E C O N O M I C S   B U L L E T I N

Delivered Pricing, Positive Externalities and Firm Dispersion

Barnali Gupta
Miami University

Abstract

This note examines firm locations in a delivered pricing model with positive production externalities. We find that, quite counter intuitively, firms will disperse rather than move closer, when production externalities are positive and reciprocal. Furthermore, we see a divergence between the private and social optimal locations, which is in contrast to the coincidence of these locations in the standard delivered pricing model.

The author thanks the editor and an anonymous referee for their comments. All errors remain the sole responsibility of the author.
Citation: Gupta, Barnali, (2008) "Delivered Pricing, Positive Externalities and Firm Dispersion." *Economics Bulletin,* Vol. 12, No. 32 pp. 1-6
Submitted: March 28, 2008. **Accepted:** November 12, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume12/EB-08L10013A.pdf

1 Introduction

A classic example of a positive production externality is in Cheung's (1973) Fable of the Bees, where an apple orchard and a bee keeper generate reciprocal positive externalities. There are also reciprocal externalities between neighboring orchards where one orchard may rent more beehives than the other. While the focus of Cheung's work is on the contractual arrangements among firms in the presence of reciprocal externalities, the results are argued given the location agglomeration of the orchards. It is widely believed and supported by both theoretical and empirical research, that positive externalities induce firms to locate closer to the source of the externality. Sharing a common resource pool, of labor or transportation, is likely to generate relatively more agglomeration, rather than firm dispersion. Examining the location choice of 751 manufacturing plants built in the U.S. since 1980, findings by Head et al (1995) for example, support the hypothesis that industry level agglomeration benefits play an important role in location decisions. In a differentiated products model, where consumers search for optimal product characteristics and firms can influence search costs through their location choice, Stahl (1982) characterizes firm agglomeration. He shows that despite being competitors, firms find it profitable to locate close to each other.

In this paper we find that, quite contrary to the expected response, each firm has an incentive to move farther away from its rival in the presence of positive and reciprocal production externalities[1]. This result is driven by the delivered pricing framework where a firm's delivered price is influenced by the rival firm's market share. In Hotelling's (1929) linear city with delivered pricing, we show that each firm locates closer to its respective market endpoint when there is a positive externality. Furthermore, we see a divergence between the private and social optimal locations, which is in contrast to the coincidence of these locations in the standard delivered pricing framework.[2] Section 2 presents the model and the results are discussed in Section 3.

2 Model

We consider a two-stage location-price duopoly, producing a homogenous good. Denote firm i's output by q_i for $i = 1$ and 2. The linear market is the interval $[0, 1]$. Let $P_i(x, a, b)$ denote the delivered price schedule offered by firm i ($i = 1, 2$), located at a and b respectively, where x and a are measured from the left endpoint and b is measured from the right endpoint, of the market. The delivered price is each firm's offer to sell and deliver to each market point x. Delivered pricing implies that firms practice spatial price discrimination across consumers.

Firm i's production cost function is $C^i(q_1, q_2) = c(q_j)\ q_i, (i = 1, 2)$ and is symmetric and twice continuously differentiable. Further:

[1] If the externality is negative, the reduced form of the cost function is concave in own output and a price equilibrium in pure strategies does not exist at any pair of locations.

[2] See Hamilton et al, (1989).

$$\begin{aligned}
i.\ C^i_i(q_1,q_2) &> 0, (i=1,2). \\
ii.\ C^i_{ii}(q_1,q_2) &= 0, (i=1,2). \\
iii.\ C^i_{ij}(q_1,q_2) &< 0, (i \neq j = 1,2). \\
iv.\ C^i_j(q_1,q_2) &< 0, (i \neq j = 1,2). \\
v.\ C^i_{jj}(q_1,q_2) &\geq 0, (i \neq j = 1,2) \\
vi.\ |C^i_i(q_1,q_2)| &> |C^i_j(q_1,q_2)|, (i \neq j = 1,2).
\end{aligned}$$

Note that (i) and (ii) imply that marginal cost of production for firm i is strictly positive and constant with respect to its own output[3]. Conditions (iii) and (iv) state that firm $i's$ marginal cost and total cost is decreasing in the other firm's output. Condition (v) assumes that this decrease occurs at an increasing rate with respect to the rival firm's output. Moreover, own marginal cost of production is greater than the (absolute value of the) marginal external benefit, so that total cost for firm i does not fall if both firms increase production. (For example, the function $C^i(q_i, q_j) = q_i/q_j$ will satisfy the above conditions on cost specifications.)

