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Equilibrium Location and Economic Welfare in Delivered Pricing Oligopoly

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Abstract

We investigate the equilibrium location pattern and welfare implication in delivered pricing model (or spatial price discrimination model) with a linear city. First, we extend a delivered pricing duopoly with Bertrand competition of Hamilton et al. (1989) to an n-firm model and explicitly solve the equilibrium location pattern. Next, we investigate welfare implication of the equilibrium location pattern. Given the Bertrand competition in the second stage we consider the welfare effect of relocations from the equilibrium locations. The equilibrium distance between firms is smaller than in the first best case, while it is too large from the second best viewpoint.

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1. Introduction

Since the seminal work of Hotelling (1929), a rich literature on spatial competition has emerged. The Hotelling model has become one of the most important methods of analyzing product differentiation. The major advantage of this approach is to endogenize product selection. Location models fall into two categories: shipping or delivered pricing (shopping or mill pricing) models are those in which firms (consumers) bear the transport costs. Although delivered pricing competition is widely observed, the literature on delivered pricing spatial competition has appeared relatively recently. Lederer and Hurter (1986) carry out a pioneering work on delivered pricing with Bertrand competition. Hamilton et al. (1989) solve the equilibrium location pattern of a Hotelling-type linear-city duopoly model with both Bertrand and Cournot competition.[1]

We generalize the Bertrand model of Hamilton et al. (1989) to an n-firm oligopoly.[2] Furthermore, we analyze welfare implications of the equilibrium location. We consider two types of problems. In one case the social planner controls both locations and prices of the firms (first best problem). In the other case the social planner controls only locations (second best problem). We find that in equilibrium the distance between the firms is too small from the first best viewpoint, while it is too large from the second best viewpoint.

2. Model

There is a linear city of length 1 where infinitely many consumers lie uniformly. There are n firms in the market and they engage in the following location-price competition. In the first stage, each firm simultaneously and independently decides where on the line to locate. Let $x_i \in [0, 1]$ be firm i's location, for $i \in \{1, ..., n\}$. After observing the rivals' locations, in the second stage each firm simultaneously and independently chooses its price level at every point (market) in the continuum $[0, 1]$ so as to maximize its profit. Without loss of generality, let $x_i \leq x_j$ for all $i < j$. For brevity, let the vector $\boldsymbol{x}$ signify the firm locations $(x_1, ..., x_n)$.

Assume that the demand function at each market is linear, i.e., $Q(x) = A - p(x)$, where A is a positive constant, and $p(x)$ and $Q(x)$ are the price and the total quantity supplied at market x, respectively. Each firm incurs a symmetric constant marginal cost of production, which we normalize to zero without loss of generality. The firms must pay transport costs. To ship a unit of the product from its plant x_i to a market at point x, firm i must pay a transport cost $t|x - x_i|$, where t is a positive constant and $|x - x_i|$ is the distance between x

[1]Greenhut and Greenhut (1975) and Norman (1981) examine Cournot competition in spatial models, but they discuss the equilibrium price pattern rather than the equilibrium location pattern.

[2]The equilibrium location pattern of the n-firm Cournot counterpart and its welfare implications are discussed by Anderson and Neven (1991) and Matsumura and Shimizu (2005), respectively. For the circular city version, see, Gupta et al. (2004), Matsumura et al. (2005), Matsushima (2001), Pal (1998), and Shimizu and Matsumura (2003).

and x_i. The consumers are assumed to have a prohibitively costly transport cost, preventing arbitrage. Finally, we assume that $A \geq 2t$. This ensures that effective competition exists at every market and monopoly pricing does not occur regardless of firm locations.

