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Wholesale price discrimination and enforcement of regulation

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Abstract

The present paper studied third-degree price discrimination in wholesale markets and its welfare property when a monopolistic manufacturer sells his/her products to two retailers who have different qualities and costs of sales. Our results revealed that price discrimination within a certain extent increases social welfare under some conditions, which would support the soft enforcement of prohibiting price discrimination by a monopolistic wholesaler.

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1. Introduction

Nowadays, there are different types of retailers with regard to retail service quality and product prices: The first type is the specialty store, which provides sufficient after sales warranty and support for products; the second type is the discount store (or web store), which achieves lower price points through rationalization and does not provide after sales warranty and support. These retailers have to face price competition along with different standards of retail service quality, and thus, their demand functions for identical products are different. Accordingly, a monopolistic manufacturer has an incentive to charge different prices from each type of retailers. That is, a monopolistic manufacturer has an incentive for third-degree price discrimination in wholesale markets. Indeed, the Times (British newspaper) dated November 15, 2005, reported that manufacturers are charging shopping websites wholesale prices between 10 and 15 per cent higher than the prices charged to high street stores. Based on this report, Aiura (2007) showed that under some conditions, the wholesale price charged from an online electronic retailer is higher than that charged from a high street retailer.[1] Because such price discrimination of a monopolistic manufacturer seems to distort competition in retailers, the antitrust laws of many countries prohibit such discrimination. In order to assess whether this prohibition supports improvement in social welfare, the present paper, which extends Aiura (2007), analyzes how social welfare would be changed by the extent of third-degree price discrimination of a monopolistic wholesaler that sells its products to two price-competing retailers who have different qualities and costs of sales: One is a high-quality, high-cost retailer, while the other is a low-quality, low-cost retailer.

In a related literature, Katz (1987), DeGraba (1990), and Yoshida (2000) studied welfare properties of price discrimination by upstream firms. Katz (1987) and DeGraba (1990) assumed that in upstream-downstream relationships, the product costs of downstream firms that compete in terms of quantities are different, and they showed that price discrimination by a monopolistic upstream firm always lowers social welfare. Yoshida (2000) assumed that not only the product costs but also productivities of downstream firms are different and showed that social welfare might improve when a monopolistic upstream firm charges a higher price to the firm that has relatively lower productivity than the other firms. Since we assume that the selling cost and service quality of downstream firms are different, our assumption is different from that of previous studies. Moreover, the previous studies focus on determining which measure is better—prohibiting price discrimination wholly or permitting price discrimination without constraint. However, in reality, the enforcement of prohibiting price discrimination by a monopolistic wholesaler is not strict, and the wholesaler would not be punished if the wholesale price gap between retailers is not large. Therefore, we consider another measure added to the two above mentioned measures—permitting price discrimination within a certain extent.

This consideration allows us to observe cases in which permitting price discrimination within a certain extent has the effect on social welfare from among the three abovementioned measures. This result would support the soft enforcement of prohibiting price discrimination by a monopolistic wholesaler. Moreover, we also observe cases of welfare improvement by third-degree price discrimination even when there is no increase in total output. Traditionally, a well-known conclusion about social welfare through third-degree price discrimination is that *an increase in total output is a necessary condition for welfare improvement by third-degree price discrimination*.[2] The present model shows one of the extension models in which the

[1] Villas-Boas (2007) empirically showed the wholesale price discrimination in the coffee market in Germany.

[2] See Schmalensee (1981), Varian (1985), and Schwartz (1990). The literature on price discrimination is surveyed by Stole (2007).

well-known conclusion is not achieved.[3]

The present paper is organized as follows. Section 2 presents some preliminaries in order to analyze the effect of manufacturer's price-discrimination on social welfare. Section 3 presents the results of this analysis. Section 4 presents the concluding remarks.

