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Bankruptcy Risk, Limited Liability and Imperfectly Enforced Emissions Taxes

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Abstract

Under reasonable conditions, noncompliance with an emissions tax has no effect on environmental outcomes or the efficient allocation of individual emissions control. Moreover, differences in individual tax violations are independent of firm-level differences. All of these desirable characteristics disappear when some firms under an emissions tax risk bankruptcy. The combination of imperfect enforcement, bankruptcy risk, and limited liability in bankrupt states produces an inefficient distribution of emissions control, higher aggregate emissions, and makes individual violations dependent on firm-level characteristics.

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1. Introduction

The most important reason for implementing an emissions tax is that it will induce the distribution of emissions control among pollution sources that minimizes their aggregate costs of achieving an aggregate emissions target. Moreover, under reasonable conditions the allocative efficiency of an emissions tax holds even when enforcement is imperfect in the sense that it is not stringent enough to keep firms from evading some portion of their tax liabilities. The reason is that firms choose their emissions to equate the tax to their marginal emissions control costs whether they are compliant or not (Harford 1978 and 1987; Sandmo 2002). Consequently, their emissions choices are independent of their compliance decisions. This implies further that aggregate emissions are independent of whether enforcement of a tax induces full or partial compliance. Thus, the environmental outcome of an emissions tax does not depend on it being perfectly enforced. Of course, firms' levels of tax evasion depend on the stringency of enforcement strategy they face, but these choices are independent of their exogenous characteristics like production and abatement technologies, and input and output choices. This is important because it frees regulators from having to gather hard-to-obtain firm-level information to target their monitoring effort.[1] Moreover, it implies that the costs of sanctioning firms are independent of this information.

We examine the performance of an emissions tax when some sources of pollution risk bankruptcy and the tax is imperfectly enforced. With the continuing application of price-based environmental policies, it is certain that regulators will confront, or have already confronted, control situations that are difficult to enforce and that involve financially distressed firms. Emissions taxes, in particular, can impose a heavier financial burden on regulated firms than other control policies like emissions trading schemes or emissions standards. Knowledge of how the financial health of regulated firms can impact the performance of an imperfectly enforced emissions tax is an important consideration in the design and evaluation of these policies.

We demonstrate that the desirable characteristics of an imperfectly enforced emissions tax disappear when some firms risk bankruptcy. In this case an emissions tax will fail to allocate individual emissions control efficiently. Thus, the main reason for implementing emissions taxes does not hold when some pollution sources are financially insecure. Moreover, firms that risk bankruptcy choose higher emissions when they are noncompliant than when they are compliant. Consequently, imperfect enforcement has a negative environmental consequence when some firms risk insolvency that is not present when all firms are financially secure. Finally, financially insecure firms choose higher violations than financially secure firms. Thus, the financial health of firms is an important element in the allocation of scarce enforcement resources. The key factor that produces these negative results is the well-known limited liability effect of debt financing—financially insecure firms ignore returns in bankrupt states because debt holders become the residual claimants. Thus, they make their decisions by optimizing only over states in which they are solvent, which distorts their choices away from those they would make if they were financially secure (Brander and Lewis 1986).

We are not the first to demonstrate that the financial health of firms and limited liability can impact the performance of regulatory designs, including environmental policies. Spiegel and Spulber (1994) investigate the interactions between the investment and financial decisions of firms and a regulator's control of their output price. Damania (2000) explores the link between pollution taxes and the financial and output decisions of firms in an oligopolistic industry, but he

[1] Malik (1990) and Stranlund and Dhanda (1999) find similar results in the context of emissions trading. Murphy and Stranlund (2006 and 2007) find strong support for these hypotheses in laboratory emissions trading experiments.

does not allow firms to be noncompliant. Similarly, we have noted that bankruptcy risk and the limited liability effect can undermine the efficiency of emissions trading in Stranlund and Zhang (2008), but we did so under the assumption that firms are perfectly compliant. Damania and Bulte (2005) relate the harvest decisions of firms in a fishery to their financial structure and imperfectly enforced regulatory controls, but they focus on fixed harvest quotas. Thus, to our knowledge we are the first to examine how the combined roles of bankruptcy risk, limited liability, and imperfect enforcement affect the performance of emissions taxes.

2. A Model of an Indebted Firm under an Imperfectly Enforced Emissions Tax

Throughout we consider an industry composed of heterogeneous, risk neutral firms whose emissions are controlled by a uniform emissions tax. Enforcement of the tax is imperfect in the sense that it is not sufficient to keep firms from attempting to evade a portion of their tax liabilities. Assume that each firm is controlled by shareholders, so the manager of each firm seeks to maximize the expected value of the firm.