The remaining elements of the model follow standard assumptions. Transport cost is linear with t being the cost per unit distance. Consumers are uniformly distributed over the market and each has a perfectly inelastic demand for one unit of the commodity with reservation price r. We assume that r is sufficiently large, to ensure that all consumers are served. If everyone buys, then $q_1 + q_2 = 1$. Furthermore, if there is some market boundary x^*, such that everyone located on the left of x^* buys from firm 1 and everyone on the right buys from firm 2, then $q_1 = x^*$ and $q_2 = 1 - x^*$. A consumer buys from the firm that offers the lowest price. If facing the same price, typically the consumer will buy from the firm with the lower delivered marginal cost. If both firms have the same delivered price and the same delivered marginal cost to a consumer at x, that consumer chooses a firm at random.

The firms play a two stage game. In stage one, they simultaneously choose locations. In stage two, each firm observes the rival's location, and simultaneously chooses delivered price schedules. At the end of stage two, consumers observe firm locations and delivered price schedules and buy according to the rules specified.

3 Results

Firm 1's delivered marginal cost to consumers in $[0, x^*]$ is strictly less than firm 2's delivered marginal cost over this range. Exactly the opposite is true for consumers in $[x^*, 1]$. By definition, a point x^* is a market division if firm 1's delivered marginal cost equals firm 2's delivered marginal cost at x^*:

$$t|x^* - a| + C^1_1(x^*, 1-x^*) - C^1_2(x^*, 1-x^*) = t|1 - b - x^*| + C^2_2(x^*, 1-x^*) - C^2_1(x^*, 1-x^*)$$

[3] The literature on delivered pricing typically assumes linear production cost. See Gupta (1994) for an analysis with convex costs.

Hence,

$$x^* = \frac{1+a-b}{2} + C_2^2(x^*,1-x^*) - C_1^1(x^*,1-x^*) + C_2^1(x^*,1-x^*) - C_1^2(x^*,1-x^*) \quad (1)$$

To a consumer at location x, each firm offers a delivered price which is the maximum of the two firms' delivered marginal cost at x. Hence, consumers in $[0, x^*]$ are offered a delivered price equal to firm 2's delivered marginal cost at x, and consumers in $[x^*, 1]$ are offered a delivered price equal to firm 1's delivered marginal cost at x. Since firm 1 has a cost advantage in $[0, x^*]$ and firm 2 in $[x^*, 1]$, firm 1 serves consumers located in $[0, x^*]$ and firm 2 serves consumers located in $[x^*, 1]$. Lemma 1 states the second stage SPNE delivered price schedules.

Lemma 1 *The SPNE price schedules are*

$$\begin{aligned} P^{1*}(x,a,b) &= P^{2*}(x,a,b) \\ &= max[t|x-a| + C_1^1(x^*,1-x^*) - C_2^1(x^*,1-x^*), \\ &\quad t|1-b-x| + C_2^2(x^*,1-x^*) - C_1^2(x^*,1-x^*)] \end{aligned}$$

Firm 1 will offer a consumer at x, a price schedule that is the maximum of its own delivered marginal cost and its rival's delivered marginal cost, undercut by some arbitrarily small δ. Hence, $P^{1*}(x,a,b) = \max[P^2(x,a,b) - \delta, t|x-a| + C_1^1(x^*,1-x^*) - C_2^1(x^*,1-x^*)]$, where δ is positive and arbitrarily small. Firm 2 likewise offers a consumer at x, the following price schedule: $P^{2*}(x,a,b) = \max[P^1(x,a,b) - \delta', t|1-b-x| + C_2^2(x^*,1-x^*) - C_1^2(x^*,1-x^*)]$, where δ' is positive and arbitrarily small. In the limit, δ, $\delta' \to 0$ and both firms offer each consumer at x the same price schedule, which is the maximum of their respective delivered marginal cost, as stated in the lemma above.