2. Preliminaries

We conduct a welfare analysis based on Aiura (2007).[4] This section introduces a price decision model according to Aiura (2007) and presents some preliminaries to enable us to conduct a welfare analysis. We consider that a monopolistic manufacturer sells his/her products to two price-competing retailers who have different qualities and costs of sales: One is a high-quality, high-cost (retailer H), and the other is a low-quality, low-cost (retailer L). The two retailers resell the manufacturer's products to end consumers.[5] The manufacturer acts as a Stackelberg price leader. In the first stage, the manufacturer sets different wholesale prices for retailers H and L under linear pricing contracts.[6] In the second stage, retailers H and L choose their respective retail prices based on the wholesale prices. w_i denotes the wholesale price, p_i denotes the retail price for retailer i ($i = H, L$), and q_i denotes the quantity of sales by retailer i ($i = H, L$). The manufacturer incurs constant marginal product cost, denoted by c_M, and retailers H and L incur constant marginal selling costs, denoted as c_H and c_L, respectively. We assume that retailer H incurs higher marginal costs than retailer L, $c_H > c_L$ and that consumers are heterogeneous with regard to the valuation of the manufacturer's product. We denote the customer's reservation price for the product by v and, for analytic simplicity, assume that it is uniformly distributed within the consumer population from 0 to 1, with a density of 1. Because retail quality is different for different retailers, we assume that the true valuation of a product through retail sales equals the valuation of the product multiplied by retail quality, and we denote the customer's reservation price for the product through retailer i by $\theta_i v$ ($\theta_i > 0$, $i = H, L$). To simplify the notation, we normalize θ_H as 1 and denote θ_L by θ. Because retailer H offers higher retail quality than retailer L, we assume that $0 < \theta_L < \theta_H$ (i.e., $0 < \theta < 1$). Retailer i offers the product at price p_i, so that a consumer whose reservation price is v derives a net consumer surplus of $\theta_i v - p_i$ by buying the product. Thus, if $\theta_i v - p_i > \theta_j v - p_j$ (for $i \neq j$, $i = H, L$ and $j = H, L$) and $\theta_i v - p_i > 0$ (for $i = H, L$), the consumer buys the product through retailer i.[7] Therefore, the demand functions for each retailer, q_H and q_L, depend on p_H and p_L. When $q_H = 0$ or $q_L = 0$, either retailer does not make a sale, and price discrimination must be unobserved. Because our purpose is to know welfare change by price discrimination, we assume $q_H > 0$ and $q_L > 0$, thus

$$q_H = 1 - \frac{p_H - p_L}{1 - \theta}, \qquad q_L = \frac{\theta p_H - p_L}{\theta(1 - \theta)}. \tag{1}$$

[3]Other extension models in which this well-known conclusion is not achieved are Yoshida (2000), Adachi (2005), and Galera and Zaratiegui (2006).

[4]Aiura (2007) does not analyze the welfare property of price discrimination by a upstream firm.

[5]We assume that the manufacturer does not permit retailers to buy and sell the manufacturer's products from other retailers. Moreover, we assume that retailers cannot change qualities and costs of sales.

[6]A monopolistic manufacturer might enforce a two-part tariff, but in this situation, it is unclear which criterion we use when deciding whether the manufacturer price-discriminates: lump-sum fee, per-unit charge, or per-unit lump-sum fee plus per-unit charge. Therefore, at the onset, we assume that a monopolistic manufacturer enforces linear pricing.

[7]If $\theta_H v - p_H \leq 0$ and $\theta_L v - p_L \leq 0$, we assume that the consumer would not buy the product and would obtain a zero surplus.