For a particular firm in the industry, given its optimal choices of inputs and outputs, the gross profit of the firm (profit excluding its tax and penalty payments) is $\pi(e,\beta)(1+z)$, where e is the firm's emissions, β is an exogenous factor that affects the firm's gross profit, and z is a continuous random variable that is independently, but not necessarily identically distributed among the firms in the industry. Each firm's gross profit function is strictly increasing and strictly concave in the firm's emissions. The random variable z captures the effects of uncertainty on the firm's gross profit, such as the effects of random shifts in the demand for its output or in factor prices. The probability density function of z is $g(z)$ with support $[\underline{z},\overline{z}]$. The expectation of z is zero so that the firm's expected gross profit is simply $\pi(e,\beta)$. The value of z is revealed only after the firm has made its emissions and compliance decisions.

Each firm's reported emissions, r, are taxed at rate t. To check whether the firms report their true emission, each of them is audited with a constant probability α that is common knowledge between the regulator and the firm. An audit reveals a firm's actual emissions without errors. A firm is in violation if its actual emissions exceed its reported emissions. Since we are concerned with the combined roles of financial insecurity and noncompliance in this paper, we limit our analysis to situations in which firms' violations are positive. If an audit reveals that a firm is in violation, a penalty $f(e-r)$ is imposed. The penalty function is the same for all firms, and it is positive, strictly increasing, and strictly convex for positive violations.

Like Brander and Lewis (1986), Damania (2000), Damania and Bulte (2005), and Stranlund and Zhang (2008), we focus the analysis on a single period in which the financial structure of the firm is fixed. A firm's financial structure is summarized by two variables; one is the firm's equity, A, and the other is the firm's debt obligation, D. The firm reimburses creditors from net profits. If the firm's losses exceed its equity, it will declare bankruptcy, shut down, and use its equity to partially pay off its debt. Apart from losing its equity, there are no other costs associated with declaring bankruptcy.

Given a realization of z, the payoff to the shareholders of the firm is

$$\pi(e,\beta)(1+z)-tr-f(e-r)-D+A, \qquad (1)$$

if it is audited by the regulator, and the payoff is

$$\pi(e,\beta)(1+z)-tr-D+A, \qquad (2)$$

if the firm is not audited. From (1) and (2) define two critical breakeven states of the random variable z at which the firm is indifferent between staying in business and ceasing production. If the firm is audited, the breakeven value of z, denoted as z^a, is determined by setting (1) equal to zero and solving for z, yielding

$$z^a = \frac{tr + f(e-r) + D - A}{\pi(e,\beta)} - 1. \tag{3}$$

The breakeven value of z when the firm is not audited is denoted z^{na}, which is determined by setting (2) equal to zero and solving for z:

$$z^{na} = \frac{tr + D - A}{\pi(e,\beta)} - 1. \tag{4}$$

Note that $z^{na} < z^a$ when the firm is noncompliant (i.e., $e - r > 0$), and $z^{na} = z^a$ when the firm is compliant ($e = r$). If the realized value of z is greater than both z^a and z^{na} the firm will be solvent whether it is audited or not, but if the realized value of z is less than both z^a and z^{na} the firm will be insolvent regardless of monitoring. If the firm is noncompliant and the realized value of z is between z^a and z^{na}, then the firm remains solvent if it is not audited, but is bankrupt if it is audited.

Note that $\int_{z^a}^{\bar{z}} g(z)dz$ and $\int_{z^{na}}^{\bar{z}} g(z)dz$ are the probabilities the firm stays in business when it is audited and when it is not, respectively. Clearly, these probabilities decrease with z^a and z^{na}. Thus, if $z^{na} \le z^a \le \underline{z}$, then $\int_{z^a}^{\bar{z}} g(z)dz = \int_{z^{na}}^{\bar{z}} g(z)dz = 1$, indicating that the firm is financially secure in the sense that it does not risk bankruptcy. However, at the other end of the range of z, if $\bar{z} \le z^{na} \le z^a$, then $\int_{z^a}^{\bar{z}} g(z)dz = \int_{z^{na}}^{\bar{z}} g(z)dz = 0$ and the firm will definitely go bankrupt. In this case it will not even bother to begin production. Despite this possibility, and the possibility that a firm will certainly be insolvent if it is audited but may not be if it is not audited, we simplify our analysis by assuming that the probabilities the firm is solvent are strictly greater than zero. This requires $z^{na} \le z^a < \bar{z}$.

