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The shape of the utility function under risk in the loss domain and the "ruinous losses" hypothesis: some experimental results

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Abstract

This paper reports some preliminary experimental results as regards the shape of the utility function for losses when elicited over a wide interval of consequences. Individual utility functions are elicited using the trade-off method, which, unlike standard elicitation procedures, is robust to probability weighting (and avoids most cognitive biases). Even though most utility functions exhibit the usual convex shape, nearly 25% of them appear to be inverse-S shaped, with convexity over moderate losses changing to concavity as losses grow. Though not conclusive (due mainly to the small size of our subject pool), this result brings some new support to the old idea that ruinous or unacceptable losses may induce some abrupt change in the shape of the utility function. Most importantly, it paves the way for more systematic investigation of the "ruinous losses" hypothesis.

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1. Introduction

A huge body of theoretical as well as experimental literature has been devoted to the investigation of the shape and properties of the utility function under risk. In the most recent years, the utility function has been much investigated experimentally in both the gain and loss domains, as well as at the intersection between the two¹. In the loss domain, convexity appears to be the slightly dominant pattern, with much diversity at the individual level (Abdellaoui 2000; Fennema and van Assen 1999; Pennings and Smidts 2003; Lattimore, Baker and Witte 1992). Levy and Levy (2002) even challenge the idea of an S-shaped utility function as assumed in Prospect Theory, since their data rather support Markowitz (1952)'s hypothesis of an inverse-S shape. Still, most of the above-mentioned studies involve quite small monetary amounts. Since the shape of the utility function over large losses has not been much investigated, the question arises of what may happen when losses grow dramatically, getting ruinous and raising "the danger of insolvency" (Mao 1970, p. 354). For instance, Kahneman and Tversky (1979) raise the question of "the effect of special circumstances on preferences" (p. 278). They suggest that "the utility function of an individual who needs \$60,000 to purchase a house may reveal an exceptionally steep rise near the critical value" (underlined by the author) and, more generally, that the utility function "does not always reflect 'pure' attitudes to money, since it could be affected by additional consequences associated with specific amounts. Such perturbations can readily produce convex regions in the value function for gains and concave regions in the value function for losses. The latter case may be more common since <u>large losses</u> often necessitate changes in life style." (underlined by the author)

Actually, a very few (and old) experimental studies have investigated professionals' behaviour and/or utility function in the neighbourhood of ruin. Though rather scarce, they all support the stimulating idea that ruinous or unacceptable losses may induce some change in behaviour/utility. For instance, in Grayson (1960)'s study (cited in Libby and Fishburn 1977, p. 283), most utility curves in the loss domain appear to exhibit a steep drop below a certain point. Libby and Fishburn (1977) conclude from their literature review on utility functions that "sometimes a significant change point lies at a positive or negative return (e.g., a target return or a return approaching ruin." (underlined by the author) Laughhunn, Payne and Crum (1980) run an experimental study with managers to investigate both their risk preferences for below-target returns and the way these preferences "may be sensitive to ruinous loss considerations." The data appear to support the authors' expectation that, "faced with the possibility of ruinous loss, managers are likely to become less risk seeking and may even revert to risk averse behaviour".

However, all these studies suffer from a strong limitation: their Expected Utility (EU) background². In some studies (see Grayson 1960 for instance), utility was directly elicited assuming EU preferences. This implies that the data should not be trusted unless all the subjects were EU-maximizers. Similarly, when only behaviour was investigated (as in Laughhunn, Payne and Crum 1980 for instance), conclusions as regards the shape of the utility function were drawn from the equivalence – that holds *only* under EU – between risk aversion (resp. seeking) and concave (resp. convex) utility. Since huge experimental evidence

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¹ Concavity has been shown to prevail in the gain domain (see however Lattimore, Baker and Witte 1992 and Levy and Levy 2002). Loss aversion has also been well documented (e.g. Abdellaoui, Bleichrodt and Paraschiv 2007; Abdellaoui, Bleichrodt and L'Haridon 2008; see however Schmidt and Traub 2002 and Vieider 2009 for somewhat different findings).

