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An independence result concerning the arrival rate of and the provision of transport to tourists

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Abstract

What is the relationship between the arrival rate of tourists at a transport providing firm's depot and the long run expected wait time experienced by individual tourists in this depot? We use a renewal theoretic framework in this note to answer this question. Our analysis yields two results. First, we show that the long run expected wait time is independent of the arrival rate of the tourists. Second, we explain why this long run expected wait time is equal to the average residual life of a particular renewal process.

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1. Introduction

There is no denying the fact that tourism is now one of the world's largest growth industries. The recent work of Olson (2003) and Batabyal (2007) tells us that tourists from all over the world generally place a high premium on visiting naturally attractive sites such as fiords, lakes, and wildlife reserves. It is not standard practice for individuals wishing to tour these kinds of naturally attractive places to arrange for their own transport. Instead, private firms—and sometimes public entities—provide the pertinent transportation services. The reader will understand that these kinds of private firms also frequently provide transport to tourists interested in visiting sites within cities such as aquariums, museums, and zoos.

Research conducted by Nordstrom (2005) and by Chaudhary and Batabyal (2008) shows that the demand for visits to locations that are tourist attractions is probabilistic. This means that a firm providing transport to tourists is typically operating in a *stochastic* environment. Hence, a key problem confronting such a firm concerns the *provision* of transport. For concreteness, assume that our transport providing firm uses buses[1] to operate its tours to a particular location. Further, suppose that these buses carrying tourists to their chosen destination leave our firm's depot at fixed points in time. To use the language of Chaudhary and Batabyal (2008), this firm schedules its buses by time.

Now, issues pertaining to the provision of transport in general and to the provision of transport to tourists in particular have been studied by several researchers. Dessouky *et al.* (2003) have analyzed an optimization model and have shown that with only marginal increases in operating costs and service delays, it is possible to cut down the environmental effects of transportation appreciably. Louca (2006) has empirically studied the long run relationships between, on the one hand, income derived from the tourism industry in Cyprus and tourist arrivals, and, on the other, three categories of what he calls supply side expenditures. Focusing on Mauritius, Khadaroo and Seetanah (2007) point out that the demand for visits to this island depend greatly on the island's land and air transport infrastructure. Wolfler Calvo and Colorni (2007) have used a heuristic method to analyze an emerging transport system in which a fleet of vehicles, without fixed routes and schedules, carries individuals from a desired pickup point to the desired delivery point. Batabyal (2008) has studied the scheduling problem faced by a firm providing transport to tourists visiting a particular location during the slack season. Wang (2008) has used integer programming to optimize the location and the number of battery exchange stations for electric scooters that can reduce the environmental impacts of tourism transport. Batabyal and Beladi (2008) have conducted a stochastic analysis of visitor classification, vehicle breakdowns, and the provision of transport for tourism. Finally, Chaudhary and Batabyal (2008) have conducted a probabilistic analysis of two ways of providing transport to tourists.

The papers we have just discussed in the previous paragraph have certainly increased our understanding of several aspects of the provision of transport in the context of tourism. Even so, to

[1] The reader should note that our subsequent analysis does not depend on the specific *means* of transport.

the best of our knowledge, the extant literature has paid virtually *no* theoretical attention to the following important question concerning the provision of transport to tourists. What is the relationship between the arrival rate of tourists at a transport providing firm's depot and the long run expected wait time experienced by individual tourists in this depot? Given this lacuna in the literature, we use a renewal theoretic framework[2] in this note to answer this question. Our analysis yields two results. First, we demonstrate the counterintuitive result that the long run expected wait time is *independent* of the arrival rate of the tourists. Second, we explain why this long run expected wait time is *equal* to the average residual life of a particular renewal process.

The rest of this note is organized as follows. Section 2.1 describes the renewal-reward model and the renewal-reward theorem in particular. This theorem will form the centerpiece of our subsequent determination of the long run expected wait time in section 2.3. Section 2.2 uses renewal theory to describe a simple and parsimonious model in which tourists arrive in accordance with a homogeneous Poisson process[3] and then wait at our transport providing firm's bus depot. Section 2.3 first computes the long run expected wait time of an individual tourist in the depot and then shows that this wait time is independent of the arrival rate of the tourists. Section 2.4 explains why the expected wait time computed in section 2.3 is equal to the average residual life of a particular renewal process. Finally, section 3 concludes and discusses ways in which the research in this note might be extended.