Using q_H and q_L, the profits of each retailer, π_H and π_L, can be written as

$$\pi_H = (p_H - w_H - c_H)q_H, \tag{2}$$
$$\pi_L = (p_L - w_L - c_L)q_L, \tag{3}$$

and the profits of the manufacturer, π_H, can be written as

$$\pi_M = (w_H - c_M)q_H + (w_L - c_M)q_L. \tag{4}$$

Before we assess the effect of price discrimination on social welfare, we need to derive retail and wholesale pricing. By backward induction, we first derive retail pricing when the wholesale price is given. The first-order conditions of (2) and (3) with respect to the retail price give the retail price equilibrium as follows:

$$p_H^* = 1 - \frac{2(1 - w_H - c_H) + (\theta - w_L - c_L)}{4 - \theta}, \tag{5}$$
$$p_L^* = \theta - \frac{\theta(1 - w_H - c_H) + 2(\theta - w_L - c_L)}{4 - \theta}. \tag{6}$$

Using p_H^* and p_L^*, we derive the wholesale price. If the manufacturer can exercise price discrimination, the first-order conditions of (4) with respect to the wholesale prices (w_H and w_L) give the the best wholesale prices to maximize the manufacturer's profit as follows:

$$w_H^* = c_M + \delta_H/2, \qquad w_L^* = c_M + \delta_L/2, \tag{7}$$

where δ_i denotes $\theta_i - c_i - c_M$ (for $i = H, L$). Moreover, δ_H and δ_L must satisfy that $[\theta/(2-\theta)]\delta_H < \delta_L < (2 - \theta)\delta_H$, in order to satisfy that $q_H > 0$ and $q_L > 0$. The previous derivations are closely derived by Aiura(2007).

If the manufacturer cannot exercise price discrimination, the manufacturer has to charge a uniform wholesale price from both retailers; that is, the manufacturer has to satisfy $w_H = w_L$. The first-order conditions of (4) with respect to $w_u = w_H = w_L$ give the the best wholesale price as follows:

$$w^{u*} = c_M + \frac{\theta\delta_H + 2\delta_L}{2(2 + \theta)}, \tag{8}$$

where δ_i denotes $\theta_i - c_i - c_M$ (for $i = H, L$). Moreover, δ_H and δ_L must satisfy that $[3\theta/(2 + 2\theta - \theta^2)]\delta_H < \delta_L < [(8 - \theta - \theta^2)/6]\delta_H$, in order to satisfy that $q_H > 0$ and $q_L > 0$. A more detailed derivation of w_u^* is given in Appendix A. Social welfare is measured by the total surplus, i.e., the sum of profits of the manufacturer and retailers and the sum of consumers' surpluses, which equals

$$\begin{aligned} SW &= \left(\int_{1-(q_H+q_L)}^{1-q_H} \theta v dv + \int_{1-q_H}^{1} v dv \right) - [(c_H + c_M)q_H + (c_L + c_M)q_L] \\ &= 1 - \frac{1}{2}(q_H^2 + \theta q_L^2) - (1 - q_H)(1 - \theta q_L) - [(c_H + c_M)q_H + (c_L + c_M)q_L]\,. \end{aligned} \tag{9}$$

Next section shows the change of SW by price discrimination of the manufacturer.

3. Welfare effect by price discrimination

3.1 The parallel between permitting and prohibiting price discrimination

The simplest method to determine whether price discrimination by the manufacturer is desirable on social welfare grounds is to calculate $SW|_{w_H=w_H^*,w_L=w_L^*} - SW|_{w_H=w_L=w^{u*}}$. Using (1) and (5) – (9), we derive

$$SW|_{w_H=w_H^*,w_L=w_L^*} - SW|_{w_H=w_L=w^{u*}} = -\frac{(\delta_H-\delta_L)[(20+9\theta-2\theta^2)\delta_H-(24+7\theta-4\theta^2)\delta_L]}{8(1-\theta)(2+\theta)^2(4-\theta)}. \quad (10)$$

Because $0 < \theta < 1$ and $(20+9\theta-2\theta^2) < (24+7\theta-4\theta^2)$, (10) implies that $SW|_{w_H=w_H^*,w_L=w_L^*}$ is more than $SW|_{w_H=w_L=w^{u*}}$ if δ_H is a little more than δ_L; otherwise, $SW|_{w_H=w_H^*,w_L=w_L^*}$ is less than $SW|_{w_H=w_L=w^{u*}}$. Moreover, (7) shows that the difference of w_H^* and w_L^* is directly proportional to the difference of δ_H and δ_L, and thus, we derive the following proposition.

