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A note on horizontal mergers in vertically related industries

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Abstract

We analyze horizontal mergers in vertically related industries. In a successive Cournot oligopoly model, we first compare the profitability of mergers in the upstream and in the downstream sectors. We characterize conditions on the concavities of the input supply function and the final demand function such that, ceteris paribus, an upstream merger is more protable than a downstream merger. We then provide a simple comparison of the relative losses of firms in an industry induced by a merger in the other sector when the degrees of concavity of the upstream and downstream demand functions are constant. We finally discuss the various mechanisms in action under non-constant degrees of concavity.

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1 Introduction

Horizontal mergers impact consumers, competitors and even firms of vertically related sectors (upstream suppliers or downstream customers). Besides the direct effect of mergers on the transfers between industries or between firms and consumers, the evolution of the market structure in an industry may impact that of the vertically related sectors, for instance by affecting the incentives to merge in these sectors.

Although this indirect effect is crucial for competition policy, very little literature has been devoted to assessing its impact. In the E.U. and the U.S., Competition Authorities generally consider a merger as less harmful when the demand stems from sufficiently concentrated firms: the underlying idea is that buyers' market power will translate into bargaining power towards their suppliers. A stream of literature has developed this theme since the seminal work of Galbraith (1952).[1] However, a more detailed analysis of the effect of the vertical position of firms on their incentives to merge and on the welfare consequences of mergers is necessary.

Some explanations have been suggested for possible differences in merger incentives at different levels of a vertical chain. Inderst and Wey (2003) explore the motivations for horizontal mergers in a bilateral duopoly when pricing behavior on the final good market is not affected (*i.e.* industry profits are invariant to the choice of market structure). By contrast, in a successive Cournot oligopoly with linear wholesale pricing, market concentration at each level affects demand and prices: when mergers induce size effects, Allain and Souam (2006) show that, *ceteris paribus*, a merger in one sector reduces the incentives to merge in the other sector. In a similar framework, but without size effects, Ziss (2005) shows that, when the upstream marginal cost and the degree of concavity of final demand are constant, the profitability of a horizontal merger is the same, *ceteris paribus*, in both sectors.

The aim of the present note is to generalize some results of Ziss (2005) and Allain and Souam (2006) with an elastic input supply function in the upstream market and an elastic final demand. Our first contribution is to compare the profitability of mergers in the two sectors. We characterize conditions on the concavities of the input supply function and the final demand function such that an upstream merger is more profitable than a downstream merger (for the same number of firms upstream and downstream, and the same number of merging firms). Our second contribution consists in analyzing the impact of a merger in one industry on the joint profit in the other industry. With an inelastic input supply function, Allain and Souam (2006) show that, *ceteris paribus*, the losses incurred by downstream firms due to a merger in the upstream sector are worse than the losses incurred by upstream firms due to a merger downstream. As a consequence, even though mergers are more profitable downstream than upstream, they also are more harmful to welfare when they occur among upstream firms. Here we extend this result when the input supply function is elastic and

[1]The impact of horizontal market structure on the bargaining power of the firms with their vertical partners has been studied in different frameworks, for instance Horn and Wolinsky (1988) with exclusive contracts, Fumagalli and Motta (2001) with two-part tariffs, von Ungern-Sternberg (1996) and Dobson and Waterson (1997) in a Nash bargaining framework and Inderst and Wey (2003) in a bilateral oligopoly. Ziss (2005) and Allain and Souam (2006) test the countervailing power hypothesis in a market framework.

show that in some cases upstream losses may become worse than downstream losses.

2 The model

We consider a successive oligopoly model where firms within each industry compete *à la Cournot*. A homogeneous good is produced by m upstream firms, which compete in quantities. These producers sell their good in an intermediate market to n downstream firms, which transform it into a final good that they sell to final consumers. The downstream firms also compete *à la Cournot*. Vertical restrictions are not allowed. The contract governing trade between the two stages of production is linear and the wholesale price is determined by the market clearing condition on the intermediate market. We assume that, at each level of the vertical chain, the firms only support supplying costs denoted w^U and w^D.

More precisely, the underlying game is the following.[2]

Stage 1: The upstream firms $j \in \{1, ..., m\}$ simultaneously commit to produce a quantity q_j^U of intermediate good. Upstream technology is as follows: each firm transforms one unit of input into one unit of intermediate good, the only cost is the input price $w^U(Q)$ (known by all). The upstream firms put their production on the intermediate market.

