equilibrium compared to boundary equilibria; (ii) interior and boundary equilibria do not differ in the timeframe needed for convergence; (iii) the speed of convergence is higher in the boundary equilibrium.

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## 1. Introduction

The p-beauty contest game is a well known and extremely simple game (Keynes, 1936; Nagel, 1995; Duffy and Nagel, 1997; Canerer et al., 1998; Weber, 2003) where *n* players are asked to choose a number from a closed interval [*L, H*]. The winning player will be the one that gets closer to a target number *G*. Such target number is defined as the average of all guesses plus a constant (*d*), multiplied by a real number (*p*) known to all players. Formally, we have: $G = p\left(\frac{1}{n}\sum_{i=1}^{n} g_i + d\right)$. In its simplest form the game parameterisation is set as follows: $0 \le p < 1$, $g_i \in [0,100] \subset \mathbf{R}$ is subject *i*'s guess and *d* is a constant set equal to 0.

Under such definition of *G* the game-theoretic solution is a unique Nash equilibrium where all players choose 0. In fact, playing 0 is the only strategy that survives the procedure of *iterated elimination of dominated strategies* (IEDS). Moreover, under such a standard parameterisation the game converges to the same value, and within the same number of iterations, if players follow Nagel's *iterative naïve best replies* (INBR) strategy.[1]

The game becomes more complicated if we set $d \neq 0$; in this case the game might well exhibit an interior equilibrium (i.e. different from 0 or 100) and for specific values of *p*, the solution of the game obtained using the two different strategies (IEDS and INBR) involves different numbers of iterations needed to reach the equilibrium.

Güth et al. (2002) proposed a game where *d* was initially set equal to 0 and subsequently equal to 50. This allowed them to analyse the p-beauty contest from a different perspective, comparing, among other things, interior and boundary equilibria. They showed that the convergence toward the equilibrium is faster when the equilibrium is interior.

In this paper we shall confute Güth et al. (2002) finding showing that the speed of convergence is actually not necessarily faster for interior equilibria. We shall distinguish between 'speed of convergence towards the game-theoretic equilibrium' and 'timeframe required for convergence' (i.e. the number of iterations). These two concepts are indeed different as they crucially depend on the initial distance of a system from its equilibrium level. In the p-beauty contest game, such distance is captured by the initial 'deviations of the guesses from the game-theoretic equilibrium'. We will prove that, for specific game's parameterisations, the speed of convergence can be slower in interior equilibria than in boundary equilibria.

The paper is structured as follows: in section 2 we present the experimental game and pose our research hypothesis. In section 3 we present the design of the experiment and in section 4 our findings. We conclude the paper in section 5.

## 2. Aim and setting of the experiment

The aim of our experiment is testing the validity of Güth et al. (2002) finding that interior equilibria (e.g. *d=50*) yield smaller deviations of the guesses from the game-theoretic

[1] See Morone and Morone (2008) for a description of both strategies.

equilibrium when compared to boundary equilibria;[2] this finding leads the authors to conclude that "swifter convergence to the equilibrium [is found] when the equilibrium is interior" (2002: 225).
Schematically, we can summarise Güth et al. experiment's parameterisations and equilibria ($s^*$), in the following table:

| | Parametrisation | Game-Theoretic euilibrium | Slient point à la Schelling | Number of iterations required for convergence IEDS | INBR |
|---|---|---|---|---|---|
| Treatment 1 | $p=1/2$, $d=0$ $g\in[0,100]$ | $s^*=0$ (boundery equilibrium) | 50 | ∞ | ∞ |
| Treatment 2 | $p=1/2$, $d=50$ $g\in[0,100]$ | $s^*=50$ (interior equilibrium) | 50 | 27 | 1 |