We are now ready to specify the decision problem of the manager of a firm. Recall that a manager is risk neutral and chooses his or her firm's emissions and emissions report to maximize the expected value of the firm. Assuming that the manager chooses positive emissions, emissions report, and violation, his or her decision problem is to choose $e > 0$ and $r > 0$ to maximize

$$\begin{aligned} V = \alpha &\left\{ \int_{z^a}^{\bar{z}} \left[\pi(e,\beta)(1+z) - tr - f(e-r) - D + A \right] g(z)dz \right\} \\ &+ (1-\alpha) \left\{ \int_{z^{na}}^{\bar{z}} \left[\pi(e,\beta)(1+z) - tr - D + A \right] g(z)dz \right\}. \end{aligned} \tag{5}$$

The firm's expected value function consists of two parts; the firm's expected value given that it is audited multiplied by the probability of an audit (the top line of V) plus the expected value of the firm given that it is not audited times the probability that it is not audited (the bottom line of V). Equity holders of the firm earn a return when the firm does not go bankrupt, but are left empty-handed when the firm is insolvent. Thus, shareholder-controlled financially insecure firms ignore states of bankruptcy when making decisions, because the payoff in these states is zero. This is why the value function V does not include terms for $z < z^a$ and $z < z^{na}$. This is the essence of the limited liability effect, which has important consequences for the choices of a firm that operates under an emissions tax.

To determine these choices we assume throughout that V is strictly concave in e and r. Using the facts that z^a and z^{na} are determined from $\pi(e,\beta)(1+z^a)-tr-f(e-r)-D+A=0$ and $\pi(e,\beta)(1+z^{na})-tr-D+A=0$, respectively, the following first-order conditions are both necessary and sufficient to identify a firm's optimal emissions and emissions report:

$$V_e=\pi_e(e,\beta)\left[\alpha\int_{z^a}^{\bar{z}}(1+z)g(z)dz+(1-\alpha)\int_{z^{na}}^{\bar{z}}(1+z)g(z)dz\right]-\alpha f'(e-r)\int_{z^a}^{\bar{z}}g(z)dz=0; \qquad (6)$$

$$V_r=\alpha f'(e-r)\int_{z^a}^{\bar{z}}g(z)dz-t\left[\alpha\int_{z^a}^{\bar{z}}g(z)dz+(1-\alpha)\int_{z^{na}}^{\bar{z}}g(z)dz\right]=0. \qquad (7)$$

Now combine equations (6) and (7) to obtain

$$\begin{aligned}&\left[\pi_e(e,\beta)-t\right]\left[\alpha\int_{z^a}^{\bar{z}}g(z)dz+(1-\alpha)\int_{z^{na}}^{\bar{z}}g(z)dz\right]\\&\qquad+\pi_e(e,\beta)\left[\alpha\int_{z^a}^{\bar{z}}zg(z)dz+(1-\alpha)\int_{z^{na}}^{\bar{z}}zg(z)dz\right]=0.\end{aligned} \qquad (8)$$

Our analysis of the effects of bankruptcy risk on imperfectly enforced emissions taxes is based on equations (7) and (8).

3. Imperfectly Enforced Emissions Taxes When Firms Are Financially Secure

We first use the model to review some fundamental conclusions about imperfectly enforced emissions taxes when firms do not risk bankruptcy. There are no new results in this section; some have been shown directly by Harford (1978 and 1987) and Sandmo (2002), while others can be gleaned from the works of Malik (1990) and Stranlund and Dhanda (1999) who focused on emissions trading. We present all of the results in a single proposition. Like all the proofs of this paper we have relegated the proof of Proposition 1 to the appendix.

Proposition 1: If no firm that operates under an emissions tax risks bankruptcy, then:
(1) The allocation of individual emissions control is efficient despite imperfect enforcement.
(2) Each firm's choice of emissions is independent of their compliance decision.
(3) Aggregate emissions are unaffected by imperfect enforcement.
(4) Firms' violations are independent of their exogenous characteristics.

The key to these results is that a noncompliant financially secure firm keys its emissions and violation decisions directly to the emissions tax by satisfying the three-way equality, $t=\pi_e(e,\beta)=\alpha f'(e-r)$. As is standard, allocative efficiency means that the distribution of emissions maximizes expected aggregate gross profit at the level of aggregate emissions control that is achieved by the tax.[2] This requires that individual expected marginal gross profits are equal. This outcome is achieved when all firms are financially secure because each of them chooses their emissions so that $t=\pi_e(e,\beta)$. Moreover, this condition holds whether a firm is compliant or not, indicating that a firm's optimal choice of emissions does not depend on whether the emissions tax is enforced perfectly or imperfectly. Consequently, industry emissions are not affected by imperfect enforcement. Lastly, that a financially secure firm chooses its violation to solve $t=\alpha f'(e-r)$ implies that its optimal violation is a function of the tax and the

[2] Since abatement costs are usually thought of as the reduction in a firm's profit from reducing its emissions, maximizing industry gross profit given some level of aggregate emissions control is fully equivalent to minimizing the aggregate abatement costs of reaching this level of control.

enforcement parameters, but does not depend on the exogenous parameters of its profit function. Thus, firms' violations are independent of differences in their exogenous characteristics, suggesting that a regulator finds no value in targeting its enforcement effort.

