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Horizontal merger of differentiated firms with product consolidation: the effect of change in substitutability

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Abstract

This paper studies an effect of a horizontal merger where a product consolidation by the merged firm may alter the substitutability in the industry. We show that as the number of firms in the industry increases, this type of merger becomes profitable for merging firms, while unprofitable for non-merging firms. Furthermore, we show that with a moderate level of change in the substitutability, the merger of a larger number of firms with a product consolidation is not necessarily profitable for merging firms.

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1 Introduction

In the industrial organization literature, the effect of a horizontal merger in the same industry has long received attention. A merger between firms producing homogeneous goods can be relatively easily modeled, since it is clear that the merged firm produces the same good. However, if two differentiated firms each producing a single brand merge into the single firm, what is the product line of the new firm? One simple option would be that it continues to produce both pre-merger brands, see Deneckere and Davidson (1985) as an example. However, it would also be a realistic option that the firm produces a new single brand which utilizes a brand synergy effect of the product consolidation. In fact, we can easily see this type of merger in the real-world: for example, the recent merger between telecommunications carriers in the US, such as the merger of Sprint and Nextel, or AT&T and BellSouth, realized the integration of two pre-merger networks into a new single network. Another example is the merger of Sony and Ericsson for cell phone production, which realized the consolidation of pre-merger product lines and created the new product line as the Sony-Ericsson brand.

We should note that this type of merger has an important effect in terms of the substitutability in the industry. That is, consolidation of two differentiated brands into the new single brand definitely alters the substitution relationships between products in the industry. If such a consolidation has a positive effect, this implies that although the number of brands is reduced with the consolidation, it has a brand synergy effect, which succeeds in more horizontal differention with other non-merging firms. Such a positive effect would be likely to occur when some positive externality arises from a product/brand consolidation. For example, like the example of AT&T-BellSouth, the consolidation of two telecommunication networks usually causes a positive externality. Also, the merger with a consolidation of two less-differentiated brands might contribute to eliminate consumers' confusion as a negative externality. On the other hand, the merger of two highly-differentiated firms might yield synergy of knowledge, resulting in creating a new valuable product. In fact, in the example of Sony-Ericsson, it is well known that the combination of Ericsson's technology and Sony's design creates a new attractive cell phone. Based on this motivation, in this paper we investigate the effect of a merger which has a benefical change in substitutability.

While our study is related to recent papers which discuss post-merger product repositioning including Gandhi et.al (2008), we analyze the setting based on the simple Cournot oligopoly model as proposed by Salant, Switzer and Reynolds (S-S-R) in their 1983 paper.

Specifically, although the S-S-R model assumes the market where each firm produces a homogeneous good, we extend it to the differentiated case and then formulate a merger with a product consolidation. Based on the equilibrium profits, we compare the profitabilities for both insiders (merging firms) and outsiders (non-merging firms) of three cases: the pre-merger case, the case of merger without a product consolidation and that with a product consolidation. Much of the literature, including Farrell and Shapiro (1990) and Davidson and Mukherjee (2007), explores the merger effect based on the S-S-R model from the viewpoint of the synergy effect in the cost side. However, we instead investigate the merger effect from the demand side such as a brand synergy.

We show that even in the presence of a slight effect in the substitutability, as the number of firms existing in the industry increases, the merger with a product consolidation is profitable for insiders. On the other hand, this merger is not beneficial for the outsiders as the number of firms in the industry increases. Furthermore, we show that with a moderate level of change in the substitutability, the merger of a larger number of firms with a product consolidation is not necessarily profitable for insiders. These findings are in sharp contrast with those of S-S-R and the related works referred to above.

2 Model Description

Let $N = \{1, 2, \ldots, n\}$ be the set of potentially symmetric firms. To exclude the trivial case, we hereafter assume $n \geq 3$. Under a pre-merger situation, the following linear inverse demand system is employed:

$$p_i = 1 - q_i - \beta \sum_{j \neq i} q_j, \ i = 1, 2, \ldots, n, \tag{1}$$

where $p_i (\geq 0)$ is the price and $q_i (\geq 0)$ is the output level of firm i's brand. $\beta (0 < \beta \leq 1)$ is the substitutability parameter which implies the degree of differentiation. We assume that firms engage in Cournot competition. To focus on the synergy effect in the demand side, we here neglect any production and setup costs. With these settings, we consider the following three cases with regard to the industrial structure.