Table I: Güth et al. (2002) summary of parameters and results

As we can see, the authors presented two comparable cases and showed how the treatment where the game-theoretic equilibrium is interior, requires less iterations for convergence. However, we can observe that for the interior equilibrium case there is the simultaneous coincidence, around the game-theoretic equilibrium, of two possible focal points: the $d$ value as well as the salient point *à la* Schelling.[3] Moreover, in the case of $d=50$, if we believe that subjects follow Nagel's *iterative naïve best replies* strategy, they would converge towards the equilibrium after one iteration.
The picture changes when we look at the boundary equilibrium treatment. Now, the salient point *à la* Schelling and the $d$ value differ, as the salient point is always set in the middle of the interval [0,100]. Moreover, both theoretical strategies (i.e. IEDS and INBR) predict an infinite number of iteration for complete convergence to the equilibrium.[4]
Departing from these observations we intend to test whether the short timeframe required for convergence is an actual property of interior equilibria or whether it is just arising from the *ad hoc* specification chosen by Güth et al. (2002). We do so considering a new set of problems' characterisation defined by different parameterisations of the game. Specifically, we shall compare the original parameterisation adopted by Güth et al. with a similar setting where we vary the value of $p$ (set equal to 1/3) and the value of $d$ (set equal to 0, 33 and 50). It is worth noting that, like in the original experimental setting, these new parameterisations produce both interior and boundary equilibria.
If Güth et al.'s result is robust to different model parameterisations, we will always observe a shorter timeframe required for convergence in the game with interior equilibrium; otherwise, we shall confute the validity of their results for problems' parameterisations different from those originally selected by the authors.

## 3. The design of the experiment

In each treatment of the experiment there are $n = 32$ subjects divided into 8 groups, each of 4 subjects. In each group subjects have to guess a number in the real interval $[L, H]$. The

[2] Note that a plausible explanation of this finding is that experimental subjects often try to avoid extreme choices (see, for instance, Rubinstein et al., 1997).
[3] Nagel (1995) suggested that the salient point à la Schelling would be 50 (i.e. the middle of the interval). However, we believe there are other possible focal points; for instance $d$ could be perceived as such by experimental subjects.
[4] Please see table A1 in the annex where we report the converging patterns generated by IEDS and INBR.

closer their guess is to the target, the higher is the pay-off. The general form of the pay-off function is: $u(g_i) = C - c\left|g_i - p\left(\frac{1}{n}\sum_{j=1}^{n} g_j + d\right)\right|$.

The experiments were run in May 2008 at ESSE (Economia Sperimentale al Sud d'Europa) at the University of Bari. The software of the computerised experiment was developed in z-Tree (Fischbacher, 1998). Groups were formed randomly at the beginning of the experiment and were kept invariant over the whole experiment (i.e. 10 periods).

## 4. Results

In this section we will analyse the results obtained in our experiments. However, before moving to our new findings we shall present the results obtained when running the experiment using exactly the same parameterisation adopted by Güth et al. (2002). This will serve to cast away any doubt on the presence of any source of difference between our experimental design and the one adopted by Güth et al.

In figure 1 we report the average values of treatments 1 and 2;[5] the converging patterns obtained in our preliminary set of experiments replicate exactly those obtained by Güth et al., as it shows smaller deviations of the guesses from the game-theoretic equilibrium and a shorter time frame for convergence in the treatment with boundary equilibrium.[6] In figure 2 we report two further treatments where $p$ is kept equal to 1/2 and $d$ is set first equal to 33 and, subsequently, to 100. These two treatments produce very similar results to those reported in figure 1. Again, the system converges within a smaller number of iterations to the interior equilibrium, consistently displaying a smaller deviation of the guesses from the game-theoretic equilibrium.[7]

[5] As mentioned earlier, each treatment was repeated 10 times. Hence, all values reported in figures 1 and 2 are averages of 10 rounds.

[6] Deviation from the equilibrium are significantly smaller in treatment 2 than in treatment 1 ($p < 0.01$ for rounds 1-10 and 1-5, $p < 0.1$ for rounds 6-10).

[7] Deviation from the equilibrium are significantly smaller in treatment 3 than in treatment 4 ($p < 0.01$ for rounds 1-10, 1-5, and 6-10)

