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Equilibrium wage dispersion and the role of endogenous search effort revisited

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Abstract

This paper considers whether extending the on-the-job search model to account for endogenous search effort can help explain the shape of the wage distribution. More precisely, I show that it can be the case that the model generates hump-shaped wage distributions if preferences are non-separable between consumption and leisure.

1   Introduction

Since Burdett and Mortensen [1998] (BM henceforth), it is has been well known that the on-the-job search model is characterized by equilibrium wage dispersion, despite the fact that workers are *ex ante* homogenous. Nevertheless, this benchmark implies that the equilibrium wage density is increasing, in contrast to the observed hump-shaped wage distribution.

Taking into account *ex ante* exogenous heterogeneity in job productivity and worker ability improves the predictive power of the model. This allows a good fit with the wage distribution (see Bontemps, Robin and Van den Berg [2000] and Postel-Vinay and Robin [2002]). However, the impact of *ex post* heterogeneity is not clear cut. On the one hand, endogenous specific human capital investments not only generate *ex post* productivity dispersion, but it can also account for hump-shaped equilibrium wage distributions (see Mortensen [2000], Quercioli [2005], Rosholm and Svarer [2004] or Chéron, Hairault and Langot [2008]). On the other hand, the assumption of endogenous search effort no longer helps to circumvent the empirical shortcomings of the BM model (see Mortensen [2003]). The intuition for this latter result is as follows. Because the likelihood of finding a better offer decreases with the wage earned, this leads workers to reduce their search intensity whenever their earnings are increasing. The search intensity ultimately falls to zero at the top of the wage distribution. This provides incentives to firms to post high wages, because the decrease in the search effort for high levels of wages raises the expected job duration.

This paper considers an extension of the BM model in order to generate a non-strictly decreasing search effort. To that end, we develop an on-the-job search model with two types of jobs and non-separable preferences between consumption and leisure. Computational experiments emphasize the potentially significant role of an endogenous search effort. According to realistic parameter values for preferences, it turns out that the model can imply hump-shaped wage distributions despite the fact that workers are *ex ante* homogenous.

2   Model

2.1   Environment and labor market flows

We distinguish two types of job which can be thought of as corresponding to differences in occupational categories. Let p_i be the marginal productivity in occupational category $i = 1, 2$, and assume $p_1 < p_2$.

Time is continuous. Homogenous workers exit unemployment at rate λ_0, and start working in type-1 jobs. Then, they work h hours, face job destruction at rate δ, and search on-the-job not only for better opportunities in p_1-jobs, but also for p_2-jobs, according to a search intensity s. A transition from a p_1-job to a p_2-job typically features a promotion by switching jobs[1]. The search is sequential and non-directed, and we let $s\lambda_i \ \forall i = 1, 2$ denote the arrival rates of jobs, either of type-1 or of type-2, increasing with the search effort. In p_1 jobs, wages are posted by firms according to the standard wage posting game. Cahuc, Postel-Vinay and Robin [2006] indeed emphasize that the earnings of low-skilled workers are mainly related to this wage setting, whereas the impact of bargaining power is greater for high-skilled workers. We then consider that once an employee reaches a p_2-job position, he earns productivity p_2. That is, we are assuming that the bargaining power of workers in p_2-jobs is one and, as a consequence, the worker stops searching on-the-job. Nevertheless, those jobs also destroy at rate δ.[2]

Let us normalize to one the sum of unemployed workers and employees in p_1-jobs ; the steady state unemployment rate is given by $u = \frac{\delta}{\delta+\lambda_0}$.[3] Then, let G and F denote the cumulative distribution functions of earnings and wage offers in p_1-jobs ; the stock of employees earning w or less is $(1-u)G(w)$. The outflows from this stock are related to *(i)* job destructions, *(ii)* p_1-wage offers greater than w, and *(iii)* p_2-job offers. On the other hand, the inflows into the stock $(1-u)G(w)$ consist of unemployed workers who draw a wage offer below w. In steady-state, $G(w)$ is derived from the following condition:

$$u\lambda_0 F(w) = (1-u)\left\{ G_1(w)\delta + \lambda_1[1 - F(w)] \int_{\underline{w}}^{w} s(y)g(y)dy + \lambda_2 \int_{\underline{w}}^{w} s(y)g(y)dy \right\}$$

where $g(w) \equiv G'(w)$ is the density of wage earnings.