Consider firm i, where i is in $\{2, ..., n-1\}$. At the market x where $|x - x_{i-1}| \geq |x - x_i|$ and $|x - x_i| \leq |x - x_{i+1}|$, only firm i supplies and the price $p(x)$ becomes $\min\{t|x - x_{i-1}|, t|x - x_{i+1}|, p_i^M(x_i)\}$, where $p_i^M(x_i) \equiv (A - t|x - x_i|)/2$ is the monopoly price by firm i. Under the assumption $A \geq 2t$, the price $p(x)$ becomes $\min\{t|x - x_{i-1}|, t|x - x_{i+1}|\}$. Thus, among the locations that firm i serves, firm i supplies at the price $t|x - x_{i-1}|$ at the markets closer to firm $i - 1$ than to firm $i + 1$, and firm i sets the price at $t|x - x_{i+1}|$ at other markets. Firms 1 and n have only one neighbor each. Thus, for $x \in [0, (x_1 + x_2)/2]$ firm 1 sets its price at $t(x_2 - x)$, and for $x \in [(x_{n-1} + x_n)/2, 1]$ firm n supplies at price $t(x - x_{n-1})$. The profit of firm i at market x is given by

$$\begin{aligned}
\pi_i(x) &= (t|x - x_{i+1}| - t|x - x_i|)(A - t|x - x_{i+1}|) \\
&\qquad\qquad \text{if } \forall j \ |x - x_i| \leq |x - x_j|, \ \text{ and } \ |x - x_{i+1}| \leq |x - x_{i-1}|, \\
&= (t|x - x_{i-1}| - t|x - x_i|)(A - t|x - x_{i-1}|) \\
&\qquad\qquad \text{if } \forall j \ |x - x_i| \leq |x - x_j|, \ \text{ and } \ |x - x_{i-1}| \leq |x - x_{i+1}|, \\
&= 0 \qquad\qquad \text{otherwise.}
\end{aligned} \tag{1}$$

Each firm i chooses its location to maximize total profit Π_i, which is given by

$$\Pi_i(\mathbf{x}) = \int_{x \in [0,1]} \pi_i(x; \mathbf{x}) dx. \tag{2}$$

3. Equilibrium Location

We solve for the equilibrium location pattern in this n firm game. From here on, let $a = A/t$. Let the superscript 'E' denote equilibrium locations. For $n = 2$, Hamilton et al. (1989) provide the solution. In the duopoly, there is a direct interaction between the location of firm 1 and firm 2. For $n = 3$, however, there is only an indirect interaction between the location strategies taken by firm 1 and firm 3. That is, any movement by firm 1 (keeping the restriction $x_1 \leq x_2$) has no effect on the profit level of firm 3. Similarly, for $n \geq 3$, any movement by firm i within $x_i \in [x_{i-1}, \ x_{i+1}]$ has no impact on incentives for firms other than firm $i - 1$ and firm $i + 1$. Taking this into account, the following proposition summarizes the equilibrium location pattern when $n \geq 3$.

Proposition 1: *In equilibrium, firms* 1 *and* n *locate at*

$$x_1^E = \frac{2n(n-1)a - 2n - 1 - (n-1)\sqrt{4n^2a^2 - 12na + 8a + 7}}{2(n^2 - 6n + 2)}, \quad x_n^E = 1 - x_1^E.$$

Firm $i \in \{2, ..., n-1\}$ *locates at* $x_i^E = x_1^E + (i-1)(1 - 2x_1^E)/(n-1)$.

Proof: First we look for several necessary conditions for equilibrium. Using (1) and (2), we can derive the following three conditions: (i) For all $i \in \{2, ..., n-1\}, x_i^E = (x_{i-1}^E + x_{i+1}^E) / 2$. (ii) $x_1^E \neq 0, \quad x_n^E \neq 1$. (iii) $x_1^E = 1 - x_n^E$. Thus, the firms do not locate at the ends of the linear market and the locations are symmetric. The inner $n-2$ firms equally divide the region $[x_1^E, x_n^E]$ into $n-1$ parts. The length of the region is $1 - 2x_1^E$, so the following holds:

$$x_2^E = \frac{(1 - 2x_1^E)}{n-1} + x_1^E. \tag{3}$$

We also have the first order condition for firm 1's profit. That is,

$$\frac{\partial \Pi_1}{\partial x_1} = \frac{t^2}{8}\Big[4(x_2 - 3x_1)a - 7x_1^2 + 14x_1x_2 - 3x_2^2\Big] = 0$$

must hold. Using these two equations, solving for x_1^E and x_2^E and looking for answers in the appropriate range yield the desired result. The locations for firms $3, ..., n-1$ are derived using the above logic used to obtain (3). x_n^E is obtained from condition (iii).