² Obviously, the literature dealing with the empirical failures of the EU model and their theoretical implications was not much developed at that time.

now exists that most people are actually *not* EU-maximizers, the reliability of these old findings as regards the utility function is also compromised.

Standard elicitation methods have been widely shown to suffer from strong biases if the decision maker is not an EU-maximizer. On the other hand, a method that is compatible with most non-EU preferences was proposed about ten years ago by Wakker and Deneffe (1996). This method, called the trade-off (TO) method, is axiomatically founded (see Wakker and Deneffe 1996; Wakker and Tversky 1993). Its main originality and interest is that it remains valid even when probability weighting is at play. Besides, since it involves binary lotteries and holds probabilities constant, it is immune from both the certainty and scale compatibility effects. The TO method has been widely used to elicit the utility function in the gain domain (Abdellaoui 2000; Abdellaoui, Barrios and Wakker 2007; Bleichrodt and Pinto 2000; Booij and van de Kuilen 2007) as well as in the loss domain (Abdellaoui 2000; Fennema and van Assen 1999; Etchart-Vincent 2004; Booij and van de Kuilen 2007). However, to the best of my knowledge, it has never been used to elicit the utility function over large losses (except, but to a quite limited extent, in Etchart-Vincent 2004). So the question remains whether the shape of the utility function should be expected to change from convexity to concavity when large losses are involved. The aim of the paper is precisely to offer some preliminary experimental results regarding this hypothesis outside the EU framework, using the TO method to elicit each subject's utility function over a wide interval of losses.

The main result of the study is the following. Even though most (67%) elicited individual utility functions classically appear to be either concave or convex (most of them being convex as expected in the loss domain), a strong minority of them (nearly 25%) exhibits an inverse-S shape, with convexity over moderate losses changing to concavity as losses grow.

The remainder of the paper is organized as follows. The main features of the experimental design are briefly described in Section 2. Section 3 reports the results, which are further discussed in Section 4.

2. Experimental design

30 subjects participated in the experiment. All of them were undergraduate wage-earning students at the Department of Economics and Management at Ecole Normale Supérieure de Cachan (France). The subjects were paid for their participation (they received a flat-rate of 15 euro, around US\$ 20), but no performance-based payment was used.

It should be noted that the primary aim of our experimental study was *not* to investigate the utility function. It was actually designed to allow for the investigation of the probability weighting function (the core results of the study are reported in Etchart-Vincent 2009). But to that purpose, the first step was to carefully elicit the utility function at the individual level, using personal interviews³. More details about the experimental design are given in Etchart-Vincent (2009).

Following Wakker and Deneffe (1996, p. 1144)'s suggestion, the TO method was implemented using a probability of one-third,. The starting point of the standard sequence (x_0) , as well as the reference outcomes (r and R) and the number of elicited values n were the same for all the subjects. They were calibrated so that we could get for the first elicited value x_1 (resp. the last elicited value x_n) a mean value around a month earnings (resp. around the price of a medium-sized car). The study was thus designed to involve significant losses (i.e.

³ Our results as regards the utility function thus appear to be an unexpected 'by-product' of the study.

losses that the subjects were expected both to be familiar with and to dislike) but not genuinely ruinous losses⁴. Using n = 6, $x_0 = -150$ euro (-US\$ 210), r = 300 euro (US\$ 420) and R = 1500 euro (US\$ 2 100), the mean values obtained for x_1 and x_6 in our 30-subject sample were -1260 euro (-US\$ 1 750) and -12850 euro (-US\$ 17 860) respectively.

3. Results

The classification of individual utility functions as concave, convex, linear or inverse-S shaped was obtained on raw data using Abdellaoui (2000)'s procedure (see Figure 1). The value $\phi_j = x_j - x_{j-1}$ was first calculated for j = 1, ..., 6. Then, the value $\psi_j = \phi_j - \phi_{j-1}$ was computed for j = 2, ..., 6. For the purpose of classification, ψ_j was considered as null if $-15 \in \psi_j \le 15 \in (\text{resp. } -75 \in \psi_j \le 75 \in (\text{resp. } -75 \in (\text{resp. } -7$