2.2   Intertemporal values of the workers

Let $\mathcal{U}$, W_1 and W_2 denote the expected discounted lifetime income of unemployed workers, and employees in p_1 and p_2-jobs, respectively. Unemployed workers earn unemployment benefits z and search for p_1-job offers. For the sake of simplicity, we consider a binding minimum wage $\underline{w}$ high enough to

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[1]For instance, a Technician who finds a Manager's position in another firm.

[2]Allowing for a lower bargaining power for workers on p_2-job position would add some complexities but without modifying, at least qualitatively, our analysis of the wage distribution impact of the search effort function.

[3]Because all equilibrium wage offers are greater than the unemployed worker's reservation wage, the contact rate λ_0 gives the unemployment exit rate.

imply that it is always in the unemployed worker's interest to accept job offers. These value functions write as follows:

$$
\begin{aligned}
r\mathcal{U} &= z + \lambda_0 \int_{\underline{w}}^{\overline{w}} \left[W_1(y) - \mathcal{U} \right] f(y) dy \\
rW_1(w) &= \max_{s \geq 0} \left\{ V(w, 1 - h - s) + \delta \left[\mathcal{U} - W_1(w) \right] \right. \\
&\quad \left. + s\lambda_1 \int_{w}^{\overline{w}} \left[W_1(y) - W_1(w) \right] f(y) dy + s\lambda_2 \left[W_2(p_2) - W_1(w) \right] \right\} \\
rW_2(p_2) &= V(p_2, 1 - h) + \delta \left[\mathcal{U} - W_2(p_2) \right]
\end{aligned}
$$

where r denotes the interest rate, the function V is increasing and concave in both arguments, and $f(w) \equiv F'(w)$ is the density of wage offers. Obviously, it is always in workers' interest to accept a wage greater than their current earnings. Because $p_2 > p_1$, it is also true that $\overline{w} < p_2$ which implies $W_2(p_2) > W_1(\overline{w})$, so that it is always in workers' interest to accept a p_2-job offer.

2.3 The wage setting process

The unconditional probability that an offer w is accepted by a randomly contacted worker, denoted by $h_1(w)$, is defined by:

$$
h_1(w) \equiv \frac{\lambda_0 u + (1-u)\lambda_1 \int_{\underline{w}}^{w} s(y)g(y)dy}{\lambda_0 u + (1-u)\lambda_1 \int_{\underline{w}}^{\overline{w}} s(y)g(y)dy} = \frac{\delta + \lambda_1 \int_{\underline{w}}^{w} s(y)g(y)dy}{\delta + \lambda_1 \int_{\underline{w}}^{\overline{w}} s(y)g(y)dy}
$$

A job-worker separation occurs either because of a job destruction or because of a job-to-p_1/p_2-job transition, so that the employer's value of a continuing match, $J_1(w)$, solves the following asset pricing equation:

$$
rJ_1(w) = p_1 - w - \left\{ \delta + \lambda_1 [1 - F(w)]s(w) + \lambda_2 s(w) \right\} J_1(w)
$$

It is worth emphasizing that this expected job value highly depends on the way the search effort is related to p_2-job opportunities. Importantly, there is still an incentive for the highest-paid workers to search on-the-job for the p_2-position.

The wage posting policies of the firms solve the conventional problem $w = \arg\{\max_{w \geq \underline{w}} h_1(w) J_1(w)\}$ from which we derive the distribution of wage offers, $F(w)$.

2.4 Optimal search effort

The optimal search effort of the worker is characterized by the following first-order condition:

$$V_2(w, 1 - h - s) \;=\; \lambda_1 \int_w^{\overline{w}} \left[W_1(y) - W_1(w) \right] f(y) dy + \lambda_2 \left[W_2(p_2) - W_1(w) \right]$$

so that $s = s(w)$. Differentiating this optimal condition once again with respect to w yields:

$$V_{22}(w, 1 - h - s(w)) s'(w) = V_{21}(w, 1 - h - s(w)) + (\lambda_2 + \lambda_1 [1 - F(w)]) W_1'(w) ght) W_1'(w) \tag{1}$$

where $V_{22} \leq 0$ and $V_{21} \gtreqless 0$. Usually (see Mortensen [2003]), it is both assumed $\lambda_2 = 0$ and a separable utility function, that is $V_{21}(w, 1 - h - s) = 0$; in this case it unambiguously emerges that $s(\overline{w}) = 0$ and $s'(w) \leq 0 \;\forall w$.

Our paper departs from these two parameter restrictions. This implies that it is still worthwhile for the highest paid workers in p_1-jobs to search on-the-job, because they look forward to a p_2-job. Furthermore, according to $V_{21} \gtreqless 0$, the relation between the search effort and wages becomes unclear:

- On the one hand, as usual, the higher the wage of the worker, the lower the probability of finding a p_1-job offer with a higher wage.