To show that this location outcome is indeed an equilibrium, we have to check for the following possible deviations:

(a) An inner firm (firms $2, \cdots, n-1$) moving within its current neighbors. For example, firm 2 moving from x_2^E to somewhere between x_1^E and x_3^E.

(b) An inner firm moving outside the range between its neighbors but not at the edges of the linear city (between 0 and x_1^E and between x_n^E and 1).

(c) An inner firm moving to near an edge of the linear city.

(d) An outer firm (firm 1 or firm n) moving while keeping its neighbor. For example, firm 1 moving from x_1^E to somewhere between 0 and x_2^E.

(e) An outer firm moving to the other end of the market. For example, firm 1 moving from x_1^E to somewhere between x_n^E and 1.

(f) An outer firm moving to become an inner firm.

We have shown that if the firms do not realign their relative locations (so that $x_1 \leq x_2 \leq \cdots \leq x_n$ holds), the location pattern given in Proposition 1 is the only possible equilibrium location pattern. Thus we have already considered deviations (a) and (d). Deviation (d) is more profitable than deviation (e), as the potential neighbor is located further away from the end of the market, giving more room for the deviating firm. Deviation (f) is profitable only if deviation (b) is profitable, as we show in Proposition 2 that the outer firms make more profit than the inner firms. Deviation (b) is profitable only if deviation (c) is profitable, since the latter allows for a higher price and more market served. Finally, we show that deviation (c) is not profitable for the inner firm. The profits of firm 2 before and after deviation can be respectively rewritten as follows:

$$\begin{aligned}
\Pi_2^E &= 2t^2 \int_{(x_1^E + x_2^E)/2}^{x_2^E} (x - x_1^E - (x_2^E - x))(a - (x - x_1^E))dx, \\
\Pi_2^D &= t^2 [\int_0^{x_2^D} (x_1^E - x - (x_2^D - x))(a - (x_1^E - x))dx
\end{aligned}$$

$$+ \quad \int_{x_2^D}^{(x_2^D+x_1^E)/2} (x_1^E - x - (x - x_2^D))(a - (x_1^E - x))dx],$$

$$\text{where} \;\; x_2^D = \frac{-6a + 7x_1^E + 2\sqrt{9a^2 - 14ax_1^E + 7x_1^E}}{7}.$$

We proved that $\Pi_2^E \geq \Pi_2^D$ by the following three steps.
(Step 1) We prove that if $x_2^E - x_1^E > x_1^E - x_2^D$, then

$$\int_{(x_1^E+x_2^E)/2}^{x_2^E} (x - x_1^E - (x_2^E - x))(a - (x - x_1^E))dx \geq \int_{x_2^D}^{(x_2^D+x_1^E)/2} (x_1^E - x - (x - x_2^D))(a - (x_1^E - x))dx.$$

(Step 2) We prove that if $x_2^E - x_1^E > x_1^E + x_2^D$, then

$$\int_{(x_1^E+x_2^E)/2}^{x_2^E} (x - x_1^E - (x_2^E - x))(a - (x - x_1^E))dx \geq \int_0^{x_2^D} (x_1^E - x - (x_2^D - x))(a - (x_1^E - x))dx.$$

(Step 3) We prove that both $x_2^E - x_1^E > x_1^E - x_2^D$ and $x_2^E - x_1^E > x_1^E + x_2^D$ hold.
The detailed derivations in the three steps are available from authors upon request. **Q.E.D.**

Table I summarizes the numerical results for x_1^E and x_2^E for selected values of a and n.