Before we move to examining how these results change when some firms in an industry risk bankruptcy, we should be clear about how the results depend on two assumptions we maintain throughout this paper. The first is that each firm submits a positive emissions report. It is straightforward to show that a financially secure firm that reports zero emissions chooses its actual emissions so that $t \geq \pi_e(e, \beta)$. If this inequality is strict for some firms, then the expected marginal gross profits of the firms will not be equalized and aggregate expected gross profits will not be maximized. The possibility that a tax regulation will be so poorly enforced that some firms report zero emissions seems rather remote. One may also wonder whether a real firm would ever reports zero emissions, given that this would send such an obvious signal of noncompliance to the regulator.

The other assumption that is necessary for Proposition 1 is that the probability a firm will be monitored does not depend on its emissions or emissions report. If firms are monitored with probability $\alpha(e, r)$ and $\alpha_e \neq -\alpha_r$, they will choose their emissions so that their expected marginal gross profits differ from the tax. This will cause expected aggregate gross profit to be less than maximum. For this reason we do not examine such a monitoring strategy in this paper.

4. Imperfectly Enforced Emissions Taxes When Firms Are Financially Insecure

In this section we demonstrate how the results in Proposition 1 are modified in the presence of financially insecure firms. We begin with the allocative efficiency of an emission tax when some firms in an industry risk bankruptcy.

Proposition 2: If some firms under an emissions tax risk bankruptcy the distribution of individual emissions control will not be efficient.

The allocative inefficiency of an emissions tax when some firms in an industry risk bankruptcy is due to the fact that these firms will choose emissions so that their expected marginal gross profits are less than the tax $\left(\text{i.e., } t > \pi_e(e, \beta)\right)$, while financially secure firms choose their emissions so that their expected marginal gross profits are equal to the tax $\left(t = \pi_e(e, \beta)\right)$. This outcome is inefficient because allocative efficiency requires that all firms' expected marginal gross profits be equal. Not only will the expected marginal gross profits of firms that risk bankruptcy differ from those of firms that are financially secure, expected marginal gross profits among financially distressed firms will likely differ because the values of z^a and z^{na} vary among these firms, and the densities $g(z)$ may vary as well. Since the emissions tax will not equate the firms' expected marginal gross profits, expected industry gross profit will not be maximized. Thus, the main reason for implementing emissions taxes does not hold in situations involving financially insecure firms.

The result that firms that risk bankruptcy choose their emissions so that their expected marginal gross profits are less than the tax implies that they choose higher emissions than if they were financially secure. This is due to the limited liability effect—since firms that risk bankruptcy do not consider bankrupt states in their decisions, they optimize only over those states in which they will be solvent. Optimization over this restricted range of the random variable z causes them to choose higher emissions.

It is important to note that this result of allocative inefficiency holds whether financially insecure firms are also noncompliant or not. In fact, imperfect enforcement causes financially insecure firms to choose even higher levels of emissions.

Proposition 3: A firm that risks bankruptcy will choose higher emissions if an emissions tax is imperfectly enforced than if it is perfectly enforced.

Proposition 4 follows directly from Proposition 3.

Proposition 4: Imperfect enforcement of an emissions tax leads to higher aggregate emissions when some firms under the tax risk bankruptcy.

Recall from Proposition 1 that, under reasonable circumstances, imperfect enforcement has no effect on the emissions of financially secure firms. Thus, a regulator does not need to be concerned about the environmental impacts of imperfect enforcement. However, Proposition 4 reveals that this result does not hold when an emissions tax is applied to firms that risk insolvency—imperfect enforcement weakens the ability of an emissions tax to improve environmental outcomes when it is applied in a situation involving financially insecure firms.

Bankruptcy risk and imperfect enforcement also makes the allocation of enforcement effort more complicated. Part (4) of Proposition 1 reveals that differences in the violations of financially secure firms are independent of differences in their exogenous characteristics. Thus, a regulator does not need to gather information about individual firms to target its enforcement effort—only the tax and the enforcement parameters, all of which are known by the regulator, determine a firm's violations. This is no longer true when some firms risk insolvency. In particular, the violations of financially insecure firms will differ from the violations of financially secure firms.