Lemma 2 *There exists a symmetric location equilibrium.*

Using the SPNE delivered price schedule from Lemma 1 in firm 1's profit function, we have:

$$\pi_1(a,b) = \int_0^{x^*} [t\,|1-b-x| + C_2^2(x^*,1-x^*) - C_1^2(x^*,1-x^*) - t\,|x-a|]dx - C^1(x^*,1-x^*)$$

Taking the second derivative and simplyfying we get:

$$\begin{aligned} \frac{\partial^2 \pi_1}{\partial a^2} &= \frac{2t^2[-t + (C_{21}^2(x^*,1-x^*) + C_{12}^1(x^*,1-x^*) + C_{21}^1(x^*,1-x^*) + C_{12}^2(x^*,1-x^*))]}{2t - [C_{21}^2(x^*,1-x^*) + C_{12}^1(x^*,1-x^*) + C_{21}^1(x^*,1-x^*) + C_{12}^2(x^*,1-x^*)]} \\ &\quad + [C_{21}^2(x^*,1-x^*) - C_{22}^2(x^*,1-x^*) - C_{11}^2(x^*,1-x^*) + C_{12}^2(x^*,1-x^*) \\ &\quad - C_{11}^1(x^*,1-x^*) + C_{12}^1(x^*,1-x^*) - C_{21}^1(x^*,1-x^*) - C_{22}^1(x^*,1-x^*)](\frac{\partial x^*}{\partial a})^2 \end{aligned}$$

Using conditions $(iii), (iv), (v)$ as specified for the cost function, it follows that

$$\frac{\partial^2 \pi_1}{\partial a^2} < 0$$

Strict concavity of π_1 guarantees a unique value of a which maximizes π_1 for each b. Hence the reaction function $a(b)$ is a continuous function from $[0,1]$ into $[0,1]$. From Brouwer's fixed point theorem, there must exist a b^* such that $a^* = a(b^*) = b^*$. Since both firms have identical cost and demand functions, $b^* = b(a^*)$ must also hold. Hence (a^*, b^*) defines a symmetric location equilibrium.

Proposition 3 establishes the incentive for each firm to disperse from its rival's given location.

Proposition 3 *If firms 1 and 2 are located at the first and third quartiles respectively and generate positive reciprocal externalities, then each firm has an incentive to unilaterally move toward its closest market endpoint, given the rival's location.*

The first derivative of firm 1's profit function is:

$$\begin{aligned}\frac{\partial \pi_1(a,b)}{\partial a} &= -2at + tx^* + [t(1+a-b-2x^*)]\frac{\partial x^*}{\partial a} \\ &\quad +[C_2^2(x^*,1-x^*) - C_1^2(x^*,1-x^*)]\frac{\partial x^*}{\partial a} \\ &\quad -[C_1^1(x^*,1-x^*) - C_2^1(x^*,1-x^*)]\frac{\partial x^*}{\partial a} \\ &\quad +[C_{21}^2(x^*,1-x^*) + C_{12}^2(x^*,1-x^*)]x^*\frac{\partial x^*}{\partial a}\end{aligned} \tag{2}$$

From (1), $x^* = \frac{1}{2}$ *at* $a = b = \frac{1}{4}$. Moreover,

$$\frac{\partial x^*}{\partial a} = \frac{t}{2t - [C_{21}^2(x^*,1-x^*) + C_{12}^1(x^*,1-x^*) + C_{21}^1(x^*,1-x^*) + C_{12}^2(x^*,1-x^*)]} > 0$$

Hence

$$\frac{\partial \pi_1(a,b)}{\partial a} < 0 \text{ at } a = 1/4.$$

Consider firm 1's incentive to move, holding firm 2's location fixed at the third quartile. By moving toward the endpoint, firm 1 effectively increases firm 2's market share and hence the positive external effect generated by that firm. In turn, this lowers firm 1's delivered marginal cost to its retained customers. Moreover, firm 1's lower market share also reduces the marginal external benefit for firm 2, which raises that firm's delivered marginal cost to its customers. Effectively that translates into a higher price that firm 1 can charge its retained customers. Both effects serve to increase firm 1's profit. The sum of these two effects dominates the negative effect of a smaller market share for firm 1. As a result, firm 1 has an incentive to move outside the market quartile and toward the endpoint. The next proposition establishes the first stage location equilibrium.