Table I: Equilibrium locations for firms 1 and 2

	$a=2$		$a=3$		$a=5$		$a=10$		$a=100$	
n	x_1^E	x_2^E	x_1^E	x_2^E	x_1^E	x_2^E	x_1^E	x_2^E	x_1^E	x_2^E
3	0.1781	0.5000	0.1737	0.5000	0.1707	0.5000	0.1686	0.5000	0.1669	0.5000
4	0.1319	0.3773	0.1294	0.3765	0.1275	0.3758	0.1262	0.3754	0.1251	0.3750
5	0.1046	0.3023	0.1029	0.3015	0.1008	0.3004	0.1002	0.3001	0.1001	0.3000
10	0.0512	0.1509	0.0508	0.1506	0.0505	0.1504	0.0502	0.1502	0.0500	0.1500
40	0.0126	0.0376	0.0126	0.0375	0.0125	0.0375	0.0125	0.0375	0.0125	0.0375
100	0.0050	0.0150	0.0050	0.0150	0.0050	0.0150	0.0050	0.0150	0.0050	0.0150

The analytical outcomes of the equilibrium profit levels are difficult to parse due to the square root signs. However, we can show a result on the relative size of profits for the firms.

Proposition 2: *In equilibrium,* $\Pi_j > \Pi_k$*, where* $j \in \{1, n\}$ *and* $k \in \{2, 3, \cdots, n-1\}$*.*

Proof: We compare the equilibrium profit levels for firms 1 and $k(\in [2, n-1])$, applying the location pattern shown in Proposition 1. They are given as follows:

$$\begin{aligned} \Pi_1(x_1^E, x_{-1}^E) &= \int_0^{x_1^E} t(x_2^E - x - (x_1^E - x))(A - t(x_2^E - x))dx \\ &+ \int_{x_1^E}^{(x_1^E+x_2^E)/2} t(x_2^E - x - (x - x_1^E))(A - t(x_2^E - x))dx. \end{aligned} \tag{4}$$

$$\begin{aligned}\Pi_k(x_k^E, x_{-k}^E) &= \int_{(x_{k-1}^E+x_k^E)/2}^{x_k^E} t(x - x_{k-1}^E - (x_k^E - x))(A - t(x - x_{k-1}^E))dx \\ &\quad + \int_{x_k^E}^{(x_k^E+x_{k+1}^E)/2} t(x_{k+1}^E - x - (x - x_k^E))(A - t(x_{k+1}^E - x))dx. \end{aligned} \tag{5}$$

The second terms in (4) and (5) are equal in equilibrium, since the firms are located equidistantly. Thus, to prove that firm 1 has a larger profit than firm 2, we need to show that the first term in (4) is larger than that in (5) in equilibrium.

The integrand in each term is the local profit, with $(x_2^E - x)$ and $(x - x_{k-1}^E)$ the price. The former in the range $[0, x_1^E]$ is larger than the latter in the range $[(x_1^E + x_2^E)/2, x_2^E]$. Since by assumption the price is lower than the monopoly level, the integrand in (4) is larger than that in (5). Now, all we need to show is that the length of the range in (4), that is $[0, x_1^E]$, is larger than that in (5), $[(x_{k-1}^E + x_k^E)/2, x_k^E]$.

$$\begin{aligned} (x_1^E - 0) - \left(x_k^E - \frac{x_{k-1}^E + x_k^E}{2}\right) &= \frac{3x_1^E - x_2^E}{2} > 0 \iff 3x_1^E > x_2^E = x_1^E + \frac{1 - 2x_1^E}{n-1} \\ \iff x_1^E > \frac{1}{2n}, \qquad\qquad x_1^E - \frac{1}{2n} &= \frac{n-1}{n(2an^2 - 3n + 2 + n\sqrt{4n^2a^2 - 12na + 8a + 7})} > 0. \end{aligned}$$

Thus we have the desired result. **Q.E.D.**

The firms on the edges make larger profits than the firms located between them, as the former firms face only one neighbor, giving them more market area and less competition.