Proposition 1. *We assume that the demand of each retailer is positive whether governments permit or prohibit price discrimination. If retailer H is charged at a higher wholesale price than retailer L and the difference between the two wholesale prices is sufficiently small, the prohibiting price discrimination decreases social welfare; otherwise, prohibiting price discrimination increases social welfare.*

Moreover, we derive that $(q_H+q_L)|_{w_H=w_H^*,w_L=w_L^*} = (q_H+q_L)|_{w_H=w_L=w^{u*}}$. Therefore, Proposition 1 is not identical to the traditionally well-known conclusion regarding third-degree price discrimination by a monopolistic seller—an increase in total output is a necessary condition for welfare improvement by third-degree price discrimination.

3.2 The case of permiting price discrimination within a certain extent

Subsection 3.1 showed which is better prohibiting price discrimination wholly or permitting price discrimination without constraint. However, the real enforcement of prohibiting price discrimination by a monopolistic wholesaler is not strict. In this subsection, we consider that the manufacturer can exercise price discrimination within tolerance limits.

We assume that a monopolistic wholesaler face profit-maximizing problem subject to the following linear constraint:

$$\begin{cases} 0 \le w_H - w_L \le t \le w_H^* - w_L^*, & \text{if } w_H^* > w_L^* \\ 0 \le w_L - w_H \le t \le w_L^* - w_H^*, & \text{if } w_H^* < w_L^* \end{cases}, \quad (11)$$

where t is some constant. The solution to this problem depends on t, and is denoted by $w_H^{c*}(t)$ and $w_L^{c*}(t)$.[8] In this subsection, as well as in subsection 3.1, if we assume that $q_i > 0$ $(i = H, L)$, we derive following Lemma with regard to $w_H^{c*}(t)$ and $w_L^{c*}(t)$.

Lemma 1. *When* $q_i|_{w_H=w_H^{c*}(t),w_L=w_L^{c*}(t)} > 0$ *(*$i = H, L$*) for any* $t \in [0,1]$ *is assumed,*

$$\begin{cases} w_H^{c*}(t) = c_M + \dfrac{\theta\delta_H+2\delta_L}{2(2+\theta)} + \dfrac{2}{2+\theta}t \\ w_L^{c*}(t) = c_M + \dfrac{\theta\delta_H+2\delta_L}{2(2+\theta)} - \dfrac{\theta}{2+\theta}t \end{cases}, \text{ if } \delta_L < \delta_H < \frac{2+2\theta-\theta^2}{3\theta}\delta_L, \quad (12)$$

[8] Since $t = 0$ implies that there is no price discrimination, $w_H^{c*}(0) = w_L^{c*}(0) = w^{u*}$. Conversely, since $t = |w_H{}^* - w_L{}^*|$ implies that there is price discrimination without constraint, $w_H^{c*}(|w_H{}^*-w_L{}^*|) = w_H^*$ and $w_L^{c*}(|w_H{}^*-w_L{}^*|) = w_L^*$.

$$\begin{cases} w_H^{c\,*}(t) = c_M + \dfrac{\theta\delta_H + 2\delta_L}{2(2+\theta)} - \dfrac{2}{2+\theta}t \\ w_L^{c\,*}(t) = c_M + \dfrac{\theta\delta_H + 2\delta_L}{2(2+\theta)} + \dfrac{\theta}{2+\theta}t \end{cases}, \text{ if } \frac{6}{8-\theta-\theta^2}\delta_\mathrm{L} < \delta_\mathrm{H} < \delta_\mathrm{L}.$$

Proof: Appendix B.