Stage 2: In the intermediate market, a *market maker* sets a (public) wholesale price w^D at which he buys the whole quantity produced by upstream firms, and at which he commits to supply the demand addressed by the downstream firms.

Stage 3: The downstream firms $i \in \{1, ..., n\}$ simultaneously express their demands q_i^D for the intermediate good. Transactions are done between the *market maker* and the downstream firms, which subsequently transform the good and sell it (simultaneously) to the final consumers. Downstream technology is as follows: each firm transforms one unit of intermediate good into one unit of output, the only cost is the input price w^D.

The objective functions of the players are the following. The firms maximize their profits and the *market maker* minimizes the absolute value of the difference between the supply and the demand of the intermediate good, i.e. $\underset{w}{Min}(\sum_{j=1}^{m} q_j^U - \sum_{i=1}^{n} q_i^D)^2$. We assume the *market maker* to be benevolent and not paid (as, for instance, the walrasian auctioneer). Finally, we assume that, when demand at the intermediate level exceeds supply, the *market maker* has to buy the excess on an external market not available to the downstream firms. This implies that the firms are never rationed.[3]

The downstream inverse demand function $P(Q)$ is twice continuously differentiable, and decreasing in the total quantity supplied, Q. Throughout the paper we denote f_x the partial derivative of function f w.r.t. x. We define $\gamma^D(Q) \equiv Q\dfrac{P_{QQ}(Q)}{P_Q(Q)}$ the degree of concavity of the downstream demand.

[2] Although this model is relatively standard in the literature on vertical relationships (cf. Salinger, 1988), the game and necessary assumptions are seldom formally presented.

[3] This point guarantees the existence and the uniqueness of the subgame perfect equilibrium of the whole game solved as usual by backward induction.

A new feature introduced by this paper is the assumption that the input price at the upstream level $w^U(Q)$ is a decreasing function of the total quantity of good sold to the upstream firms (i.e. $w_Q^U < 0$). We define $\gamma^U(Q) \equiv Q\dfrac{w_{QQ}^U(Q)}{w_Q^U(Q)}$ the degree of concavity of the upstream input supply function. This reflects the fact that the upstream firms may also be supplied by strategic firms in an imperfectly competitive industry.[4] Finally, we assume that $\gamma^D(Q) > -2$. This implies that the marginal revenue at the downstream level is decreasing and guarantees that the second order condition of the two-level Cournot model is verified for every market structure.[5]

The profit of downstream firm i is $\pi_i^D(m,n) = \left[P(Q) - w^D\right] q_i^D$ and that of upstream firm j is $\pi_j^U(m,n) = \left[w^D - w^U\right] q_j^U$. Note that each profit depends on the market structure on both the upstream and downstream markets.

We solve the game by backward induction. In stage 3 the F.O.C yields $P(Q) - w^D + P'(Q)q_i^D = 0$ for $i = 1, ..., n$. At the symmetric equilibrium, the wholesale price on the intermediate market is thus given by:

$$w^D(Q,n) = P(Q) + P'(Q)\frac{Q}{n}.$$

This condition implicitly defines the wholesale demand function faced by the upstream firms.

3 Comparing merger profitability in the upstream and downstream sectors

Our first aim is to compare the profitability of mergers at both levels of the vertical chain. As in Salant *et al.* (1983) or Farrell and Shapiro (1990), we consider that a merged firm is similar to its competitors: competition remains symmetric after a merger. A merger of s firms is thus said to be profitable if the *post merger* profit of the merged firm (weakly) exceeds the sum of the *pre merger* profits of the insiders, *i.e.* if and only if $\frac{\pi^D(m,n-s+1)}{\pi^D(m,n)} \geq s$, while a merger of s upstream firms is profitable if and only if $\frac{\pi^U(m-s+1,n)}{\pi^U(m,n)} \geq s$. We therefore define, for each merger in any sector, the profitability ratio as the ratio of the post-merger profit of the merged firm to the pre-merger profit of one insider. We compare these profit ratios in order to determine in which sector (upstream or downstream) the merger of s firms is more profitable. We assume that the number of firms in an industry can be treated as a continuous variable. The profit ratios may thus be written as follows:

$$\frac{\pi^D(m,n-s+1)}{\pi^D(m,n)} = \exp[\int_{n-s+1}^{n} -\frac{\pi_n^D(m,t)}{\pi^D(m,t)}dt], \tag{1}$$

$$\frac{\pi^U(m-s+1,n)}{\pi^U(m,n)} = \exp[\int_{m-s+1}^{m} -\frac{\pi_m^U(t,n)}{\pi^U(t,n)}dt]. \tag{2}$$

[4] We also consider the case $w_Q^U > 0$ and discuss how this assumption affects our results.