4. Welfare Implications

We consider welfare implications. We denote consumer surplus at each location by $cs(x) = \frac{1}{2}(Q(x))^2$. Consumer surplus from the whole market is denoted by

$$CS = \int_0^1 cs(x)dx,$$

total producer surplus by $\Pi = \sum_i \Pi_i$, and total welfare by $W = CS + \Pi$.

Consider the first best outcome. The social planner controls both prices and locations of firms. Then social welfare is maximized by marginal cost pricing of each firm and the location pattern that effectively minimizes transport costs. Thus, the first best is achieved by $x_i^{FB} = (2i-1)/(2n)$, $i \in \{1, ..., n\}$, and dividing the market so that each firm i only serves markets $[(i-1)/n, i/n]$. Note that half of the firms located to the left (right) of 1/2 are all located to the right (left) of the first best locations. That is, $x_i^E > (2i-1)/(2n)$ if and only if $i < (n+1)/2$.

We now examine the second-best problem. Given the Bertrand competition in the second stage, we discuss the welfare effect of relocation.

Proposition 3:

$$(a)\qquad \frac{\partial \Pi}{\partial x_1}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} < 0,\quad \frac{\partial CS}{\partial x_1}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} > 0,\ \text{and}\ \frac{\partial W}{\partial x_1}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} > 0.$$

$$(b)\qquad \frac{\partial \Pi}{\partial x_i}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} = \frac{\partial CS}{\partial x_i}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} = \frac{\partial W}{\partial x_i}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} = 0,\ (i=3,\ldots,n-2).$$

$$(c)\qquad \frac{\partial \Pi}{\partial x_2}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} \geq 0,\quad \frac{\partial CS}{\partial x_2}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} \leq 0,\ \text{and}\ \frac{\partial W}{\partial x_2}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} \leq 0$$

and the equalities are satisfied if and only if $n = 3$.

Proof: We here prove case (a). The proof of the other cases can be proceeded similarly and is available from the authors upon request. In case (a), note that a slight movement by firm 1 only affects a part of the market area served by firm 2. Thus we only need to look at those markets in the following welfare analysis.

From the first order condition for firm 1's equilibrium location we have

$$\frac{\partial \Pi}{\partial x_1}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} = \frac{\partial \Pi_2}{\partial x_1}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} = -\int_{(x_1^E+x_2^E)/2}^{(x_1^E+x_3^E)/2} t^2\Big(a-2(x-x_1^E)+|x-x_2^E|\Big)dx < 0.$$

Note that $\partial\pi_2(x)/\partial p(x) = t(a-2(x-x_1^E)+|x-x_2^E|)$ must be positive because the price $p(x) = t(x-x_1^E)$ is lower than the monopoly price $t(a+|x-x_2^E|)/2$. Finally, we have

$$\frac{\partial CS}{\partial x_1}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} = \int_{(x_1^E+x_2^E)/2}^{(x_1^E+x_3^E)/2} \frac{\partial t(x-x_1^E)}{\partial x_1^E}\frac{\partial cs(x)}{\partial p(x)}dx = \int_{(x_1^E+x_2^E)/2}^{(x_1^E+x_3^E)/2} t^2\Big(a-(x-x_1^E)\Big)dx > 0,$$

$$\text{and}\quad \frac{\partial W}{\partial x_1}\Big|_{\mathbf{X}=(x_1^E,\ldots,x_n^E)} = \int_{(x_1^E+x_2^E)/2}^{(x_1^E+x_3^E)/2} t^2\Big((x-x_1^E)-|x-x_2^E|\Big)dx > 0.\quad \textbf{Q.E.D.}$$

Note that the case for firm n $(n-1)$ is the mirror image of case a (c). Proposition 3(a) implies that a slight increase in the distance between the edge of the linear-city and the outside firm from their equilibrium distance increases consumer surplus, reduces the joint profit of the two firms, and increases the total social surplus. The direction of the welfare improving relocation in the second best setting is opposite from the first best one. In the second best case, a decrease in the distance between firms accelerates competition, particularly near the edges of the linear city, and thus improves welfare.