- On the other hand, if $V_{21}(w, 1 - h - s) < 0$, a higher wage decreases the marginal value of leisure (reduces the marginal cost of the search). Then, it could be the case that this effect is high enough to account for an increasing relationship between the search effort and wage earnings.[4]

3 Equilibrium Search Effort and the Shape of the Wage Distribution

3.1 Labor market equilibrium and calibration

The labor market equilibrium can be summarized by a system which jointly defines $\{F(w), s(w)\}$. Let us consider the particular case where $r \to 0$. We have:

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[4]On the contrary, if $V_{21}(w, 1 - h - s) > 0$, the search effort is unambiguously increasing with wages.

$$f(w) = \frac{\delta + \lambda_1[1 - F(w)]s(w) + s(w)\lambda_2}{2s(w)\lambda_1(p_1 - w)} + \frac{s'(w)[\lambda_2 + \lambda_1(1 - F(w))]}{2s(w)\lambda_1}$$

$$s'(w) = \frac{V_{21}(w, 1 - h - s(w)) + \frac{\lambda_1[1-F(w)]}{\delta + s(w)\lambda_1[1-F(w)]+s(w)\lambda_2}V_1(w, 1 - h - s(w))}{V_{22}(w, 1 - h - s(w))}$$

where $F(w)$ and $s(w)$ satisfy the following boundary conditions:

$$F(\underline{w}) = 0$$

$$V_2(\underline{w}, 1 - h - s(\underline{w})) = \left(\frac{\lambda_2}{\delta + s(\underline{w})\lambda_2}\right)[V(p_2, 1 - h) - V(\underline{w}, 1 - h - s(\underline{w}))]$$
$$+ \lambda_1\left(1 - \frac{s(\underline{w})\lambda_2}{\delta + s(\underline{w})\lambda_2}\right)\int_{\underline{w}}^{\overline{w}}\left(\frac{[1 - F(y)]V_1(y, 1 - h - s(y))}{\delta + s(y)\lambda_1 + s(y)\lambda_2}\right)dy$$

and $\overline{w}$ is given by $F(\overline{w}) = 1$.

We consider the following CRRA specification of the utility function:

$$V(w, 1 - h - s) = \begin{cases} \frac{(w^\alpha(1-h-s)^{1-\alpha})^{1-\rho}}{1-\rho} & \text{(if } \rho \neq 1) \\ \alpha \log(w) + (1 - \alpha)\log(1 - h - s) & \text{(if } \rho = 1) \end{cases}$$

with $\rho \geq 0, \alpha \in [0, 1]$, and where the value of ρ with respect to 1 determines the sign of the cross-partial $V_{12}(w, 1 - h - s)$. Our main purpose is then to emphasize on the explaining role played by the search effort. Therefore, simulations aim at showing how sensitive is the equilibrium wage offer distribution with respect to parameter ρ. Other parameters are set in a fairly standard way to provide illustrative simulations (see Table 1).

h	α	$\underline{w}$	p_1	p_2	δ	λ_0	λ_1	λ_2
1/3	2/3	1	5	10	0.1	1	0.3	$\lambda_1/10$

3.2 Computational experiments

Figure 1 shows our simulation results for search effort and wage offer density functions according to plausible empirical values of risk-aversion parameter ρ[5]. When $\rho = 1$, separability between consumption and leisure leads to

[5]For instance King and Rebelo [1999] consider $\rho = 3$ in their Handbook of Macroeconomics' paper.