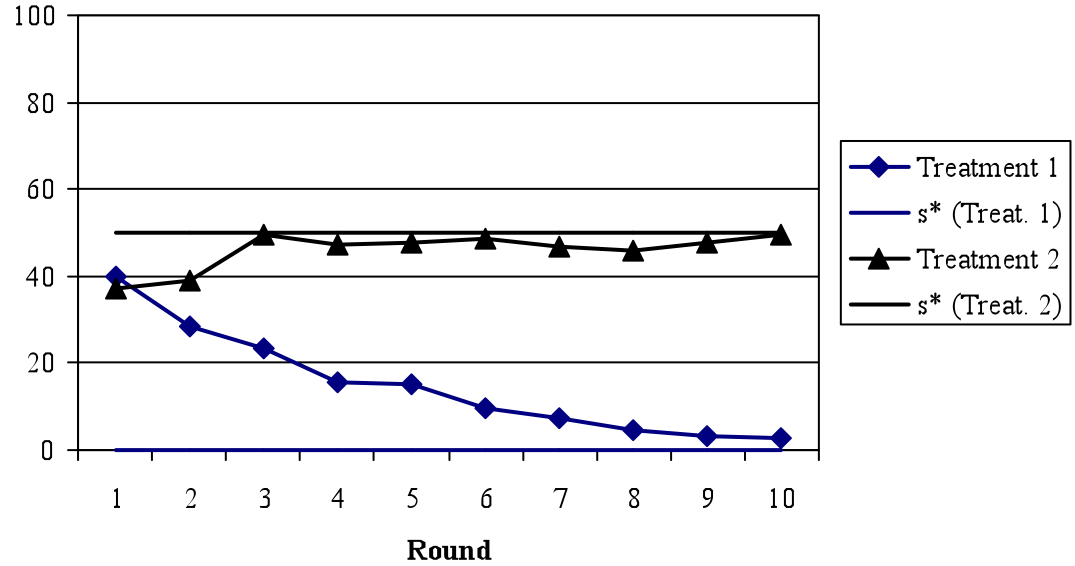


Figure 1. Treatment averages (p=1/2; d=0 and d=50)

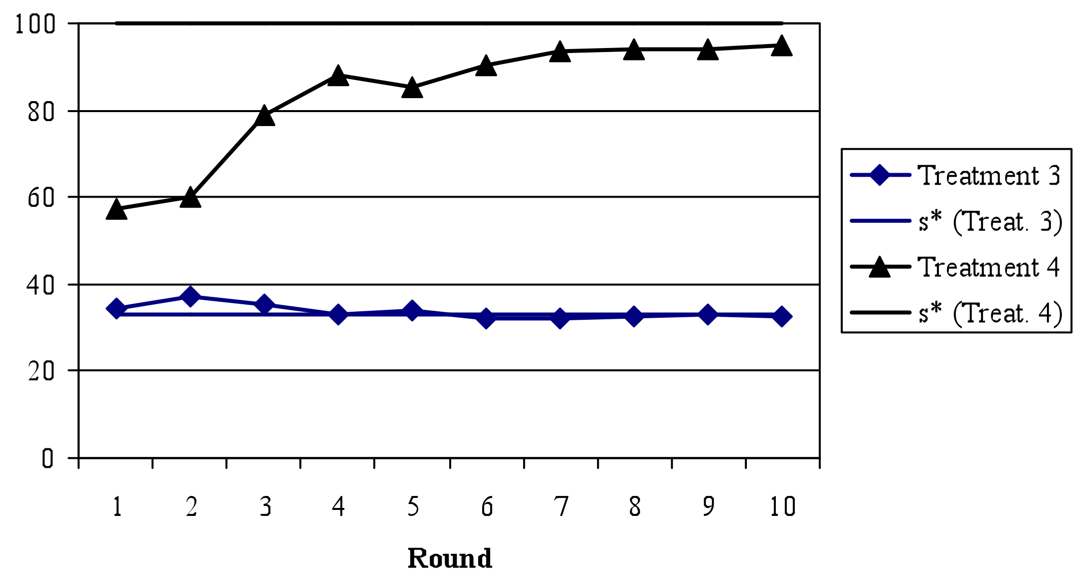


Figure 2. Treatment averages (p=1/2; d=33 and d=100)

We summarise the parameterisation and the results of these two last treatments in table II. As we can see, in this case the boundary equilibrium does not present the simultaneous coincidence of the two focal points around the equilibrium value (i.e. *d* and the salient point *à la* Schelling are different).

| | Parametrisation | Game-Theoretic euilibrium | Slient point à la Schelling | Number of iterations required for convergence IEDS | INBR |
|---|---|---|---|---|---|
| Treatment 3 | $p$=1/2, $d$=33 $g\in[0,100]$ | $s^*$=33 (boundery equilibrium) | 50 | 27 | 26 |
| Treatment 4 | $p$=1/2, $d$=100 $g\in[0,100]$ | $s^*$=100 (interior equilibrium) | 50 | 1 | 27 |

Table II[9]: Summary of parameters and results

These first four treatments seem to confirm Güth et al. findings. We shall now move on to consider three other treatments where $p$ is now set equal to 1/3 and $d$ is set equal to 0, 33 and 50, respectively.[8] We report a summary of these new treatments' parameterisation and results in table III.