Finally, we compare the locations for the three location patterns we analyzed. Table 2 describes the relationship among the first best, second best, and equilibrium locations of the four firm case for different values of a. Let the superscript 'SB' and 'FB' denote the second best and the first best locations respectively.

From this table we can infer the following. First, similarly to the first best case, $x_1^{SB}+x_2^{SB} = 1/2$ holds. Therefore, firm 1 supplies to markets $[0, 1/4]$ and firm 2 supplies to markets

Table II: Equilibrium and second best locations for firms 1 and 2 when $n = 4$.

	$a = 2$	$a = 3$	$a = 5$	$a = 10$	$a = 100$
x_1^E	0.1319	0.1294	0.1275	0.1262	0.1251
x_1^{SB}	0.1345	0.1309	0.1284	0.1266	0.1252
x_1^{FB}	0.1250	0.1250	0.1250	0.1250	0.1250
x_2^E	0.3773	0.3765	0.3758	0.3754	0.3750
x_2^{SB}	0.3655	0.3691	0.3716	0.3734	0.3748
x_2^{SB}	0.3750	0.3750	0.3750	0.3750	0.3750

$[1/4, 1/2]$. Second, $x_1^{FB} < x_1^E < x_1^{SB}$ and $x_2^{SB} < x_2^{FB} < x_2^E$ hold. Third, as a increases, all three values converge. This is because $a \equiv A/t$ going to ∞ implies that the unit transport t is negligible. This result is consistent with the standard findings in location theory.

We offer an intuition for these inferences. Consider the 4-firm case. Both in the first best and the second best cases, welfare is maximized when inefficient transport cost is minimized. This occurs when firm 1 supplies to the first quarter and firm 2 supplies to the second quarter of the market. This is the reason why $x_1^{SB} + x_2^{SB} = 1/2$ holds.

Suppose that the firms locate so that $x_1 + x_2 = 1/2$. Given the firm's locations, in the first best pricing case the price $p(x)$ is equal to firm 1's marginal cost (unit transport cost) for $x \in [0, 1/4]$. Thus the quantity is decreasing in x for $x \in [x_1, 1/4]$. On the other hand, in the second best pricing case (as well as the equilibrium case), the price $p(x)$ is equal to firm 2's marginal cost (unit transport cost) for $x \in [0, 1/4]$. Thus the quantity is increasing in x for $x \in [x_1, 1/4]$.

Consider a change from the first best case of $x_1^* = 1/8$ ($x_2^* = 3/8$) to $x_1 = 1/8 + \varepsilon$ ($x_2 = 3/8 - \varepsilon$), where ε is small and positive. It lowers the market prices for all $x \in [0, 1/2]$ in the second best pricing case. This competition-accelerating effect does not exist in the first best pricing case. This is one of the factors yielding the difference between the first best and the second best locations. In addition, the move raises transport costs for markets $x \in [0, x_1]$ and $x \in [x_2, 1/2]$, and it lowers them for markets $x \in (x_1, x_2)$. In the second best (first best) pricing case, the quantity supplied for markets close to 1/4 is high (low). The cost reducing effect dominates (is dominated by) the cost rising effect in the second best (first best) pricing case. Note that $1/4 \in (x_1, x_2)$. This also yields the difference between the first best and the second best locations.

In this note we adopt the standard assumption of this field in this paper, such as uniform distribution of consumers. As Tabuchi and Thisse (1995) show, the non-uniform distribution of consumers can change the equilibrium locations drastically in the mill pricing model. Similar principle might apply to the delivered pricing model. Investigating this problem in the delivered pricing model remains for future research.

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