Proposition 4 *There exists a symmetric equilibrium such that the firms choose to locate outside the quartiles and closer to the market endpoints.*

Recall (2):

$$\frac{\partial \pi_1(a,b)}{\partial a} = -2at + tx^* + [t(1+a-b-2x^*)]\frac{\partial x^*}{\partial a} + [C_2^2(x^*,1-x^*) - C_1^2(x^*,1-x^*)]\frac{\partial x^*}{\partial a}$$
$$-[C_1^1(x^*,1-x^*) - C_2^1(x^*,1-x^*)]\frac{\partial x^*}{\partial a}$$
$$+[C_{21}^2(x^*,1-x^*) + C_{12}^2(x^*,1-x^*)]x^*\frac{\partial x^*}{\partial a}$$

At a symmetric equilibrium:

$$-2at + \frac{t}{2} + \frac{[C_{21}^2(x^*,1-x^*) + C_{12}^2(x^*,1-x^*)]}{2}\frac{\partial x^*}{\partial a} = 0$$

or,

$$a^* = \frac{1}{4} + \frac{[C_{21}^2(x^*,1-x^*) + C_{12}^2(x^*,1-x^*)]}{2}\frac{\partial x^*}{\partial a} \tag{3}$$

Since $C_{21}^2(x^*,1-x^*), C_{12}^2(x^*,1-x^*) < 0$ and $\frac{\partial x^*}{\partial a} > 0$, it follows that $a^* < 1/4$.

From (3), note that if a firm's marginal cost of production is constant and independent of the external marginal benefit, $C_{21}^2(x^*,1-x^*) = 0, (i \neq j = 1,2)$, then the quartiles would constitute a symmetric equilibrium location pair, even with a positive reciprocal externality, $C_j^i < 0, (i \neq j = 1,2)$. That is so because given its rival's location at the market quartile, the firm cannot affect its rival's marginal cost and hence its own delivered price, by relocating away from the quartile. If it were to disperse outside the quartile, it would only lose market share, without any offsetting increase in delivered price. This is similar to the outcome with the standard constant marginal cost assumption.The incentive to relocate is generated by a firm's ability to influence its rival firm's delivered marginal cost and hence its own delivered price, via $C_{21}^2(x^*,1-x^*), C_{12}^2(x^*,1-x^*) < 0$, manifested in the positive reciprocal externality. In this case, we see from (3), that the firms locate outside their respective quartiles, so that they are farther away from each other.

It should be noted that the social cost minimizing locations with the positive externality, are at the market quartiles, since the entire market is served and therefore total output does not change as the firms relocate. Hence the social cost minimizing locations coincide with the locations that minimize transport cost. Therefore, contrary to the usual result with delivered pricing, the profit maximizing locations do not coincide with the social cost minimizing locations.

References

[1] Cheung, S. N.S. (1973) "The Fable of the Bees: An Economic Investigation" *Journal of Law and Economics* 16, 11-33.

[2] Gupta, B. (1994) "Competitive Spatial Price Discrimination with Strictly Convex Production Costs" *Regional Science and Urban Economics* 24, 265-272.

[3] Hamilton, J.H., J. Thisse and A. Weskamp (1989) "Spatial Discrimination: Bertrand versus Cournot in a model of Location Choice" *Regional Science and Urban Economics* 19, 87-102.

[4] Head, K., J. Reiss and D. Swenson (1995) "Agglomeration Benefits and Location Choice: Evidence from Japanese Manufacturing Investments in the U.S." *Journal of International Economics* 38, 223-247.

[5] Hotelling, H. (1929) "Stability in Competition" *Economic Journal* 39, 41-57.

[6] Stahl, K. (1982) "Differentiated Products, Consumer Search and Locational Oligopoly" *Journal of Industrial Economics* 31, 97-113.