Lemma 1 and (7) show that $w_H^{c\,*}(t)$ ($w_L^{c\,*}(t)$) is a monotonically increasing (decreasing) function when $w_H^* > w_L^*$, and $w_H^{c\,*}(t)$ ($w_L^{c\,*}(t)$) is a monotonically decreasing (increasing) function when $w_H^* < w_L^*$. Moreover, we can observe that q_H (q_L) monotonically decreases, q_L (q_H) monotonically increases, and $q_H + q_L$ is constant as t increases when $w_H^* > w_L^*$ ($w_H^* < w_L^*$). In other words, each retail sale of retailers H and L changes as t increases, but the total retail sales remain constant. Therefore whether social welfare improves or worsens depends on the shift in the consumers' choice of retailers.

When $q_i|_{w_H = w_H^{c\,*}(t), w_L = w_L^{c\,*}(t)} > 0$ ($i = H, L$) for any $t \in [0, 1]$, differentiating SW with respect to t yields

$$\frac{\partial SW}{\partial t} = [(1-\theta)(1-q_H) - (c_H - c_L)]\frac{\partial q_H}{\partial t}.$$

If $\delta_H < \delta_L$, we can observe that $[(1-\theta)(1-q_H) - (c_H - c_L)] < \delta_H - \delta_L < 0$ and $\partial q_H/\partial t > 0$, and thus, we derive

$$\frac{\partial SW}{\partial t} < 0 \text{ for } 0 \le t \le w_L^* - w_H^*, \ \text{ if } \frac{6}{8-\theta-\theta^2}\delta_\mathrm{L} < \delta_\mathrm{H} < \delta_\mathrm{L}. \tag{13}$$

If $\delta_H > \delta_L$, although $\partial q_H/\partial t < 0$, the sign of $[(1-\theta)(1-q_H) - (c_H - c_L)]$ cannot be determined. Substituting (12) into $[(1-\theta)(1-q_H) - (c_H - c_L)]$, we obtain

$$\frac{(8+5\theta-\theta^2)\delta_H - 2(5+2\theta-\theta^2)\delta_L}{2(2+\theta)(4-\theta)} + \frac{1}{(2+\theta)}t.$$

Therefore, we derive the following:

$$\frac{\partial SW}{\partial t} > 0 \text{ for } 0 \le t \le w_H^* - w_L^*, \quad \text{if } \delta_L < \delta_H < \frac{14+3\theta-2\theta^2}{12+4\theta-\theta^2}\delta_L, \tag{14}$$

$$\begin{cases} \dfrac{\partial SW}{\partial t} < 0 \quad \text{for } \bar{t} < t \le w_H^* - w_L^* \\ \dfrac{\partial SW}{\partial t} = 0 \quad \text{for } t = \bar{t} \\ \dfrac{\partial SW}{\partial t} > 0 \quad \text{for } 0 \le t < \bar{t} \end{cases}, \text{ where } \bar{t} = \frac{-(8+5\theta-\theta^2)\delta_H + 2(5+2\theta-\theta^2)\delta_L}{2(4-\theta)}, \tag{15}$$

$$\text{if } \frac{14+3\theta-2\theta^2}{12+4\theta-\theta^2}\delta_L \le \delta_H \le \frac{2(5+2\theta-\theta^2)}{8+5\theta-\theta^2}\delta_L,$$

$$\frac{\partial SW}{\partial t} < 0 \text{ for } 0 \le t \le w_H^* - w_L^*, \quad \text{if } \frac{2(5+2\theta-\theta^2)}{8+5\theta-\theta^2}\delta_L < \delta_H < \frac{2+2\theta-\theta^2}{3\theta}\delta_L. \tag{16}$$

(13) – (16) show that SW monotonically increases as t increases if δ_H is a little more than δ_L, SW first increases and then decreases as t increases if δ_H is moderately more than δ_L;

Otherwise, SW monotonically decreases as t increases. This implication and (7) derive the following proposition.