Proposition 5: Noncompliant firms that risk bankruptcy choose higher violations than if they were financially secure.

Armed with Proposition 5, a regulator who is motivated to use its resources to detect and punish firms that tend toward higher violations may wish to target its enforcement effort at financially insecure firms. Doing so, of course, requires the regulator to gather information on the financial health of all firms. At best, gathering this information will add to the cost of enforcing an emissions tax. Moreover, the higher aggregate violations that are produced by bankruptcy risk and imperfect enforcement may lead to higher costs of sanctioning noncompliant firms. Thus, bankruptcy risk places more pressure on scarce enforcement resources.

Note that Proposition 5 does not address part (4) of Proposition 1 directly. There we showed the independence of parametric differences in firms' profit functions on differences in their violations. This independence does not hold when firms risk bankruptcy. Moreover, the comparative static relating a parametric change in a financially insecure firm's profit to its violation choice is generally a complicated function with an indeterminate sign. To see these points, let us simplify the problem by assuming that a firm only risks bankruptcy if it is audited. Under this assumption, we derive the following in the appendix:

$$sign\left[\partial\left(e-r\right)/\partial\beta\right] = sign\left[\pi_{e\beta}\left(t-\pi_e(1+z^a)\right)+\pi_{ee}\pi_\beta\left(1+z^a\right)\right]. \tag{9}$$

Let us make the reasonable assumptions that $\pi_\beta > 0,\ \pi_{e\beta} > 0,$ and $\pi_{ee} < 0.$ Moreover, recall from the discussion following Proposition 2 that $t > \pi_e$. Under these conditions, the sign of $\partial(e-r)/\partial\beta$ is indeterminate because of variation in the sign and the magnitude of z^a. For example, if $z^a \le -1$, then $\partial(e-r)/\partial\beta < 0$. However, it is possible that $\partial(e-r)/\partial\beta > 0$ if z^a is a larger enough positive number.

Therefore, unlike financially secure firms, it is clear that a financially insecure firm's violation is dependent on exogenous elements that affect its gross profit function. However, the exact nature of this relationship depends on all the parameters of the firm's decision problem, including the enforcement strategy it faces, its financial structure, and the uncertainty about its profit. Since some of these factors involve hidden information, a regulator will have a difficult time targeting its enforcement effort based on information that determines a firm's profit function.

5. Conclusions

We have examined the combined roles of bankruptcy risk, limited liability, and imperfect enforcement on the performance of an emissions tax. In the absence of bankruptcy risk in a population of regulated firms, emissions taxes retain their beneficial characteristics even when they are not enforced perfectly. Under reasonable conditions, an imperfectly enforced emissions tax produces an efficient allocation of individual emissions control, the aggregate level of emissions control is the same as under a perfectly enforced tax, and differences in individual violations are independent of firm-level differences. All of these characteristics disappear when some firms risk bankruptcy: the allocation of emissions control is inefficient, imperfect enforcement causes higher aggregate emissions, and financially insecure firms choose higher violations. Thus, the combined effects of bankruptcy risk and imperfect enforcement produce higher expected aggregate costs of emissions control, worse environmental quality, and more pressure on scarce enforcement resources.

Regulatory options to limit these losses are probably limited to options that reduce the risks of bankruptcy among financially insecure firms. One option is to provide direct subsidies to distressed firms (Damania and Bulte 2006). This option is fraught with difficulties, including the political difficulty of subsidizing polluting firms, and the moral hazard problem that would surely result because firms would have an incentive to exaggerate their risk of bankruptcy to obtain the subsidy. A more reasonable option might be to allow firms to pollute up to a certain level for free before the tax kicks in. This would reduce firms' tax payments, thereby reducing the bankruptcy risk of financially insecure firms, and may lead to more efficient outcomes.

It may also be possible to use tax rates and enforcement stringency to achieve more efficient outcomes. However, determining how this can be done requires determining the comparative statics of how changes in the tax, monitoring, and penalties affect firms' choices of emissions and violations. In our model these comparative statics always have indeterminate signs. Moreover, these results depend on the private information of firms, including information about their profit functions and their financial health. Thus, even though one can imagine that regulators could minimize the inefficiencies associated with bankruptcy risk with judicious choices of tax rates and enforcement strategies, the information requirements of doing so are quite severe.