| | Parametrisation | Game-Theoretic euilibrium | Slient point à la Schelling | Number of iterations required for convergence IEDS | INBR |
|---|---|---|---|---|---|
| Treatment 3 | $p$=1/3, $d$=0 $g\in[0,100]$ | $s^*$=0 (boundery equilibrium) | 50 | ∞ | ∞ |
| Treatment 4 | $p$=1/3, $d$=33 $g\in[0,100]$ | $s^*$=16.5 (interior equilibrium) | 50 | 17 | 16 |
| Treatment 5 | $p$=1/3, $d$=50 $g\in[0,100]$ | $s^*$=25 (interior equilibrium) | 50 | 17 | 16 |

Table III[9]: Summary of parameters and results

The picture emerging from these new treatments is rather different from what we obtained so far. First and foremost, we do not observe any significant difference in the timeframe of convergence towards the equilibrium across treatments: in fact, in all cases (i.e. one boundary and two internal) it takes for the system approximately the same number of rounds (about 6-7) to converge. However, we can easily observe that the initial deviation of the guesses from the game-theoretic equilibrium is smaller in the two treatments that converge towards interior equilibria. This finding suggests that the conclusions obtained by Güth et al. are only partially confirmed. While interior equilibria treatments initially produce smaller deviation, the speed of convergence is higher in the boundary equilibrium treatment.

[8] Note that we do not consider the treatment with $p$=1/3 and $d$=100 as it converges to an interior equilibrium ($s^*$=50) and, therefore, is not comparable with $p$=1/2 and d=100.

[9] Please see table A2 in the annex where we report the converging patterns generated by IEDS and INBR.

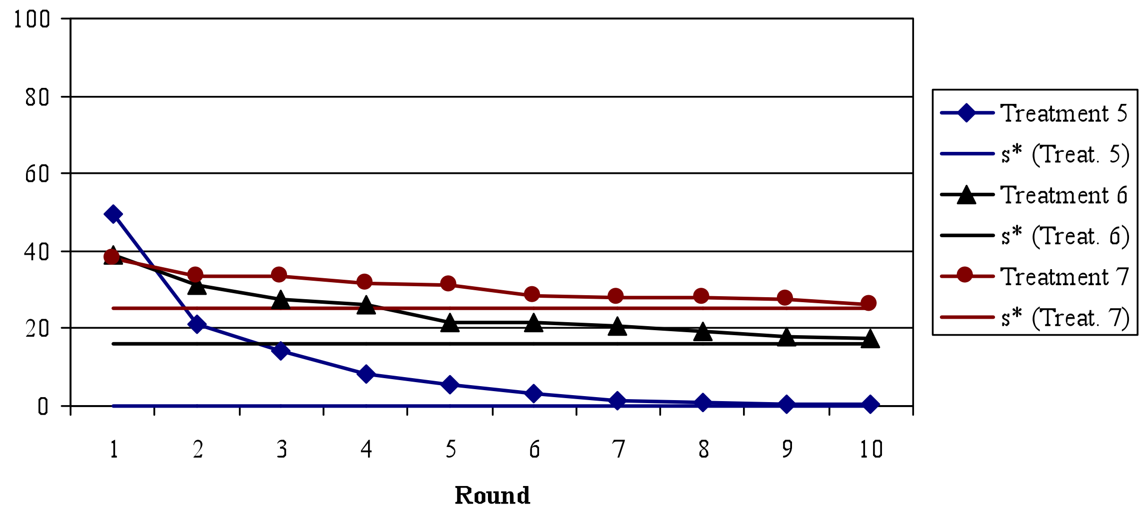


Figure 3. Treatment averages (p=1/3; d=0, d=33 and d=50)

In fact, as observed above, all three treatments converge towards their theoretical equilibria in the same time frame;[10] therefore, the treatment which starts converging from a further point (i.e. the one which displays a higher deviation from the game-theoretic equilibrium) must converge faster, as it clearly emerges from figure 3.

Hence, we can conclude that the initial deviation from the game theoretic equilibrium is always greater in boundary equilibria, independently from the initial parameterisation of the experiment. We share Güth et al.'s view that this finding might depend on the tendency to choose interior instead of extreme, boundary strategies. However, along this finding we also observe that under the new experiment parameterisation the timeframe required for convergence is the same for boundary and interior equilibria and, consequently, the speed of convergence is higher in boundary equilibria. In fact, if two runners reach the same target in the same timeframe, the one starting from further away must run faster in order to cover a larger distance in the same time.