Proposition 2. *We assume that the demand of each retailer is positive whether governments permit or prohibit price discrimination. If retailer H is charged at higher wholesale price than retailer L and the difference between the two wholesale prices is sufficiently small, permitting price discrimination within a certain extent has a more desirable effect on social welfare than prohibiting price discrimination. otherwise, prohibiting price discrimination has the most desirable effect on social welfare.*

Proposition 2 supports the soft enforcement of prohibiting price discrimination by a monopolistic manufacturer who sells to retailers, but this is exclusive to the situation when the manufacturer charges a higher price to retailer H (high-quality, high-cost retailer). Moreover, both Propositions 1 and 2 imply the possibility of welfare improvement without increasing the total output. The intuitive interpretation of welfare improvement without increasing the total output is as follows: When $\delta_H > \delta_L$ and the difference between δ_H and δ_L is small, for most consumers, choosing retailer L is desirable on social welfare grounds. However, the duopoly in the retail market leads to a situation wherein the retail sales by retailer L are less than what is desired on social welfare grounds. When permitted to price-discriminate, a monopolistic manufacturer increases the wholesale price for retailer H and decreases the wholesale price for retailer L; accordingly, the retail sales by retailer L increase. Therefore, price discrimination by a monopolistic manufacturer cancels out the undersupply of retailer L by duopoly and improves social welfare.

4. Concluding remarks

The present paper studied third-degree price discrimination in wholesale markets and its welfare property when a monopolistic manufacturer sells his/her products to two retailers who have different qualities and costs of sales. We observed that price discrimination within a certain extent increases social welfare under some conditions, which would support the soft enforcement of prohibiting price discrimination by a monopolistic wholesaler. Since this increase in social welfare does not accompany an increase in the total retail sales, the present model shows one of the extension models in which the well-known conclusion regarding third-degree price discrimination is not achieved.

Some extensions are worth mentioning. First, we could consider oligopolistic retailers (i.e., two retailers) and not duopolistic retailers (i.e., more than two retailers). Even in an oligopolistic retail market, third-degree price discrimination by a monopolistic wholesaler would achieve the same results as in the present paper. Second, because the demand function considered in the present paper was linear, the total demand was unaffected by price discrimination. However, when the demand function is nonlinear, the total demand is affected by price discrimination; therefore, social welfare trends would change. There may be cases wherein welfare improves with decreasing total output. Finally, we only assumed that a monopolistic manufacturer employs linear pricing. Thus, we need to consider non-linear pricing, too. Moreover, since the pricing scheme and price level may be determined by negotiations between the manufacturer and retailers, we need to consider this negotiations as well. These important and interesting topics are left for future research.

Appendix A: *Derivation of* w_u^*

Substituting p_H^* and p_L^* into (4),

$$\pi_M = \frac{1}{\theta(1-\theta)(4-\theta)}\{[\theta(2-\theta)(1-w_H-c_H)-\theta(\theta-w_L-c_L)](w_H-c_M)+ [-\theta(1-w_H-c_H)+(2-\theta)(\theta-w_L-c_L)](w_L-c_M)\}.$$

The first-order condition to maximize π_M with respect to $w^u = w_H = w_L$ is

$$\frac{\partial \pi_M}{\partial w^u} = \frac{\theta(1-2w^u-c_H+c_M)+2(\theta-2w^u-c_L+c_M)}{\theta(4-\theta)} = 0,$$

which gives w^{u*} as (8). Moreover, because we assume that $q_i|_{w_H=w_L=w^{u*}} > 0$ $(i = H, L)$, $\delta_H = 1-c_H-c_M$ and $\delta_L = \theta-c_L-c_M$ must satisfy that $[3\theta/(2+2\theta-\theta^2)]\delta_H < \delta_L < [(8-\theta-\theta^2)/6]\delta_H$, in which the second-order condition is satisfied.