Appendix

Proof of Proposition 1: If a firm does not risk bankruptcy then $z^{na} \le z^a \le \underline{z}$, which implies

$$\int_{z^a}^{\overline{z}} g(z)dz = \int_{z^{na}}^{\overline{z}} g(z)dz = 1 \text{ and } \int_{z^a}^{\overline{z}} zg(z)dz = \int_{z^{na}}^{\overline{z}} zg(z)dz = 0.$$

The latter relationships are due to our assumption that the expectation of z is equal to zero. Now substitute these into equations (7) and (8) to obtain $t = \alpha f'(e-r)$ and $\pi_e(e,\beta) = t$, respectively. Allocative efficiency requires that the industry's expected gross profit be maximized given the level of aggregate emissions that is induced by the emissions tax. As is well-known the necessary conditions for this maximization problem imply that the firms' marginal expected gross profits are equal. This is achieved because $\pi_e(e,\beta) = t$ for every firm, each firm faces the same emissions tax, and hence, $\pi_e(e,\beta)$ is equal for every firm. This proves part (1) of the proposition. To prove part (2) note that a firm's choice of emissions is $e(t,\beta)$, the implicit solution to $\pi_e(e,\beta) = t$. Since this decision does not depend on monitoring or penalties, it is independent of the firm's compliance decision. Part (3) follows directly from part (2): if individual firms' emissions are unaffected by imperfect enforcement, aggregate emissions are unaffected as well. To prove part (4), write the firm's optimal emissions report as $r(\beta)$. (Writing the firm's report in this way is not meant to suggest that it only depends on β; it also depends on the emissions tax, monitoring, and the penalty function). Given the firm's optimal choice of emissions, $e(t,\beta)$, and $t = \alpha f'(e-r)$ we have $t - \alpha f'(e(t,\beta) - r(\beta)) \equiv 0$. Differentiate this with respect to β to obtain $-\alpha f''(\cdot)(e_\beta - r_\beta) = 0,$ which implies that the marginal effect on the firm's violation of a change in β is $e_\beta - r_\beta = 0.$ This indicates that the firm's choice of violation is independent of β, implying further that differences in the individual violations of financially secure firms are independent of their exogenous differences. The proof is complete. QED.

Proof of Proposition 2: Rearrange equation (8) to obtain

$$\frac{-\left[\pi_e(e,\beta) - t\right]}{\pi_e(e,\beta)} = \frac{\alpha \int_{z^a}^{\overline{z}} zg(z)dz + (1-\alpha)\int_{z^{na}}^{\overline{z}} zg(z)dz}{\alpha \int_{z^a}^{\overline{z}} g(z)dz + (1-\alpha)\int_{z^{na}}^{\overline{z}} g(z)dz}. \tag{A.1}$$

On the right hand side of (A.1), the denominator is positive because $\int_{z^a}^{\overline{z}} g(z)dz > 0$ and $\int_{z^{na}}^{\overline{z}} g(z)dz > 0$. The numerator is also positive. To understand why, recall that the expectation of z is zero so that $\int_{\underline{z}}^{\overline{z}} zg(z)dz = 0$. Since the firm risks bankruptcy, at least when it is audited, $\underline{z} < z^a$, implying $\int_{z^a}^{\overline{z}} zg(z)dz > 0$. If the firm is definitely solvent when it is not audited, $z^{na} \le \underline{z}$ and $\int_{z^{na}}^{\overline{z}} zg(z)dz = 0$. If the firm risks insolvency when it is not audited , $\underline{z} < z^{na}$ and $\int_{z^{na}}^{\overline{z}} zg(z)dz > 0$. Since $\int_{z^a}^{\overline{z}} zg(z)dz > 0$ and $\int_{z^{na}}^{\overline{z}} zg(z)dz \ge 0$, the numerator of the right side of (A.1) is strictly positive, implying further that the entire right side of (A.1) is positive.

Given that the right side of (A.1) is positive, the equality holds if and only if the left side is positive as well. Note that this will be true if and only if $\pi_e(e,\beta)>0$ and $\pi_e(e,\beta)<t$. Recall that allocative efficiency requires that each firm choose its emissions so that its expected marginal gross profit is equal to the tax. Since financially insecure firms choose their emissions so that $\pi_e(e,\beta)<t$ while those that are financially secure choose their emissions so that $\pi_e(e,\beta)=t$, the distribution of emissions in an industry that contains financially insecure firms will not be efficient. QED.