[5] *Cf.* Fauli-Oller [1997] and Ziss [2005].

Using the techniques developed by Fauli-Oller (1997) and Ziss (2005), we express these ratios as functions of the degree of concavity of the different demand functions (see Appendix A). We define the following measures of demand concavity:

- $\gamma^I(Q,n) \equiv \frac{w^D_{QQ}*Q}{w^D_Q} = \gamma^D(Q) + \frac{\gamma^{D\prime}(Q)Q}{n+1+\gamma^D(Q)}$ is the degree of concavity of the inverse intermediate demand function (or wholesale price) w^D.

- $\mu(Q,n) \equiv Q\frac{w^D_{QQ}-w^U_{QQ}}{w^D_Q-w^U_Q} = \frac{\gamma^I(Q,n)w^D_Q-\gamma^U(Q)w^U_Q}{w^D_Q-w^U_Q}$ represents the degree of concavity of the difference between the wholesale price paid by downstream firms and the input price paid by the upstream firms.

A simple case gives a first insight of the relative profitability of upstream and downstream mergers. Consider the symmetric case where the number of firms is initially the same in both sectors ($m = n$), and assume that the final demand and the input supply function have constant degrees of concavity ($\gamma^D(Q) = \gamma^D$ and $\gamma^U(Q) = \gamma^U$).[6]

Proposition 1 *Within the symmetric case ($m = n$) and under constant degrees of concavity at both levels, a merger of s downstream firms is more profitable than a merger of s upstream firms if and only if $\gamma^U \leq \gamma^D$.*

Proof. See Appendix B. ■

In this framework, comparing the ratios boils down to comparing the degree of concavity of the final demand, γ^D, with γ^U. When $\gamma^U \leq \gamma^D$, the downstream ratio is higher than the upstream one, thus a merger is more profitable and incentives to merge are higher downstream than upstream.[7]

In a framework where the downstream degree of concavity is constant and the upstream supply function is non-elastic ($\gamma^U = 0$), Ziss (2005) shows that the profitability of a merger of s firms is the same in both sectors provided that there are the same number of firms in the merging sector prior to merger. Proposition 1 extends this result to a framework where the input supply function has a constant elasticity.

In the following proposition, we give more general circumstances under which the simple comparison of the degrees of concavity at the upstream and downstream levels still can sign the difference in the profitability of mergers of s firms among n in the upstream and downstream sectors.

Proposition 2 *If the degree of concavity of downstream demand $\gamma^D(.)$ is increasing, $\gamma^U(Q) \leq \gamma^D(Q)$ is a sufficient condition for a downstream merger to be more profitable than an upstream one. By contrast if $\gamma^D(.)$ is decreasing, $\gamma^U(Q) \geq \gamma^D(Q)$ is a sufficient condition for an upstream merger to be more profitable than a downstream one.*

[6] Note that in this case, $\gamma^D(Q) = \gamma^I(Q,n) = \gamma^D$.

[7] If $w^U_Q > 0$, the results are reversed: $\mu(Q,n) \geq \gamma^D \Leftrightarrow \gamma^U \geq \gamma^D$. Note that if $w^U_Q = 0$, $\gamma^U = 0$ and the profitability is the same at both levels, as in Ziss (2005).

Proof. See Appendix C. ■

These two propositions show that the shape of the input supply function affects the merger incentives in a non trivial manner, depending on the degrees of concavity of the demand functions at different levels.