Figure 1. Method for Categorizing Individual Utility Functions

As expected (see Table I), convexity appears to prevail, with 18 (resp. 2) out of 30 individual curves being classified as convex (resp. concave). Another noticeable result is the absence of linear utility functions. Though in sharp contrast with previous findings (see Abdellaoui 2000 for instance), this result may be due to the fact that the utility function was

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⁴ Actually, the first pilots were run with larger amounts of money (reaching the price of a flat). But our about 21-year old subjects had never been in the position to deal with so much money in their life, and they seemed not to realize what it meant to incur such large losses for real. So the amounts at stake were finally calibrated to remain reasonable given the subjects' income and way of life, so that we could collect reliable data.

elicited on a wide interval of consequences (Wakker and Deneffe 1996, p. 1137). Finally, and quite interestingly, 7 out of 30 (nearly 25%) individual utility functions appear to exhibit an inverse-S shape, with a convex upper part (over small and moderate losses) and a concave lower part (over large losses). No subject had an S-shaped utility function.

Table I. The Shape of Individual Utility Functions

Convex	18
Concave	2
Linear	0
Inverse-S shaped	7
S-shaped	0
None	3
Total	30

Now, we focus on the seven inverse-S shaped utility functions. The 7 sets of 6 elicited utility values are given in Table II, along with a rough estimate of the 7 inflection values⁵. As usual, huge heterogeneity appears to prevail at the individual level. Quite strikingly, the last value x_6 was a rather moderate loss (inferior to 6500 euro) for 3 subjects (out of 7). The other four subjects actually dealt with significant losses (with an absolute value $|x_6|$ larger than 16000 euro).

Table II. Elicited Utility Values (in Euro) for the 7 Inverse-S Shaped Utility Functions

Subject	2	4	12	18	22	25	30
\mathbf{x}_0	-150	-150	-150	-150	-150	-150	-150
\mathbf{x}_1	-1400	-840	-500	-1000	-670	-1650	– 750
x ₂	-4600	-2090	-870	-5490	-1430	-4580	-1520
X ₃	-9890	-4720	-1390	-10710	-2600	-8670	-2300
X ₄	-13820	-10180	-2150	-12780	-4010	-13970	-3200
X ₅	-16770	-13450	-2880	-14830	-5260	-19190	-4050
X ₆	-19400	-16080	-3570	-16880	-6490	-22710	-4730
Inflection value (rough estimate)	-11860	-11810	-2520	-11740	-4630	-16580	-3620

For the sake of comparison, Table III gives the last utility value (x_6) elicited for each of the other 23 subjects. Interestingly, it appears that the 7 subjects whose utility function is inverse-S shaped did not deal with especially large losses as compared to the other 23 subjects. Moreover, while dealing with an especially wide interval of losses (with $x_6 = -59240$ \Leftrightarrow), Subject 24 appears to exhibit a very convex utility function. Put together, these results suggest that the 'ruin point' (or any psychological inflection point), if it may exist, may have little connection with the objective size of losses at stake.

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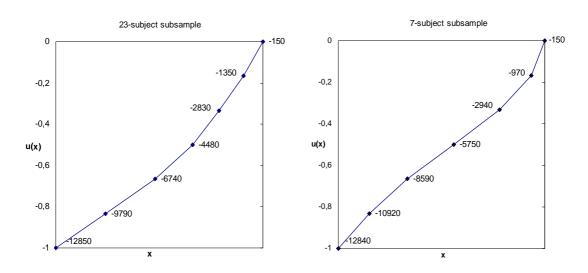
⁵ Each inflection value (that is, the abscissa of each inflection point) was approximated as the mean value between the last elicited value for which the utility function was convex and the first elicited value for which the utility function was concave.

Table III. Elicited Utility Value x_6 (in Euro) for the Other 23 Subjects

Subject	1	3	5	6	7	8	9	10
X ₆	-3510	-11040	-3670	-6590	-9580	-8290	-9270	-15250
Subject	11	13	14	15	16	17	19	20
X ₆	-7070	-6740	-7160	-1400	-6840	-59820	-4480	-4880
Subject	21	23	24	26	27	28	29	
X ₆	-23040	-15100	-59240	-4800	-8040	-3680	-16070	

To give a better overview of the data, Figure 2 displays the mean utility functions obtained for the 23-subject subsample and for the 7-subject subsample respectively. Interestingly enough, while intermediate values strongly differ between subsamples, the last elicited value x_6 is highly similar.