Proof of Proposition 3: When an emissions tax is perfectly enforced, a firm truthfully reports its emission so that $e=r$. From (3) and (4), compliance implies $z^a=z^{na}$. Under this condition the firm's expected value function (5) simplifies to

$$V\left(e,z^a=z^{na}\right)=\int_{z^{na}}^{\overline{z}}\left[\pi(e,\beta)(1+z)-te-D+A\right]g(z)dz,$$

and its optimal choice of emissions satisfies

$$V_e\left(e,z^a=z^{na}\right)=\left[\pi_e(e,\beta)-t\right]\int_{z^{na}}^{\overline{z}}g(z)dz+\pi_e(e,\beta)\int_{z^{na}}^{\overline{z}}zg(z)dz=0. \tag{A.2}$$

Suppose on the other hand that the emissions tax is imperfectly enforced so that the firm is noncompliant. Suppose further that when the firm is noncompliant it chooses emissions $\overline{e}$ to satisfy equation (8). The proof of the proposition is based on evaluating the sign of $V_e\left(e,z^a=z^{na}\right)$ at $\overline{e}$ and using the strict concavity of $V\left(e,z^a=z^{na}\right)$ in e to show that the firm's choice of emissions when it is compliant is less than $\overline{e}$.

Using (A.2), $V_e\left(e,z^a=z^{na}\right)$ evaluated at $\overline{e}$ is

$$V_e\left(\overline{e},z^a=z^{na}\right)=\left[\pi_e(\overline{e},\beta)-t\right]\left[\int_{z^{na}(\overline{e})}^{\overline{z}}g(z)dz\right]+\pi_e(\overline{e},\beta)\left[\int_{z^{na}(\overline{e})}^{\overline{z}}zg(z)dz\right]. \tag{A.3}$$

That $\overline{e}$ satisfies equation (8) allows us to write it as the identity

$$\begin{aligned}&\left[\pi_e(\overline{e},\beta)-t\right]\left[\alpha\int_{z^a(\overline{e})}^{\overline{z}}g(z)dz+(1-\alpha)\int_{z^{na}(\overline{e})}^{\overline{z}}g(z)dz\right]\\&\qquad+\pi_e(\overline{e},\beta)\left[\alpha\int_{z^a(\overline{e})}^{\overline{z}}zg(z)dz+(1-\alpha)\int_{z^{na}(\overline{e})}^{\overline{z}}zg(z)dz\right]\equiv 0.\end{aligned} \tag{A.4}$$

Use (A.4) to substitute for $\pi_e(\overline{e},\beta)-t$ in (A.3) and rearrange terms to show that

$$V_e\left(\overline{e},z^a=z^{na}\right)=\pi_e(\overline{e},\beta)\frac{\alpha\left[\int_{z^{na}(\overline{e})}^{\overline{z}}zg(z)dz\int_{z^a(\overline{e})}^{\overline{z}}g(z)dz-\int_{z^{na}(\overline{e})}^{\overline{z}}g(z)dz\int_{z^a(\overline{e})}^{\overline{z}}zg(z)dz\right]}{\alpha\int_{z^a(\overline{e})}^{\overline{z}}g(z)dz+(1-\alpha)\int_{z^{na}(\overline{e})}^{\overline{z}}g(z)dz}. \tag{A.5}$$

From the proof of Proposition 2, $\pi_e(\overline{e},\beta)>0$. The denominator of (A.5) is also positive, so the sign of $V_e\left(\overline{e},z^a=z^{na}\right)$ is equal to the sign of the term in hard brackets. Rearrange this term to show that it has the same sign as

$$\frac{\int_{z^{na}(\overline{e})}^{\overline{z}}zg(z)dz}{\int_{z^{na}(\overline{e})}^{\overline{z}}g(z)dz}-\frac{\int_{z^a(\overline{e})}^{\overline{z}}zg(z)dz}{\int_{z^a(\overline{e})}^{\overline{z}}g(z)dz}. \tag{A.6}$$

The first term of this difference is the conditional expectation of z given $z \in [z^{na}(\overline{e}), \overline{z}]$, while the second is the conditional expectation of z given $z \in [z^{a}(\overline{e}), \overline{z}]$. Since $z^{na}(\overline{e}) < z^{a}(\overline{e})$ because the firm is noncompliant at $\overline{e}$, the first term is less than the second and (A.6) is negative. Since (A.6) has the same sign as $V_e\left(\overline{e}, z^a = z^{na}\right)$, $V_e\left(\overline{e}, z^a = z^{na}\right) < 0$.

The firm's optimal choice of emissions given that it is compliant is the solution to $V_e\left(e, z^a = z^{na}\right) = 0$. Since $V_e\left(e, z^a = z^{na}\right)$ is monotonically decreasing in e and $V_e\left(\overline{e}, z^a = z^{na}\right) < 0$, the firm's choice of emissions when it is compliant is less than if it was noncompliant. Thus, imperfect enforcement causes a firm that risks bankruptcy to choose higher emissions. QED.