4 The effect of a merger on vertically related firms

In this section, we proceed further towards a welfare analysis of mergers in vertically related industries by comparing the profit losses induced by a merger of s firms at a level of the vertical chain (*e.g.* among usptream firms) on the firms at the other level (*e.g.* downstream). Ziss (2005) establishes that a horizontal merger at either level reduces total welfare. Taking size effects[8] into account, Allain and Souam (2006) show that, even though downstream mergers would be more profitable than upstream mergers of the same number of firms, the latter would be more harmful since the losses inflicted upon downstream firms by an upstream merger are larger than the losses of upstream firms following a downstream merger. This section aims to study the robustness of this result when the upstream input supply function is elastic. We thus compare the profit ratios $\frac{\pi^U(m,n-s+1)}{\pi^U(m,n)}$ and $\frac{\pi^D(m-s+1,n)}{\pi^D(m,n)}$. We define $R \equiv \frac{\frac{\pi_n^U}{\pi^U}}{\frac{\pi_m^D}{\pi^D}}$. When $m = n$, if R is larger (resp. lower) than 1, the profit losses are worse for the upstream (resp. downstream) firms (see Appendix D).

We first provide a simple condition under which downstream firms suffer more from an upstream merger than the reverse, in the framework of Ziss (2005). With $m = n$, assume that the input supply function is not elastic (i.e. $w_Q^U = 0$) and that the final demand has a constant degree of concavity (i.e. $\gamma^D = \gamma^I = \mu$).

Proposition 3 *With a non elastic input supply function, when the degree of concavity of downstream demand is constant and larger than -1 ($\gamma^D > -1$) and $m = n$, the reduction of profit of downstream firms following an upstream merger is worse than the reduction in upstream profits following a downstream merger involving the same number of firms.*

Proof. We have $-\frac{\pi_n^U}{\pi^U} = -\frac{\left(m+1+\gamma^D\right)}{n(n+1+\gamma^D)(m+1+\gamma^I)}$ and $-\frac{\pi_m^D}{\pi^D} = -\frac{2+\gamma^D}{m(m+1+\mu)}$. Thus, with $m = n$, $R = \frac{1}{2+\gamma^D}$, and $R < 1$ iff $\gamma^D > -1$. ■

We now extend these results to a more general framework. Consider first that the input supply function is elastic, with constant degrees of concavity ($\gamma^D = \gamma^I(Q,n)$ and γ^U) and $w_Q^U < 0$. We have:

$$R = \frac{w_Q^D+(n-1)w_Q^U}{w_Q^D-w_Q^U}\frac{n+1+\mu}{n+1+\gamma^D}\frac{1}{2+\gamma^D}. \tag{3}$$

R can be greater or smaller than 1. Note first that $\frac{n+1+\mu}{n+1+\gamma^D} < 1$ iff $\mu < \gamma^D$, which is equivalent in this case to $\gamma^U > \gamma^D$. Moreover, $\frac{1}{2+\gamma^D} < 1$ iff $\gamma^D > -1$. Besides, since $w_Q^U < 0$,

[8] By assuming that the merged firm is "larger" than the others as it has access to the combined productive capacity of the merger partners: see Perry and Porter (1985).

we have $\frac{w_Q^D+(n-1)w_Q^U}{w_Q^D-w_Q^U} > 1$: upstream margin in equilibrium is $\widetilde{w}^D - w^U = -\frac{Q}{m}\left(w_Q^D - w_Q^U\right) \geq 0$, thus $w_Q^D \leq w_Q^U < 0$ and $\frac{w_Q^D+(n-1)w_Q^U}{w_Q^D-w_Q^U} = 1 + \frac{nw_Q^U}{w_Q^D-w_Q^U} > 1$. Finally, with constant degrees of concavity, the elasticity of the input function tends to increase the ratio R. This makes the losses of the upstream firms relatively worse than those of the downstream firms.

This result stems from the fact that a change in upstream market structure does not have the same impact on the equilibrium wholesale price as a change in downstream market structure. Following Ziss (2005) we have:

$$\widetilde{w}_n^D = \frac{P'(Q)Q[\gamma^D(Q)-\gamma^I(Q,n)]}{n^2[m+1+\gamma^I(Q,n)]} = -\frac{P'(Q)Q^2\gamma^{D\prime}(Q)}{n^2[m+1+\gamma^I(Q,n)][n+1+\gamma^D(Q)]} = 0$$
$$\widetilde{w}_m^D = \frac{P'(Q)Q}{n}\frac{[n+1+\gamma^D(Q)]}{[m+1+\mu(Q,n)]} < 0.$$

Under constant degrees of concavity, a decrease in the number of downstream firms does not impact the equilibrium wholesale price but increases the input wholesale price, while a decrease in the number of upstream firms increases the equilibrium wholesale price and the retail price.