Figure 2. The Mean Utility Function (23-Subject Subsample and 7-Subject Subsample)



As said at the beginning of Section 2, the main aim of the study was to get the subjects' probability weighting function. For that purpose, their utility function had to be accurately elicited first (using the TO method). But then, it also had to be parametrically fitted as properly as possible (see Etchart-Vincent 2009 for some details about the way probability weights were obtained from utility values). So, the 30 individual utility functions were fitted using both the usual one-parameter POWer specification, with $U_{POW}(x) = -(-x)^{\alpha}$ (Tversky and Kahneman 1992) and the two-parameter EXpo-POWer specification, such that $U_{EXPOW}(x) = [1-\exp(-\beta(-x)^{\alpha})]/(\exp(-\beta)-1)$ with $\beta>0$ and $\alpha>0$ (Abdellaoui, Barrios and Wakker 2007; Saha 1993). The POW specification was used for the sake of comparison with previous studies⁶, but it was clearly outperformed by the EXPOW specification for all the subjects.

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⁶ The median value α^M of individual POW estimates ($\alpha^M = 0.746$) appears to be significantly lower than that found by Abdellaoui (2000) ($\alpha^M = 0.92$), Abdellaoui, Vossman and Weber (2005) ($\alpha^M = 0.96$), Fennema and van Assen (1999) ($\alpha^M = 0.837$), and Tversky and Kahneman (1992) ($\alpha^M = 0.88$) (see also, for instance, Booij and van de Kuilen 2007 for similar results on mean values), implying stronger convexity. On the other hand, our median POW parameter is remarkably similar to Abdellaoui, Bleichrodt and Paraschiv (2007)'s value of 0.73. This may be due to the fact that, in their study as in ours, utility was elicited over an unusually wide interval of losses.

Besides, neither POW nor EXPOW allow changes in convexity. So, they are not suitable for the parametric fitting of inverse-S shaped utility functions⁷. Since our experimental design strongly required that all individual utility functions be accurately fitted, we used the GE specification (with reference to Goldstein and Einhorn, who introduced it in their 1987 paper)

to fit the 7 inverse-S shaped utility functions. GE is given by $GE(x) = \frac{\delta(-x)^{\gamma}}{\delta(-x)^{\gamma} + (1+x)^{\gamma}}$; it was developed to allow the parametric fitting of probability weighting functions, which often exhibit an inverse-S shape. Unsurprisingly, GE did much better estimation work than EXPOW for the 7 functions under consideration.

4. Discussion

Obviously, our data do not provide any conclusive evidence as regards the 'ruinous losses' phenomenon. The reason for this is threefold. First, 7 subjects out of 30 is not very much. Second, a 30-subject sample is obviously too small to guarantee the statistical reliability of any result. Third, and most importantly perhaps, the study did actually not involve genuinely ruinous losses (for some subjects, it did even not involve very large losses).

Though not conclusive, our findings remain suggestive, since they bring some support to the old idea that the convex utility function for losses may become concave when losses grow. Moreover, since the data were obtained using the quite robust trade-off method, there is no reason to believe that the inverse-S shape of several utility functions would be an artefact. However, given the level of losses at stake, there is no clear evidence that the 7 subjects under consideration did actually reach either an objective 'ruin point' or even some kind of subjective 'ruin point' or psychological threshold. Information as regards the financial as well as psychological background of the subjects is obviously lacking to allow proper interpretation of the results.

Moreover, most of the individually elicited utility functions were actually not inverse-S shaped. However, this is obviously not sufficient to discredit the 'ruinous losses' hypothesis. Simply, the 6-point elicitation process may have prevented most subjects from reaching their 'ruin point'. Obviously, the 'ruin point' is unlikely to be the same for all the subjects; instead, it can be expected to be a personal feature, depending on socio-demographic, financial, and psychological characteristics. More systematic investigation of the utility function as well as of the financial and personal background of the subjects is warranted to investigate whether the 'ruinous losses' phenomenon should be considered as a general human feature or not, as well as to capture the location of the inflection point for each subject if it may exist.