## 5. Conclusions

The experiment presented in this paper follows quite closely Güth et al., 2002 as we attempt to investigate differences in the speed of convergence towards boundary and interior equilibria in the p-beauty contest game. In doing so, we distinguish between 'speed of convergence towards the game-theoretic equilibrium' and 'timeframe required for convergence'. These two concepts crucially depend upon the initial 'deviations of the guesses from the game-theoretic equilibrium'. In contrast to earlier findings (Güth et al., 2002), we obtain the following results: (i) interior equilibria treatments initially produce smaller deviation compared to boundary equilibria treatments; (ii) interior and boundary equilibria treatments do not differ in the timeframe needed for convergence; (iii) the speed of convergence is higher in the boundary equilibrium treatment.

[10] Comparing treatment 5 and treatment 6, treatment 5 and treatment 7, treatment 6 and treatment 7 we can reject, the hypothesis that deviation from the equilibrium is statistically significantly smaller in interior equilibrium treatments.

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Annex

| Treatment 1 | | Treatment 2 | |
|---|---|---|---|
| IEDS | INBR | IEDS | INBR |
| 50 | 25 | 75 | 50 |
| 25 | 12.5 | 62.5 | 50 |
| 12.5 | 6.25 | 56.25 | 50 |
| 6.25 | 3.125 | 53.125 | 50 |
| 3.125 | 1.5625 | 51.5625 | 50 |
| 1.5625 | 0.78125 | 50.78125 | 50 |
| 0.78125 | 0.390625 | 50.390625 | 50 |
| 0.390625 | 0.1953125 | 50.195313 | 50 |
| 0.1953125 | 0.09765625 | 50.097656 | 50 |
| 0.09765625 | 0.048828125 | 50.048828 | 50 |
| 0.048828125 | 0.024414063 | 50.024414 | 50 |
| 0.024414063 | 0.012207031 | 50.012207 | 50 |
| 0.012207031 | 0.006103516 | 50.006104 | 50 |
| 0.006103516 | 0.003051758 | 50.003052 | 50 |
| 0.003051758 | 0.001525879 | 50.001526 | 50 |
| 0.001525879 | 0.000762939 | 50.000763 | 50 |
| 0.000762939 | 0.00038147 | 50.000381 | 50 |
| 0.00038147 | 0.000190735 | 50.000191 | 50 |
| 0.000190735 | 9.53674E-05 | 50.000095 | 50 |
| 9.53674E-05 | 4.76837E-05 | 50.000048 | 50 |
| 4.76837E-05 | 2.38419E-05 | 50.000024 | 50 |
| 2.38419E-05 | 1.19209E-05 | 50.000012 | 50 |
| 1.19209E-05 | 5.96046E-06 | 50.000006 | 50 |
| 5.96046E-06 | 2.98023E-06 | 50.000003 | 50 |
| 2.98023E-06 | 1.49012E-06 | 50.000001 | 50 |
| 1.49012E-06 | 7.45058E-07 | 50.000001 | 50 |
| 7.45058E-07 | 3.72529E-07 | 50 | 50 |
| 3.72529E-07 | 1.86265E-07 | 50 | 50 |
| 1.86265E-07 | 9.31323E-08 | 50 | 50 |
| 9.31323E-08 | 4.65661E-08 | 50 | 50 |
| 4.65661E-08 | 2.32831E-08 | 50 | 50 |
| 2.32831E-08 | 1.16415E-08 | 50 | 50 |