Indeed, a merger in the downstream market reduces downstream output, therefore the demand for intermediate good drops. This induces a decrease of upstream production, which in turn yields an increase of the upstream input price (for $w_Q^U < 0$). The upstream firms thus bear an additional loss because w^U increases as Q decreases. By contrast, the wholesale price $\widetilde{w}^D$ remains constant since $\widetilde{w}_n^D = 0$, and the downstream firms are not affected through this channel. This effect is strenghtened when $\mu > \gamma^D$ (which boils down to $\gamma^U < \gamma^D$).[9]

Finally, if the degree of concavity of the downstream demand function is not constant, and still assuming that $m = n$, the ratio is:

$$R = \frac{n+1+\mu}{n+1+\gamma^D}\frac{1}{2+\gamma^D}\left[\frac{w_Q^D+(n-1)w_Q^U}{w_Q^D-w_Q^U} + \frac{1}{w_Q^D-w_Q^U}\frac{P'(Q)}{n+1+\gamma^I}\gamma^{D\prime}(Q)Q\right].$$

This yields the following proposition.

Proposition 4 *An increasing degree of concavity of the final demand function enhances the ratio R. This makes the losses of upstream firms after a downstream merger relatively higher compared to the losses of downstream firms after an upstream merger. By contrast, a decreasing degree of concavity tends to reduce R and makes downstream losses worse.*

When the degree of concavity of the final demand function is not constant, a downstream merger has an impact on the wholesale equilibrium price. If the degree of concavity of final demand increases, a merger downstream induces a decrease in the wholesale price equilibrium (since $\widetilde{w}_n^D > 0$) that increases the profit loss of each upstream firm, through a reduction of

[9] Note that if $w_Q^U > 0$, $\frac{w_Q^D+(n-1)w_Q^U}{w_Q^D-w_Q^U} = 1 + \frac{nw_Q^U}{w_Q^D-w_Q^U} < 1$. In this case, downstream firms lose more than upstream firms when $\gamma^U \geq \gamma^D > -1$.

upstream margin (as w^U increases and $\widetilde{w}^D$ decreases). By contrast, a downstream firm is less impacted by an upstream merger since the increase of the equilibrium wholesale price is offset by the increase of the retail price.[10]

Finally, it is worth noting how these results extend to asymmetric configurations ($m \neq n$). When, *ceteris paribus*, the number of active firms in a sector increases (e.g. m), this tends to increase R and thus makes the upstream losses relatively higher after a downstream merger. Conversely, an increase in the number of downstream firms (n) worsens the situation of the downstream firms after an upstream merger.

5 Conclusion

This note compares the motivations for horizontal mergers at different levels of a vertical chain, and the effect of a merger on the profits of the other sector when the final demand function and the upstream input supply function are elastic. We show that a simple comparison of the degrees of concavity of the input supply function and the final demand function allows to sign the difference in the relative profitability of the mergers at both levels. The relative profitability of mergers at different levels of a vertical chain thus depends crucially on the elasticity of the input supply function. We also provide a simple comparison of the relative losses of firms at some level induced by a merger at the other level when the degrees of concavity are constant, and we discuss the various mechanisms in action under non constant degrees of concavity.

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[10] Conversely, a decreasing degree of concavity induces a decrease in the ratio R, making the downstream loss relatively higher than the upstream loss after a merger.

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7 Appendix

A. Determination of $\dfrac{\pi_n^D(m,t)}{\pi^D(m,t)}$ and $\dfrac{\pi_m^U(t,n)}{\pi^U(t,n)}$

- For any downstream firm i ($i \in \{1, ..., n\}$), the first order condition (FOC) is:

$$P(Q) - w^D + P'(Q)q_i^D = 0. \tag{4}$$

At the symmetric equilibrium, $q_i^D = \frac{Q}{n}$ and the intermediate demand function faced by upstream firms is thus:

$$w^D(Q,n) = P(Q) + P'(Q)\tfrac{Q}{n}$$

Therefore, as in Ziss (2005), the derivatives of w^D w.r.t Q and n are given by:

$$w_Q^D(Q,n) = \tfrac{P'(Q)}{n}[n+1+\gamma^D(Q)] < 0 \tag{5}$$

$$w_n^D(Q,n) = -\tfrac{QP'(Q)}{n^2} > 0 \tag{6}$$

$$w_{QQ}^D = \tfrac{P''(Q)[n+1+\gamma^D(Q)]+P'(Q)\gamma^{D\prime}(Q)}{n} \tag{7}$$

$$w_{Qn}^D(Q,n) = -\tfrac{P'(Q)[1+\gamma^D(Q)]}{n^2} \tag{8}$$

Deriving (4) w.r.t. n and rearranging thus yields:

$$\frac{Q_n}{Q} = \frac{1+\frac{\gamma^D(Q)-\gamma^I(Q,n)}{m+1+\gamma^I(Q,n)}}{n[n+1+\gamma^D(Q)]}, \tag{9}$$

- For any upstream firm j ($j \in \{1, ..., m\}$), the FOC is:

$$w^D + q_j^U w_Q^D(Q,n) - w^U - q_j^U w_Q^U = 0 \tag{10}$$

At the symmetric equilibrium, the total quantity $Q(m,n)$ is thus implicitly given by:

$$w^D(Q(m,n),n) - w^U(Q(m,n)) = \tfrac{Q(m,n)}{m}\left[w_Q^U(\tfrac{Q(m,n)}{m},n) - w_Q^D(Q(m,n),n)\right]. \qquad (11)$$

Derivating w.r.t. m yields:

$$\frac{Q_m}{Q} = \frac{1}{m}\frac{1}{m+1+Q\frac{w_{QQ}^D - w_{QQ}^U}{w_Q^D - w_Q^U}}.$$

- Let us denote $\widetilde{w}^D$ the equilibrium downstream wholesale price. The profit of a downstream firm at the equilibrium is $\pi^D = \left[P(Q(m,n)) - \widetilde{w}^D\right]\frac{Q(m,n)}{n}$. We then have $-\frac{\pi_n^D}{\pi^D} = -\frac{Q_n}{Q} + \frac{P'(Q)Q_n - \widetilde{w}_n^D}{QP'(Q)}n + \frac{1}{n}$
  Furthermore, deriving the downstream wholesale price at the equilibrium w.r.t. n yields:

$$\begin{aligned}\widetilde{w}_n^D &= w_Q^D Q_n + w_n^D = w_Q^D Q\frac{1+\frac{\gamma^D(Q)-\gamma^I(Q,n)}{m+1+\gamma^I(Q,n)}}{n[n+1+\gamma^D(Q)]} - \frac{QP'(Q)}{n^2}\\ &= \frac{P'(Q)Q[\gamma^D(Q)-\gamma^I(Q,n)]}{n^2[m+1+\gamma^I(Q,n)]}.\end{aligned}$$

  Rewriting and reintegrating (9) and (6) finally yields:

$$\begin{aligned}-\frac{\pi_n^D}{\pi^D} &= \frac{2n+\gamma^D(Q)-(2+\gamma^D(Q))\frac{\left[\gamma^D(Q)-\gamma^I(Q,n)\right]}{[m+1+\gamma^I(Q,n)]}}{n(n+1+\gamma^D(Q))}\\ &= \frac{2n+\gamma^D(Q)+(2+\gamma^D(Q))\frac{\gamma^{D\prime}(Q)Q}{[n+1+\gamma^D(Q)][m+1+\gamma^I(Q,n)]}}{n(n+1+\gamma^D(Q))}\end{aligned}$$

Similarly, $-\frac{\pi_m^U}{\pi^U} = (m-1)\frac{Q_m}{Q} + \frac{1}{m} = \frac{2m+\mu(Q,n)}{m[m+1+\mu(Q,n)]}$ where $\mu(Q,n) = Q\frac{w_{QQ}^D - w_{QQ}^U}{w_Q^D - w_Q^U}$.