To be more specific, it would be especially useful to get some information about each subject's financial characteristics (income, current expenses, wealth, ongoing loans, and so on) before eliciting her utility function using the TO method. This may help calibrate the elicitation process so as to maximize the likelihood to capture the subject's inflection point (if it may exist), while keeping the maximum number n^* of elicited utility values small enough to ensure the reliability of the data⁸. Moreover, once an inflection point is detected, previously collected information regarding the financial background of the subject may also help decide whether this point can be rightly labelled/interpreted as a 'ruin point' or not.

⁷ For subjects 2 and 4, estimation using POW and EXPOW did not converge.

⁸ Indeed, if implemented for a too long time, the TO method may induce the use of some undesirable heuristics. Of course, it may still be the case that, for some subjects, no change in concavity occurs within the n^* bound.

Now, it may be of interest to try to understand why the shape of the utility function may change as losses grow. First, let us remember that this shape can be accounted for by the combined effects of two psychological features, namely the 'diminishing sensitivity' principle and the 'decreasing marginal utility' principle (Köbberling, Schwieren and Wakker 2007). The 'diminishing sensitivity' principle has to do with the numerical perception of money; as for any physical quantity, it induces convexity in the loss domain (and concavity in the gain domain). The 'decreasing marginal utility' principle has to do with the intrinsic value of money; it always induces concavity. In the loss domain, both effects contradict: a utility function will not exhibit a convex shape unless the 'diminishing sensitivity' principle prevails. This may contribute to explain why, even though the most common pattern in the loss domain appears to be convexity (Abdellaoui 2000; Currim and Sarin 1989; Etchart-Vincent 2004; Tversky and Kahneman 1992), concavity has sometimes been shown to occur (Holt and Laury 2002; Abdellaoui, Bleichrodt and L'Haridon 2008). It may also help understand why, among those studies that find convexity, most find it to be weaker than concavity in the gain domain (Fennema and Van Assen 1999; Köbberling, Schwieren and Wakker 2004; Abdellaoui, Vossman and Weber 2005).

By contrast, it might be the case that, when elicited over a wide interval of losses, the utility function exhibits either an especially strong 'diminishing sensitivity' effect or an especially low 'decreasing marginal utility' effect, or even both. This might contribute to explain the rather high degree of convexity observed in both the present study and Abdellaoui, Bleichrodt and Paraschiv (2007)'s. Conversely, it is possible that, in the neighbourhood of (objective or subjective) ruin, 'decreasing marginal utility' of money comes to overcome 'diminishing sensitivity', inducing concavity. In this respect, it could be the case that the 'diminishing sensitivity' principle globally drives the utility function for losses, except near ruin.

To investigate this point further, it would be interesting to mobilize the stimulating distinction between a deliberate decision mode and a more intuitive one (Schunk and Betsch 2006; see also Slovic *et al.* 2004 and the references therein for a similar dual cognitive approach) in order to see whether changes in the shape of the utility function as losses grow may be imputed to a basic change in the decision making process or not. This could be completed with some investigation into the brain, to see whether and how a dramatic increase in loss size is likely to affect brain activity, in relation with the emotional content of losses (for a typical approach, see Knutson and Peterson 2005).

Finally, the fact that the utility function may exhibit a concave lower part in the loss domain raises both a practical and a theoretical concern. First, as regards parametric fitting, no specification should be considered as suitable unless it is able both to fit well the function under consideration and to offer some desirable economic properties (see Abdellaoui, Barrios and Wakker 2007, p. 364). If concavity turned out to be a systematic feature of the utility function over large losses, it would certainly be of great interest to try to find some new specifications satisfying both conditions. Second, concavity over large losses appears to conflict with the basic normative requirement that the utility function should be bounded, from both above and below (see for instance Markowitz 1952, p. 154). If concavity over large losses was shown to be a dominant pattern, finding out how to solve the dilemma between the descriptive and normative requirements would become necessary.

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