Case 1: Pre-Merger Situation

Each firm i noncooperatively chooses its output level. Given the profit functions $\pi_i = p_i(q_1, \ldots, q_n) q_i$, we consider the Cournot game where each firm i determines q_i to maximize π_i. The output levels and profits at the unique Cournot-Nash equilibrium, denoted by q_i^* and π_i^*, are obtained as follows:

$$\pi_i^* = (q_i^*)^2 = (\frac{1}{2 + (n-1)\beta})^2, \ i = 1, 2, \ldots, n.$$

Case 2: Merger without Product Consolidation

We consider the situation where two firms merge and the merged firm continues to produce both pre-merger brands. Without loss of generality, firms 1 and 2 merge and we denote the integrated firm by M. All other firms continue to behave independently. We note that the inverse demand system for this situation remains (1). Let $\pi_M \equiv p_1(q_1, \ldots, q_n)q_1 + p_2(q_1, \ldots, q_n)q_2$ be firm M's profit. Then we consider the following Cournot game:

$$\max_{q_1,q_2} \pi_M \quad \text{and} \quad \max_{q_i} \pi_i \quad i = 3, 4, \ldots, n.$$

The output levels and profits at the unique Cournot-Nash equilibrium, denoted by q_i^{**} and π_i^{**}, are obtained as follows:

$$q_1^{**} = q_2^{**} = \frac{2-\beta}{2(2+(n-1)\beta-\beta^2)}, \; \pi_M^{**} = \frac{(2-\beta)(2+\beta-\beta^2)}{2(2+(n-1)\beta-\beta^2)^2},$$

$$\pi_i^{**} = (q_i^{**})^2 = (\frac{1}{2+(n-1)\beta-\beta^2})^2, \; i = 3, 4, \ldots, n.$$

Case 3: Merger with Product Consolidation

We consider the situation where the integrated firm M produces only the new single brand M. It should be noted that the reduction of the number of brands alters the substitution relationships in the market. We specifically employ the following inverse demand system:

$$p_M^S = 1 - q_M^S - \theta\beta \sum_{j \neq M} q_j^S, \;\; p_i^S = 1 - q_i^S - \theta\beta q_M^S - \beta \sum_{j \neq i,M} q_j^S, \; i = 3, 4, \ldots, n,$$

where $p_i^S(\geq 0)$ is the price and $q_i^S(\geq 0)$ is the output level of firm i's brand ($i = M, 3, 4, \ldots, n$). θ is the parameter which implies the degree of change in the substitutability after the merger. For simplicity, we assume that θ is exogenous and independent of the value of β. We here focus on the case where the product consolidation has a positive effect on consumers' utility, which contributes to higher differentiation with other brands. Thus, we assume $0 \leq \theta \leq 1$. In addition, it is supposed that the substitutability relationships between the outsiders are invariant after the merger. Our formulation includes the setting of S-S-R as a special case ($\beta = \theta = 1$).

For $i = M, 3, 4, \ldots, n$, let $\pi_i^S \equiv p_i^S(q_M^S, \ldots, q_n^S)q_i^S$ be firm i's profit functions. Then each firm i determines q_i^S to maximize π_i^S. The output levels and profits at the unique Cournot-Nash equilibrium, denoted by q_i^{***} and π_i^{***}, are obtained as follows:

$$\pi_M^{***} = (q_M^{***})^2 = (\frac{2+(1-\theta)\beta n + (2\theta-3)\beta}{4+(2-\theta^2\beta)\beta n + 2\theta^2\beta^2 - 6\beta})^2,$$

$$\pi_i^{***} = (q_i^{***})^2 = (\frac{2-\theta\beta}{4+(2-\theta^2\beta)\beta n+2\theta^2\beta^2-6\beta})^2,\ i=3,4,\ldots,n.$$

3 The Profitability of a Merger with Product Consolidation

3.1 The Profitability for Merging Firms

In this section, we discuss the profitability of a merger with a product consolidation. Firstly, we investigate it for insiders. To see this, however, we should compare in advance the profitability between cases 1 and 2. The following lemma suggests that the degree of differentiation plays an important role in this regard.