Table A1: IEDS and INBR game theoretical solution for treatments 1 and 2

| Treatment 3 | | Treatment 4 | | Treatment 5 | | Treatment 6 | | Treatment 7 | |
|---|---|---|---|---|---|---|---|---|---|
| IEDS | INBR | IEDS | INBR | IEDS | INBR | IEDS | INBR | IEDS | INBR |
| 66.5 | 41.5 | 100 | 75 | 33.333333 | 16.666667 | 44.333333 | 27.666667 | 50 | 33.333333 |
| 49.75 | 37.25 | 100 | 87.5 | 11.111111 | 5.5555556 | 25.777778 | 20.222222 | 33.333333 | 27.777778 |
| 41.375 | 35.125 | 100 | 93.75 | 3.7037037 | 1.8518519 | 19.592593 | 17.740741 | 27.777778 | 25.925926 |
| 37.1875 | 34.0625 | 100 | 96.875 | 1.2345679 | 0.617284 | 17.530864 | 16.91358 | 25.925926 | 25.308642 |
| 35.09375 | 33.53125 | 100 | 98.4375 | 0.4115226 | 0.2057613 | 16.843621 | 16.63786 | 25.308642 | 25.102881 |
| 34.046875 | 33.265625 | 100 | 99.21875 | 0.1371742 | 0.0685871 | 16.61454 | 16.545953 | 25.102881 | 25.034294 |
| 33.523438 | 33.132813 | 100 | 99.609375 | 0.0457247 | 0.0228624 | 16.53818 | 16.515318 | 25.034294 | 25.011431 |
| 33.261719 | 33.066406 | 100 | 99.804688 | 0.0152416 | 0.0076208 | 16.512727 | 16.505106 | 25.011431 | 25.00381 |
| 33.130859 | 33.033203 | 100 | 99.902344 | 0.0050805 | 0.0025403 | 16.504242 | 16.501702 | 25.00381 | 25.00127 |
| 33.06543 | 33.016602 | 100 | 99.951172 | 0.0016935 | 0.0008468 | 16.501414 | 16.500567 | 25.00127 | 25.000423 |
| 33.032715 | 33.008301 | 100 | 99.975586 | 0.0005645 | 0.0002823 | 16.500471 | 16.500189 | 25.000423 | 25.000141 |
| 33.016357 | 33.00415 | 100 | 99.987793 | 0.0001882 | 9.408E-05 | 16.500157 | 16.500063 | 25.000141 | 25.000047 |
| 33.008179 | 33.002075 | 100 | 99.993896 | 6.272E-05 | 3.136E-05 | 16.500052 | 16.500021 | 25.000047 | 25.000016 |
| 33.004089 | 33.001038 | 100 | 99.996948 | 2.091E-05 | 1.045E-05 | 16.500017 | 16.500007 | 25.000016 | 25.000005 |
| 33.002045 | 33.000519 | 100 | 99.998474 | 6.969E-06 | 3.485E-06 | 16.500006 | 16.500002 | 25.000005 | 25.000002 |
| 33.001022 | 33.000259 | 100 | 99.999237 | 2.323E-06 | 1.162E-06 | 16.500002 | 16.500001 | 25.000002 | 25.000001 |
| 33.000511 | 33.00013 | 100 | 99.999619 | 7.744E-07 | 3.872E-07 | 16.500001 | 16.5 | 25.000001 | 25 |
| 33.000256 | 33.000065 | 100 | 99.999809 | 2.581E-07 | 1.291E-07 | 16.5 | 16.5 | 25 | 25 |
| 33.000128 | 33.000032 | 100 | 99.999905 | 8.604E-08 | 4.302E-08 | 16.5 | 16.5 | 25 | 25 |
| 33.000064 | 33.000016 | 100 | 99.999952 | 2.868E-08 | 1.434E-08 | 16.5 | 16.5 | 25 | 25 |
| 33.000032 | 33.000008 | 100 | 99.999976 | 9.56E-09 | 4.78E-09 | 16.5 | 16.5 | 25 | 25 |
| 33.000016 | 33.000004 | 100 | 99.999988 | 3.187E-09 | 1.593E-09 | 16.5 | 16.5 | 25 | 25 |
| 33.000008 | 33.000002 | 100 | 99.999994 | 1.062E-09 | 5.311E-10 | 16.5 | 16.5 | 25 | 25 |
| 33.000004 | 33.000001 | 100 | 99.999997 | 3.541E-10 | 1.77E-10 | 16.5 | 16.5 | 25 | 25 |
| 33.000002 | 33.000001 | 100 | 99.999999 | 1.18E-10 | 5.901E-11 | 16.5 | 16.5 | 25 | 25 |
| 33.000001 | 33 | 100 | 99.999999 | 3.934E-11 | 1.967E-11 | 16.5 | 16.5 | 25 | 25 |
| 33 | 33 | 100 | 100 | 1.311E-11 | 6.557E-12 | 16.5 | 16.5 | 25 | 25 |
| 33 | 33 | 100 | 100 | 4.371E-12 | 2.186E-12 | 16.5 | 16.5 | 25 | 25 |
| 33 | 33 | 100 | 100 | 1.457E-12 | 7.285E-13 | 16.5 | 16.5 | 25 | 25 |
| 33 | 33 | 100 | 100 | 4.857E-13 | 2.428E-13 | 16.5 | 16.5 | 25 | 25 |
| 33 | 33 | 100 | 100 | 1.619E-13 | 8.095E-14 | 16.5 | 16.5 | 25 | 25 |
| 33 | 33 | 100 | 100 | 5.397E-14 | 2.698E-14 | 16.5 | 16.5 | 25 | 25 |

Table A2: IEDS and INBR game theoretical solution for treatments 3, 4, 5, 6 and 7