2. The Theoretical Framework

2.1. Preliminaries

The so called renewal-reward model is a very useful tool in the analysis of a whole host of applied probability models. The thing to note in this context is that many stochastic processes are regenerative in nature. In other words and as noted by Tijms (2003, p. 33), they regenerate or renew themselves from time to time so that the behavior of the process after the regeneration or renewal epoch is a probabilistic reproduction of the behavior of the process starting at time $t=0$. The time interval between two regeneration or renewal epochs is called a cycle and the sequence of "regeneration cycles" makes up a renewal process. What is salient for our purpose is that the limiting or long run behavior of a regenerative stochastic process on which a particular reward structure is imposed can be studied in terms of the behavior of the process during a single regeneration cycle.

More formally, the textbook by Ross (1996, pp. 132-137) tells us that a stochastic process $\{Z(t):t\geq 0\}$ is a counting process if $Z(t)$ represents the total number of counts that have taken place by time t. Clearly, since $Z(t-1)$, $Z(t)$, $Z(t+1)$, etc. are stochastic, the time between any two counts $Z(t)$ and $Z(t-1)$ is also stochastic. This time between any two counts is called the interarrival time. A counting process for which the interarrival times have a *general* cumulative probability distribution function is a renewal process.

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See Ross (1996, pp. 98-162) and Tijms (2003, pp. 33-80) for textbook expositions of renewal theory.

3

See Ross (1996, pp. 59-97) and Tijms (2003, pp. 1-32) for textbook discussions of the homogeneous Poisson process.

Consider a renewal process $\{Z(t){:}t \geq 0\}$ with interarrival times X_z, $z \geq 1$, which have a cumulative probability distribution function $G(\cdot)$. In addition, assume that a monetary reward R_z is earned when the *zth* renewal is completed. Let $R(t)$, the total reward earned by time t, be $\Sigma_{z=1}^{Z(t)} R_z$, and let $E[R_z]=E[R]$, and $E[X_z]=E[X]$. The renewal-reward theorem—see Ross (1996, p. 133) or Tijms (2003, p. 41)—tells us that if $E[R]$ and $E[X]$ are finite, then with probability one,

$$\lim_{t \to \infty} \frac{E[R(t)]}{t} = \frac{E[R]}{E[X]}. \tag{1}$$

In words, equation (1) is telling us that if we think of a cycle being completed every time a renewal occurs, then the long run expected reward—the left-hand-side (LHS) of equation (1)—is the expected reward in a cycle or $E[\textit{reward per cycle}]=E[R]$ divided by the expected amount of time it takes to complete that cycle or $E[\textit{length of cycle}]=E[X]$. The reader should note that the renewal-reward theorem describes a *long run* or *steady state* result and this theorem holds for positive rewards such as profits and for negative rewards such as costs. Let us now proceed to our model of transport provision.

2.2. A model of transport provision

Consider a transport providing private firm that offers tourists bus tours to a particular location that is a tourist attraction. These tourists arrive at our firm's bus depot in accordance with a homogeneous Poisson process with rate $\lambda > 0$.[4] Buses carrying tourists depart from our firm's depot in accordance with a renewal process with interdeparture time $T > 0$. The task before us now is to compute the long run expected wait time of an individual tourist. To undertake this computation, we shall use the renewal-reward model described in section 2.1 and, in more detail, in Ross (1996, pp. 132-149) and in Tijms (2003, pp. 33-80).