B. Proof of proposition 1

Using the expressions of $-\frac{\pi_n^D}{\pi^D}$ and $-\frac{\pi_m^U}{\pi^U}$ derived in Appendix A and rewriting (1) and (2) yields:

$$\begin{aligned}\frac{\pi^D(m,n-s+1)}{\pi^D(m,n)} &= \exp[\int_{n-s+1}^{n}\frac{2n+\gamma^D}{n(n+1+\gamma^D)}dt],\\ \frac{\pi^U(m-s+1,n)}{\pi^U(m,n)} &= \exp[\int_{m-s+1}^{m}\frac{2n+\mu(Q,n)}{n[n+1+\mu(Q,n)]}dt].\end{aligned}$$

The ratio is higher in the downstream market if and only if $\gamma^D \leq \mu(Q,n)$. In equilibrium the upstream firms enjoy a nonnegative profit: (11) implies $\widetilde{w}^D - w^U = -\frac{Q}{m}\left(w_Q^D - w_Q^U\right) \geq 0$, thus $w_Q^D \leq w_Q^U < 0$. Therefore $\mu(Q,n) = \frac{\gamma^D w_Q^D - \gamma^U w_Q^U}{w_Q^D - w_Q^U} \geq \gamma^D \Leftrightarrow \gamma^U \leq \gamma^D$.

C. Proof of Proposition 2

Assume that $\gamma^{D\prime}(Q) < 0$. As $\mu(Q,n) > \gamma^D(Q)$ we have $w_Q^D\left[\gamma^I(Q,n) - \gamma^D(Q)\right] \leq w_Q^U\left[\gamma^U(Q) - \gamma^D(Q)\right]$, which is equivalent to $w_Q^D\left[\frac{\gamma^{D\prime}(Q)Q}{n+1+\gamma^D(Q)}\right] \leq w_Q^U\left[\gamma^U(Q) - \gamma^D(Q)\right]$. This implies that $\gamma^U(Q) < \gamma^D(Q)$. So when $\gamma^U(Q) \geq \gamma^D(Q)$, $\mu(Q,n) \leq \gamma^D(Q)$. The upstream ratio is then higher than the downstream one and an upstream merger is more profitable than a downstream one. Using the same argument, one can show that $\mu(Q,n) < \gamma^D(Q)$ implies that $\gamma^U(Q) > \gamma^D(Q)$, under an increasing degree of concavity.

When $w_Q^U > 0$, the results are changed as follows: $(\gamma^U(Q) \leq \gamma^D(Q))$ is a sufficient condition under which an upstream merger is more profitable than a downstream one under a decreasing degree of concavity ; $(\gamma^U(Q) \geq \gamma^D(Q))$ is a sufficient condition under which a downstream merger is more profitable than an upstream one under an increasing degree of concavity.

D. Determination of $\frac{\pi^U(m,n-s+1)}{\pi^U(m,n)}$ and $\frac{\pi^D(m-s+1,n)}{\pi^D(m,n)}$

We have:

$$
\begin{aligned}
\frac{\pi^U(m,n-s+1)}{\pi^U(m,n)} &= \exp[\int_{n-s+1}^{n} -\frac{\pi_n^U(m,t)}{\pi^U(m,t)}dt] \\
\frac{\pi^D(m-s+1,n)}{\pi^D(m,n)} &= \exp[\int_{m-s+1}^{m} -\frac{\pi_m^D(t,n)}{\pi^D(t,n)}dt]
\end{aligned}
$$

Omitting the arguments for simplicity, we have $\frac{\pi_n^U}{\pi^U} = \frac{Q_n}{Q} + \frac{\widetilde{w}_n^D + \left(w_Q^D - w_Q^U\right)Q_n}{\widetilde{w}^D - w^U}$. Using the fact that at the equilibrium $\widetilde{w}^D - w^U = -\frac{Q}{m}\left(w_Q^D - w_Q^U\right)$, we get: $\frac{\pi_n^U}{\pi^U} = (1-m)\frac{Q_n}{Q} + \frac{\widetilde{w}_n^D}{\widetilde{w}^D - w^U}$. Since $\widetilde{w}_n^D = \frac{P'(Q)Q[\gamma^D(Q) - \gamma^I(Q,n)]}{n^2[m+1+\gamma^I(Q,n)]}$, we have

$\frac{\pi_n^U}{\pi^U} = \frac{\left(m+1+\gamma^D\right)}{n(n+1+\gamma^D)(m+1+\gamma^I)}\frac{w_Q^D + (m-1)w_Q^U}{w_Q^D - w_Q^U} - \frac{m}{n^2}\frac{1}{w_Q^D - w_Q^U}\frac{P'(Q)\left(\gamma^D - \gamma^I\right)}{m+1+\mu^D}$.

The same method yields $\frac{\pi_m^D}{\pi^D} = \frac{2+\gamma^D}{m(m+1+\mu)}$.