Figure 1. Illustrative simulations
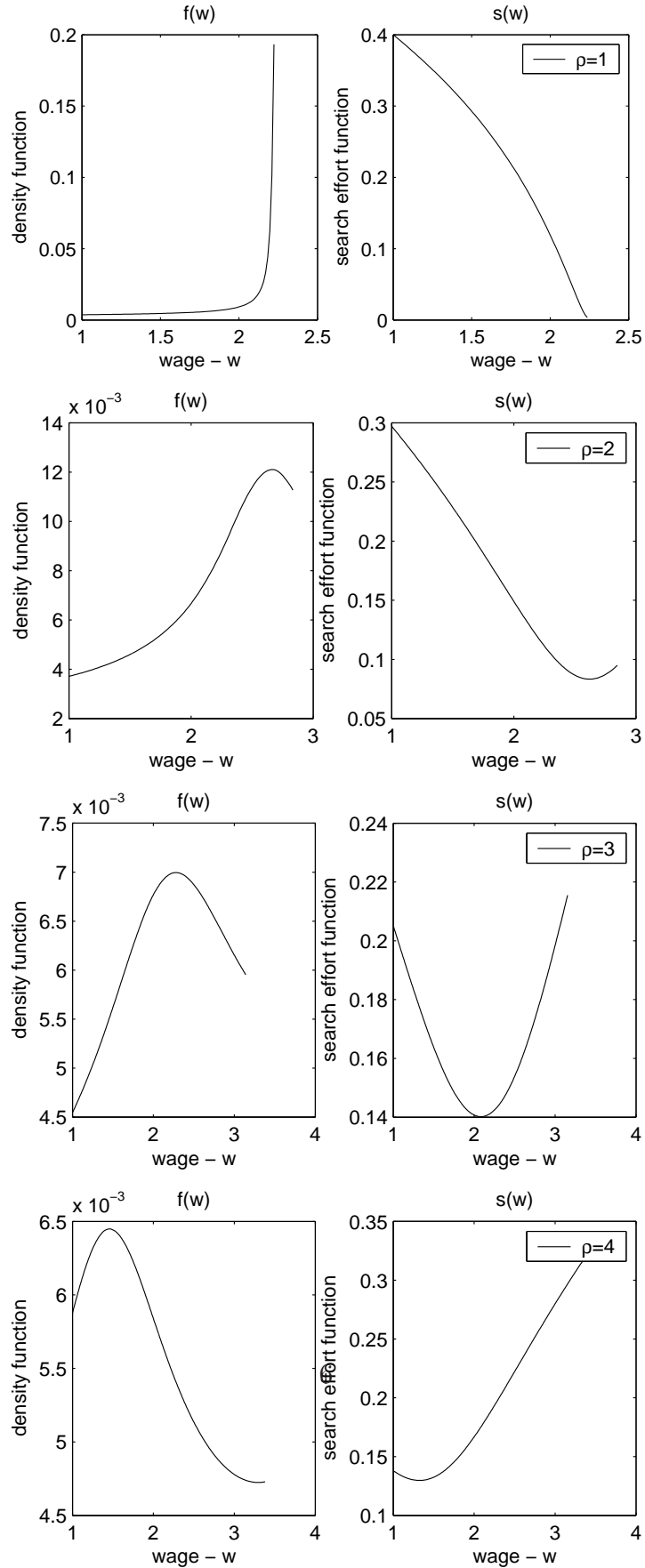
the conventional Burdett-Mortensen outcome: the search effort (earnings density) is a decreasing (increasing) function of wages.

If we consider $\rho > 1$, it turns out that the search effort can rise with wages, at least for high levels of earnings. Despite the fact that the likelihood of accepting a p_1-job offer decreases when workers' wages rise, higher earnings also account for higher consumption, and this provides incentives to workers to increase their search effort as a means of reaching p_2-jobs. Interestingly, if the strength of the latter mechanism is high enough, the wage offer density is hump-shaped.[6] Indeed, because for high level of wages workers raise their search effort, this reduces firms' expectations of job duration. As compared to an equilibrium with a decreasing search effort function, this provides incentives to firms to post lower wages. Then, it can be the case that the overall wage distribution is hump-shaped.

The main concern is then the following: how plausible is a non-strictly decreasing shape of the (unobservable) effort function? From an empirical standpoint, Bowlus and Neuman [2006] first noticed by using US data that the job-to-job transition rate rises as the wage increases. Using French data, Chéron and Ding [2008] also emphasized that the probability of a transition to a higher skill-occupation is increasing with wages. This occurs more specifically for Technician-Manager job-to-job transitions. These empirical findings cast some doubts on the empirical plausibility of the standard BM model. The fact that workers' search effort increases as wages rise, as implied by our framework, could be an explanation of those empirical findings.

4 Concluding remarks

To conclude, it seems worth discussing a direct extension of this work. One may indeed wonder to what extent labor market equilibrium properties depend on the bargaining power of workers in p_2-jobs, which here is assumed to be one. For values between zero and one, wage earnings would be a weighted average of productivity p_2 and workers' reservation wages. Accordingly, a hump-shaped p_1-distribution of wages would account for a hump-shaped distribution of those reservation wages. Consequently, this should also contribute to generating a hump-shaped p_2-distribution of wages. We leave this extension of our canonical model for future research.

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[6]Higher values for ρ could also imply a strictly decreasing wage offer density.

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