Lemma 1 *There necessarily exists $\beta^*(n)$ such that for any $\beta \in (\beta^*(n), 1]$, $\pi_M^{**} < \pi_1^* + \pi_2^*$ holds.*

Although the proof is straightforward, it would be a lengthy process to give details here. However, a complete proof can be obtained from the authors upon request. Lemma 1 shows that the merger without a product consolidation is beneficial only in a highly differentiated market. As shown in S-S-R, under a less differentiated situation, the merged firm reduces its output in order to attain a higher price. In response, however, the outsiders expand their outputs, resulting in the decrease of the equilibrium output of the merged firm. This outsiders' "free-ride" behavior is relaxed in a highly differentiated situation, so that the merger is profitable for insiders. We finally note that $\beta^*(n) \leq \beta^*(3) \approx 0.55$ holds for any $n \geq 3$.

We now investigate the profitability of case 3. Let $\Delta\pi_C \equiv \pi_M^{***} - (\pi_1^* + \pi_2^*)$ and $\Delta\pi_{NC} \equiv \pi_M^{***} - \pi_M^{**}$.

Our main proposition is immediately derived as follows:

Proposition 1 *1. When $n \leq \frac{2(\sqrt{2}-1)+\beta}{\beta}$, $\Delta\pi_C \leq 0$. Otherwise, there always exists $0 < \hat{\theta}(n) < 1$ such that for any $\theta < \hat{\theta}(n)$, $\Delta\pi_C > 0$. In addition, $\hat{\theta}(n+1) > \hat{\theta}(n)$.*

2. When $n \leq \frac{\beta^2+\beta-2+(2-\beta)\sqrt{2(\beta+1)}}{\beta}$, $\Delta\pi_{NC} \leq 0$. Otherwise, there always exists $0 < \bar{\theta}(n) < 1$ such that for any $\theta < \bar{\theta}(n)$, $\Delta\pi_{NC} > 0$. In addition, $\bar{\theta}(n+1) > \bar{\theta}(n)$.

Proof Since $\Delta\pi_C$ can be written as $(q_M^{***})^2 - 2(q_1^*)^2$, the sign of $q_M^{***} - \sqrt{2}q_1^*$ determines that of $\Delta\pi_C$. Furthermore, a direct calculation shows that this is equivalent to the sign of $F(\theta, n)$, where

$$F(\theta,n) \equiv (2+(1-\theta)\beta n+(2\theta-3)\beta)(2+(n-1)\beta) - \sqrt{2}(4+(2-\theta^2\beta)\beta n+2\theta^2\beta^2-6\beta).$$

It is straightforward from direct calculating that F is strictly convex in θ and $F(1,n) < 0$ for all $n \geq 3$. We thus have that $F(0,n) > 0 \Leftrightarrow n > \frac{2(\sqrt{2}-1)+\beta}{\beta}$ ensures the existence and uniqueness of positive $\hat{\theta}(n)$. In the following, we show that $\hat{\theta}(n+1) > \hat{\theta}(n)$ for any $n(\geq 3)$. We clearly have that if $\frac{\partial \hat{\theta}}{\partial n} > 0$ holds for any continuous $n \geq 3$, then $\hat{\theta}(n+1) > \hat{\theta}(n)$ also holds. Thus, we now assume that $F(\theta, n)$ is a continuous function of θ and n. We first derive that $\hat{\theta}(n)$ is strictly increasing at $n = 3$. In fact, $\frac{\partial F}{\partial \theta} < 0$ for any feasible n, β and θ. Also, we can directly verify that $\frac{\partial F}{\partial n} > 0$ for $n = 3$ and $\theta = \hat{\theta}(3)$. Therefore, by the implicit function theorem, we have $\frac{\partial \hat{\theta}}{\partial n} > 0$ at $n = 3$.

The fact of $\frac{\partial \hat{\theta}}{\partial n} > 0$ at $n = 3$ and continuity of $\hat{\theta}$ in $n(\geq 3)$ indeed imply that if $\hat{\theta}(n)$ is not strictly increasing, then we must have some $\theta' \geq \hat{\theta}(3)$ such that $F(\theta', n)$ has at least two distinct solutions with respect to n. However, we can prove that for any $\theta \geq \hat{\theta}(3)$, $F(\theta, n) = 0$ necessarily has the unique feasible solution with respect to n. To see this, it is sufficient to show $F(\theta, 3) \leq 0$ for any $\theta \geq \hat{\theta}(3)$, since F is the convex quadratic function in n. In fact, we can directly derive that $F(\theta, 3) = (\theta\beta - 2)(\sqrt{2}\theta\beta - 2\beta + 2\sqrt{2} - 2) \leq 0 \Leftrightarrow \theta \geq \hat{\theta}(3)$. Therefore, we have that $\hat{\theta}$ is strictly increasing in $n(\geq 3)$.