2.3. Long run expected wait time

Let W_n denote the wait time of the *nth* tourist and let N_k denote the number of tourists arriving in the depot between the $(k-1)th$ and the kth departure of a bus. Some thought tells us that the waiting time stochastic process $\{W_n\}$ is a regenerative process that regenerates itself at the epochs N_1+1, $N_1+N_2+1, \ldots$ Now, assume that a reward of W_n is received for the *nth* tourist. Then, using the renewal-reward theorem—see section 2.1 above and theorem 3.6.1 in Ross (1996, p. 133) or theorem 2.2.1 in Tijms (2003, p. 41)—it follows that, with probability one,

$$\lim_{\{n \to \infty\}} \left(\frac{1}{n}\right) \sum_{k=1}^{k=n} W_k = \frac{E[\Sigma_{j=1}^{N_1} W_j]}{E[N_1]}, \tag{2}$$

[4] The Poisson process has been used previously by Martin-Cejas (2006), Batabyal (2007), and Chaudhary and Batabyal (2008) to study problems in the economics of tourism.

where $E[\cdot]$ is the expectation operator.

In words, the long run expected wait time of an individual tourist is equal to a particular ratio of expectations and that ratio is

$$\frac{E[\textit{total wait time within two departures of a bus}]}{E[\textit{number of arrivals within two departures of a bus}]}. \tag{3}$$

We can use equation 1.1.10 in Tijms (2003, p. 17) to mathematically describe the expectation in the numerator of (3). This procedure tells us that $E[\textit{total wait time within two departures of a bus} / \textit{interdeparture time is } x] = (1/2)\lambda x^2$. In our case, the interdeparture time is T. Therefore, the numerator of (3) reduces to $(1/2)\lambda E[T^2]$.

Recall that the tourists arrive at our transport providing firm's depot in accordance with a homogeneous Poisson process with rate $\lambda > 0$. This fact and the theory of renewal-reward processes together tell us that the denominator of (3) is given by $\lambda E[T]$. Now, putting the results contained in this and the previous paragraphs together, we get an expression for the long run expected wait time of an individual tourist in our firm's depot. That expression is

$$\textit{Long run expected wait time of individual tourist} = \frac{E[T^2]}{2E[T]}. \tag{4}$$

Intuitively, we would expect this expected wait time to depend on the arrival rate of the tourists. However, inspecting equation (4) it is clear that the long run wait time of an individual tourist in our model is *independent* of the tourist arrival rate $\lambda > 0$. Mathematically, this result holds because the arrival rate $\lambda > 0$ appears multiplicatively in the two expectations describing the numerator and the denominator of (3) and hence cancels out in equation (4). We now proceed to explain why the wait time in equation (4) is equal to the average residual life of a particular renewal process.

2.4. Average residual life

In the context of this note, the particular renewal process we are interested in is the renewal process for which the so called interoccurrence times are given by the *interdeparture* times of our transport providing firm's buses. Further, from Tijms (2003, p. 37) we know that for this renewal process, the residual life is "the time elapsed from epoch t until the next renewal after epoch t." Now, the mean residual life in our model is equal to the long run expected wait time of individual tourists because the Poisson tourist arrivals occur completely *randomly* in time. Put differently, this equality holds because of the well known "Poisson arrivals see time averages" or PASTA property. This property tells us that the long run fraction of *time* a stochastic system is in a particular state is equal to the long run fraction of tourist *arrivals* who find the stochastic system in this same state.[5]

3. Conclusions

In this note, we used a renewal theoretic framework to analyze the relationship between the arrival rate of tourists at a transport providing firm's depot and the long run expected wait time experienced by individual tourists in this depot. Our analysis yielded two results. First, we demonstrated the counterintuitive result that the long run expected wait time is *independent* of the arrival rate of the tourists. Second, we explained why this long run expected wait time is *equal* to the average residual life of a particular renewal process.

The analysis in this note can be extended in a number of directions. Here are two suggestions for extending the research described here. First, it would be useful to analyze the extent to which the results in this note change when the tourists under study arrive at a transport providing firm's depot in accordance with a *nonhomogeneous* Poisson process. Second, it would be instructive to set up and solve an optimization problem for a transport providing firm in which the long run expected wait time of an individual tourist from section 2.3 is a part of the objective function being optimized. Studies of transport provision to tourists in a probabilistic environment that incorporate these features of the problem into the analysis will provide additional insights into issues in the economics of tourism that have both theoretical and practical implications.

[5] See Tijms (2003, pp. 53-58) for a textbook exposition of the PASTA property.

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