Appendix B: *Proof of Lemma 1*

Because we assume that $q_i|_{w_H=w_H^{c*}(t),w_L=w_L^{c*}(t)} > 0$ $(i = H, L)$, $(2-\theta)(1-w_H^{c*}(t)-c_H) > (\theta-w_L^{c*}(t)-c_L)$ and $\theta(1-w_H^{c*}(t)-c_H) < (2-\theta)(\theta-w_L^{c*}(t)-c_L)$ must be satisfied.

Since constraint (11) is satisfied, we obtain

$$\begin{cases} \dfrac{\partial w_H^{c*}(t)}{\partial t} - \dfrac{\partial w_L^{c*}(t)}{\partial t} = 1, \quad \text{if } w_H^* > w_L^* \\ \dfrac{\partial w_L^{c*}(t)}{\partial t} - \dfrac{\partial w_H^{c*}(t)}{\partial t} = 1, \quad \text{if } w_H^* < w_L^* \end{cases}, \tag{17}$$

Under $q_i|_{w_H=w_H^{c*}(t),w_L=w_L^{c*}(t)} > 0$ $(i = H, L)$ and constraint (11), the wholesale prices satisfy the following condition.

$$\frac{\partial \pi_M(w_H^{c*}(t), w_H^{c*}(t))}{\partial w_H} + \frac{\partial \pi_M(w_H^{c*}(t), w_H^{c*}(t))}{\partial w_L} = 0. \tag{18}$$

Since (18) is satisfied for the proximity of $t = t'$ satisfying $q_i|_{w_H=w_H^{c*}(t),w_L=w_L^{c*}(t)} > 0$ $(i = H, L)$,

$$\frac{\partial}{\partial t}\left(\frac{\partial \pi_M(w_H^{c*}(t), w_H^{c*}(t))}{\partial w_H} + \frac{\partial \pi_M(w_H^{c*}(t), w_H^{c*}(t))}{\partial w_L}\right) = 0.$$

Since $q_i|_{w_H=w_H^{c*}(t),w_L=w_L^{c*}(t)} > 0$ $(i = H, L)$, using (17), we obtain

$$\frac{\partial \pi_M(\cdot)}{\partial w_H} + \frac{\partial \pi_M(\cdot)}{\partial w_L} = \frac{\theta(1-2w_H^{c*}(t)-c_H+c_M)+2(1-2w_L^{c*}(t)-c_L+c_M)}{\theta(4-\theta)},$$

and thus,

$$\frac{\partial}{\partial t}\left(\frac{\partial \pi_M(\cdot)}{\partial w_H} + \frac{\partial \pi_M(\cdot)}{\partial w_L}\right) = -\frac{2}{\theta(4-\theta)}\left(\theta\frac{\partial w_H^{c*}(t)}{\partial t} + 2\frac{\partial w_L^{c*}(t)}{\partial t}\right) = 0. \tag{19}$$

(17) and (19) give

$$\begin{cases} \dfrac{\partial w_H^{c*}(t)}{\partial t} = \dfrac{2}{2+\theta} \\ \dfrac{\partial w_L^{c*}(t)}{\partial t} = -\dfrac{\theta}{2+\theta} \end{cases}, \text{ if } w_H^* > w_L^*, \tag{20}$$

and

$$\begin{cases} \dfrac{\partial w_H^{c\,*}(t)}{\partial t} = -\dfrac{2}{2+\theta} \\ \dfrac{\partial w_L^{c\,*}(t)}{\partial t} = \dfrac{\theta}{2+\theta} \end{cases}, \text{ if } w_H^* < w_L^*. \tag{21}$$

Because $w_H^{c\,*}(|w_H^* - w_L^*|) = w_H^*$ and $w_L^{c\,*}(|w_H^* - w_L^*|) = w_L^*$, we have Lemma 1 by solving the differential equations in (20) and (21).

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