With regard to $\Delta\pi_{NC}$, the proof follows similar steps, so details can be obtained from the authors upon request. **Q.E.D.**

Proposition 1 shows that with a change in the substitutability, the merger of two firms with a product consolidation is the most profitable structure for insiders as n increases. This result is in sharp contrast with that of S-S-R. In fact, when $\theta = 1$, the function $F(1, n)$, defined in the proof of Proposition 1 , is strictly decreasing in n. Therefore, there exists $\hat{n}$ such that for any $n \geq \hat{n}$, $\Delta\pi_C < 0$. That is, without any change in the substitutability, the merger with a product consolidation is unprofitable for insiders as the number of firms in the industry increases. On the other hand, the parameter $\theta(< 1)$ implies that the merged firm succeeds in horizontally differentiating more with *every* outsider. Therefore, as n increases, the merged firm can differentiate with more firms, resulting in the increase of its market power. We thus have that the threshold of θ for the profitable merger is increasing in n.

3.2 The Profitability for Outsiders

We next focus on the profitability of outsiders. In this regard, we investigate two relationships as follows: firstly, we compare the equilibrium profits in Case 3 between the insider and outsider. Subsequently, we explore the change of the equilibrium profit of an outsider with regard to the merger. We can immediately derive the following two propositions.

Proposition 2 *When* $n \leq \frac{3\beta+2(\sqrt{2}-1)}{\beta}$, $\pi_M^{***} \leq \pi_i^{***}$ *always holds. Otherwise, there exists* $0 < \hat{\theta}_1(n) < 1$ *such that for any* $\theta \leq \hat{\theta}_1(n)$, $\pi_M^{***} > \pi_i^{***}$. *In addition,* $\hat{\theta}_1(n+1) > \hat{\theta}_1(n)$.

Proof It can be directly verified that the sign of $G_1(n,\theta) \equiv \beta(2+\sqrt{2}-n)\theta + 2(1-\sqrt{2}) + (n-3)\beta$ determines that of $\pi_M^{***} - \pi_i^{***}$. If $n \leq 2+\sqrt{2}$, then $G_1(n,\theta) < 0$ for any θ. On the other hand, if $n > 2+\sqrt{2}$, G_1 is decreasing in θ and $G_1(n,1) < 0$. Therefore, $G_1(n,0) \leq 0 \Leftrightarrow n \leq \frac{3\beta+2(\sqrt{2}-1)}{\beta}$ proves the first part of the proposition. We next consider the case of $n > \frac{3\beta+2(\sqrt{2}-1)}{\beta}$. Then it is straightforward that $\frac{\partial\hat{\theta}_1}{\partial n} = \frac{(\sqrt{2}-1)(2-\beta)}{\beta(n-2-\sqrt{2})^2} > 0$. **Q.E.D.**

Proposition 3 *Suppose* $\theta < 1$. *Then, there necessarily exists* $\bar{n}(> 3)$ *such that for any* $n < \bar{n}$, $\pi_i^{***} > \pi_i^*$ *and* $n > \bar{n}$, $\pi_i^* > \pi_i^{***}$ $(i = 3, \ldots, n)$.

Proof By a direct calculation, it follows that the sign of $G_2(n,\theta) \equiv (2-\theta\beta)(2+\beta(n-1)) - (4-\theta^2\beta^2 n+2\beta n+2\theta^2\beta^2-6\beta)^2 = -(1-\theta)\theta\beta^2 n+(1-2\theta)\theta\beta^2+2(2-\theta)\beta$ determines that of $\pi_i^{***} - \pi_i^*$. We have that G_2 is linearly decreasing in n and $G_2(3,\theta) = \beta(2-\theta)(2-\theta\beta) > 0$, which ensures the existence of $\bar{n}$ satisfying $G_2(\bar{n},\theta) = 0$. **Q.E.D.**

If $\theta = 1$, then the function G_2, defined in the proof of Proposition 3 , is always positive, which implies that the merger is always profitable for the outsiders. As previously mentioned, S-S-R pointed out that this result is due to the outsiders' free-ride behavior. However, our results show that with a sort of brand synergy effect, the merger can prevent such free-rides and therefore outsiders do not necessarily benefit from the merger. From Proposition 3 , one might point out that when n is small, the merger is rather profitable for outsiders. However, we should note that this is due to a different mechanism from the outsiders' free-ride behavior above mentioned. In our situation, the product consolidation contributes to the horizontal differentiation between the merged firm and outsiders. Therefore, when the number of firms in the industry is relatively small, outsiders also benefit from the differentiation with the merged firm. However, as shown in the previous subsection, the merged firm increases its market power as the number of firms in the industry increases. As a result, while the merged firm can easily expand its output, outsiders are compelled to compete with each other in the market which is "shrunk" by the merger firm. Therefore, outsiders reduce their output levels, resulting in loss of their profits.