Proof of Proposition 5: Define

$$M = \left[\alpha \int_{z^a}^{\overline{z}} g(z)dz + (1-\alpha)\int_{z^{na}}^{\overline{z}} g(z)dz\right] \Big/ \int_{z^a}^{\overline{z}} g(z)dz .$$

Use M to rewrite (7) as $\alpha f'(e-r) = tM$. If the firm does not risk bankruptcy, $z^{na} < z^a \le \underline{z}$, which implies $\int_{z^a}^{\overline{z}} g(z)dz = \int_{z^{na}}^{\overline{z}} g(z)dz = 1$ and $M = 1$. Thus, as we showed in the proof of Proposition 1, a noncompliant financially secure firm chooses its violation so that $\alpha f'(e-r) = t$.

On the other hand, if a noncompliant firm risks bankruptcy, either $\underline{z} < z^{na} < z^a < \overline{z}$ or $z^{na} < \underline{z} < z^a < \overline{z}$. In both cases $\int_{z^a}^{\overline{z}} g(z)dz < \int_{z^{na}}^{\overline{z}} g(z)dz$. Since the numerator of M is a linear combination of $\int_{z^a}^{\overline{z}} g(z)dz$ and $\int_{z^{na}}^{\overline{z}} g(z)dz$, the fact that the former is less than the latter implies $\alpha \int_{z^a}^{\overline{z}} g(z)dz + (1-\alpha)\int_{z^{na}}^{\overline{z}} g(z)dz > \int_{z^a}^{\overline{z}} g(z)dz$ and $M > 1$. Therefore, a noncompliant financially insecure firm chooses its violation so that $\alpha f'(e-r) > t$. Since $f(e-r)$ is strictly convex, this implies that a noncompliant firm that risks bankruptcy chooses a higher violation than if it did not risk bankruptcy. QED

Derivation of Equation (9): Under the assumption that the firm risks bankruptcy only when it is audited, $z^{na} \le \underline{z}$ and $z^a \in (\underline{z}, \overline{z})$. Under these conditions the first order conditions (6) and (7) become

$$V_e = \pi_e(e,\beta)\left[\alpha \int_{z^a}^{\overline{z}} (1+z)g(z)dz + (1-\alpha)\right] - \alpha f'(e-r)\int_{z^a}^{\overline{z}} g(z)dz = 0; \qquad \text{(A.7)}$$

$$V_r = \alpha f'(e-r)\int_{z^a}^{\overline{z}} g(z)dz - t\left[\alpha \int_{z^a}^{\overline{z}} g(z)dz + (1-\alpha)\right] = 0. \qquad \text{(A.8)}$$

In the usual manner of deriving comparative statics, calculate $\partial(e-r)/\partial\beta = e_\beta - r_\beta = \left[V_{r\beta}(V_{er}+V_{ee}) - V_{e\beta}(V_{re}+V_{rr})\right]\Big/|H|$, where $|H|$ is the determinant of the Hessian matrix of V. We require $|H| > 0$, so $sign\left[\partial(e-r)/\partial\beta\right] = sign\left[V_{r\beta}(V_{er}+V_{ee}) - V_{e\beta}(V_{re}+V_{rr})\right]$. Calculate the second derivatives of V from (A.7) and (A.8), substitute them into $V_{r\beta}(V_{er}+V_{ee}) - V_{e\beta}(V_{re}+V_{rr})$ and collect terms to obtain:

$$sign\left[\partial\left(e-r\right)/\partial\beta\right]$$
$$= sign\left\{\alpha g(z^a)(f'-t)\left[\alpha\int_{z^a}^{\overline{z}}(1+z)g(z)dz+(1-\alpha)\right]\left[\pi_{e\beta}(z_e^a+z_r^a)-\pi_{ee}z_\beta^a\right]\right\}=0 \qquad \text{(A.9)}$$

Rewrite (A.8) as $V_r = (f'-t)\alpha\int_{z^a}^{\overline{z}} g(z)dz - t(1-\alpha) = 0$, which indicates that $f'-t>0$ is required for $V_r = 0$. Furthermore, $\alpha\int_{z^a}^{\overline{z}}(1+z)g(z)dz+(1-\alpha)>0$ since $\int_{z^a}^{\overline{z}}(1+z)g(z)dz>0$. Therefore, (A.9) can be simplified to $sign\left[\partial\left(e-r\right)/\partial\beta\right] = sign\left[\pi_{e\beta}(z_e^a+z_r^a)-\pi_{ee}z_\beta^a\right]$. Finally, substitute z_e^a, z_r^a and z_β^a from (3) to obtain equation (9).

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