4 The Profitability of a Larger Merger

In this section, we extend our analysis to the case of a merger of more than two firms. Given specifically the merger of firms $1, \dots, m (m \geq 2)$ into the new single brand M, the inverse demand model of Case 3 is modified as follows:

$$p_M^L = 1 - q_M^L - \theta\beta \sum_{j \neq M} q_j^L, \quad p_i^L = 1 - q_i^L - \theta\beta q_M^L - \beta \sum_{j \neq i, M} q_j^L, \ i \neq M,$$

where $p_i^L (\geq 0)$ is the price and $q_i^L (\geq 0)$ is the output level of firm i's brand ($i = M, m+1, \dots, n$). We immediately obtain the unique Cournot-Nash equilibrium profits as follows:

$$\pi_M^L = (\frac{2 + (1-\theta)(n-m)\beta - \beta}{4 + (2-\theta^2\beta)(n-m)\beta - 2\beta})^2, \quad \pi_i^L = (\frac{2 - \theta\beta}{4 + (2-\theta^2\beta)(n-m)\beta - 2\beta})^2, \ i \neq M.$$

In the following, we focus on the merger under a less differentiated industry. As shown in Section 3.1, the merger without product consolidation is not a beneficial structure in a less differentiated industry. We thus compare the equilibrium profit π_M^L with $m\pi_i^*$, which is the total profits earned by m firms in Case 1. Let $\Delta\pi_L \equiv \pi_M^L - m\pi_i^*$. Although the analysis of $\Delta\pi_L$ is complicated for a general case of β, we can derive an interesting result for an almost homogeneous case as follows:

Proposition 4 *Suppose $n \geq 4$ and $\beta \approx 1$. Then for some range of θ, there exist $\underline{m}$ and $\overline{m}$ such that $\Delta\pi_L < 0$ for $\underline{m} < m < \overline{m}$, while $\Delta\pi_L > 0$ for $2 \leq m < \underline{m}$ and $\overline{m} < m \leq n$.*

Proof We assume $\beta = 1$ as a limit case. Then, as in the previous propositions, we can construct the specific function $H(m, \theta) = (1 + (1-\theta)(n-m))(n+1) - \sqrt{m}(2 + (n-m)(2-\theta^2))$, which determines the sign of $\Delta\pi_L$. It can be directly verified that $\frac{\partial^2 H}{\partial m^2} > 0$ for any feasible m and $\frac{\partial H}{\partial m} < 0$ at $m = 2$ for any $n \geq 4$. Also, it is straightforward that H is strictly decreasing in θ. In addition, we have $H(2, 0) > 0$, $H(2, 1) < 0$ and $H(n, \theta) > 0$ for any θ. These results ensure that there exist $\underline{\theta}$ and $\overline{\theta}$ such that for any $\theta \in [\underline{\theta}, \overline{\theta}]$, $H(m, \theta) = 0$ has two solutions with respect to m in $2 \leq m \leq n$. **Q.E.D.**

In the S-S-R setting ($\beta = \theta = 1$), the lower threshold $\underline{m}$ does not exist, which implies that a smaller merger is necessarily unprofitable. On the other hand, as they pointed out, the upper threshold $\overline{m}$ is over 80% of all firms in the industry, that is, a profitable merger requires the participation of very many firms in the industry. This is of course due to the severe "free-ride problem". In contrast, in our setting, a smaller merger is rather beneficial for the merging firms, since it succeeds in a horizontal differentiation with more outsiders. On the other hand, however, a larger merger is also profitable, since it results in reducing the number of players in the industry and thus can increase its market power. Therefore, we would expect that there might exist the worst case with regard to a merger size, where both of these positive effects disappear. In fact, Proposition 4 shows that if θ is at an intermediate level, the merger with a moderate size is not beneficial. We note that from the proof of Proposition 4 , it follows that the merger with any size is beneficial when a brand synergy effect is very high ($\theta \approx 0$), since the impact of a horizontal differentiation is so high that the "free-ride problem" vanishes.

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