E C O N O M I C S B U L L E T I N

Fertility-related pensions and fertility disincentives

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Abstract

Since recent studies have argued that a pro-natalist effect could be obtained by introducing fertility-related pension systems for contrasting, especially in European countries, the plague of below-replacement fertility and the resulting problem of financing the widespread pay-as-you-go (PAYG) pension benefits, we built up an overlapping generations (OLG) general equilibrium model with endogenous fertility, to investigate whether and how a fertility-related pension reform increases population growth. We show that if the capital's share in production is high enough, such a reform, in contrast with the suggestions of the preceding literature, always reduces the long-run fertility rate.

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1 Introduction

Many developed countries are currently plagued by below-replacement fertility (e.g., Germany, Italy, Japan and Spain). Moreover, population ageing raised the problem of financing the widespread pay-as-you-go (PAYG) pension benefits. Recent studies have argued that a pro-natalist effect as well as a welfare-improving result could be obtained by introducing fertility-related pension systems (FR-PAYG) (e.g., Kolmar, 1997; Abio et al., 2004; Sinn, 2004; Fenge and Meyer, 2004, 2005). Early discussions of the idea of pension payments linked with the individuals' desired number of children are in Bental (1989), Nishimura and Zhang (1992, 1995), Zhang and Zhang (1995), Kolmar (1997), and more recently in van Groezen et al. (2003), Abio et al. (2004), Fenge and Meier (2004, 2005). While the first four articles, as well as Sinn (2004), considered children as an investment good, the other authors introduced children as a consumption good. Our model belongs to the latter strand of literature, and its general structure is relatively close to that one developed by van Groezen et al. (2003). However, differently from the latter model, we use a general rather a partial equilibrium context and investigate the effects of the introduction of a FR-PAYG pension reform.[1] Furthermore, in line with Kolmar (1997) and van Groezen et al. (2003), we assume the cost of raising children to be a fixed amount of consumption good, in contrast with Abio et al. (2004) and Fenge and Meier (2004, 2005), who introduced the cost of child-rearing as an opportunity cost of the parents' home time which is increasing in their working income.[2] Moreover, we also consider the child-subsidy support policy as in van Groezen et al. (2003) and Fenge and Meier (2005), which is, on the contrary, neglected by the other papers above cited.

We note that pension systems linking benefits with the individuals' desired number of children have been recently proposed in many developed countries, or, in some cases, they have already been – at least partially – adopted, as a means of increasing (reducing the drop in) population growth.[3] The aim of a FR-PAYG reform should be, loosely speaking, to enhance the number of children which seems to be insufficient to keep balanced a publicly provided PAYG pension scheme. In this paper we investigate whether and how the introduction of a FR-PAYG pension reform reaches its goal. We show that if the capital's share in production is sufficiently high, such a reform, in contrast with the prevailing literature, does not ever reach its goal, i.e. it always disincentives fertility behaviour.

The remainder of the paper is organised as follows. In section 2 we develop the model. In section 3 the comparative static analysis is presented, and the main analytical results are discussed and numerically illustrated with an example. Finally, Section 4 bears conclusions.

2 The model

2.1 Government

Following van Groezen et al. (2003), we suppose the government runs two distinct balanced budget policies in every period for both childcare expenditure and social security.

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[1] The importance to this extension dates back to Cigno (1993), who argued that the effects on fertility induced by PAYG pension systems may be quite different in a closed economy or in a large open economy in comparison with those in a small open economy. For instance, Fenge and Meier (2005) concluded that "…an interesting extension of the current analysis would be to drop the assumption of a small open economy", and later "…tackling the problem addressed here in a framework with endogenous factor prices is of interest." (p. 45).

[2] We note that the qualitative results of this paper do not depend on this assumption. In a companion paper we show that such results hold even under the hypothesis of time cost of child-rearing.

[3] As noted by Zhang and Zhang (1995), in countries such as North America, Europe and Japan there exist elements of social security which depend on individual fertility (e.g., survivor benefits for dependents as well as portions of old-age pension payments). Nishimura and Zhang (1995) reported that: i) in France, child supplement is 10 per-cent of pensions if the insured reared three or more children; ii) in the United Kingdom, child supplement is £ 8.40 per child; iii) in the United States, dependents' allowance is 50 per-cent of workers' pension to each child. More recently, Fenge and Meier (2005, p. 29) argued that "while many countries have already adopted some type of child benefit in the pension formula, the size of this benefits is everywhere much smaller than the contributions of the child to the pension system".

The child subsidy is supposed to be entirely financed by levying and adjusting over time lump-sum taxes on the young-adult generation. Therefore, the per-capita time-t childcare government constraint is simply:

$$\beta n_t = \tau_t , \tag{B1}$$

where the left-hand side represents the total childcare expenditure and the right-hand side the tax receipts, with $\beta > 0$ being a (constant) subsidy per child and $\tau_t > 0$ is a lump-sum tax. Notice that agents act in an atomistic way and do not take the government budget (1) into account when deciding on the desired number of children.

As regards the social security system, the government balances its budget in every period according to:

$$p_t = \theta w_t \cdot [(1-\omega)\bar{n}_{t-1} + \omega n_{t-1}], \tag{B2}$$

where the left-hand side represents the social security expenditure and the right-hand side the tax receipts, with $0 < \theta < 1$ being a constant proportional-to-wage contribution rate paid by young-aged individuals, $0 \leq \omega \leq 1$ is a parameter measuring the relative importance of the (individual's) household's number of children on pension payments (if $\omega = 0$ we have a pure PAYG system; if, on the contrary, $\omega = 1$ we have a fully FR-PAYG, see, for instance, Kolmar, 1997; Abio et al., 2004; Fenge and Meier, 2005).[4] Eq. (2) shows that pension benefits depends, with a share ω of the contribution, on the individual's desired number of children and, with a share $1-\omega$, on the aggregate growth factor of the population. Following Fenge and Meier (2005, p. 34), we will call the policy variable ω the child factor. Notice that $\bar{n}$ represents the average fertility rate of the economy while n is the private number of children freely chosen by the agents.

2.2 Individuals

Identical agents (N_t) are supposed to have a finite lifetime and overlap over three periods: childhood, young adulthood and old-age. During childhood individuals do not make economic decisions and thus they consume a fixed fraction of the time endowment from their parents. Adult individuals belonging to generation t have a homothetic and separable utility function defined over young-aged consumption ($c_{1,t}$), old-aged consumption ($c_{2,t+1}$), and the number of children (n_t), as in Galor and Weil (1996).

Only young-adult individuals join the workforce, and the labour supply is supposed to be constant and normalised to unity. During adulthood, each individual receives a working income at the competitive rate w_t which is used to consume, to pay taxes, to raise children and to save. We assume that raising children requires a fixed cost $e > 0$ per child (measured in units of market goods). Moreover, we suppose parents receive a direct monetary transfer for each child $\beta \in (0, e)$ – provided by the government at balanced budget – to support child-rearing activities.

During old-age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}. Furthermore, each old-age individual is entitled to a publicly provided pension benefit (p_{t+1}) financed at balanced budget by the government.

Therefore, the representative individual born at time t is faced with the following program:

$$\max_{\{c_{1,t}, c_{2,t+1}, n_t\}} U_t(c_{1,t}, c_{2,t+1}, n_t) = \ln(c_{1,t}) + \chi \ln(c_{2,t+1}) + \phi \ln(n_t), \tag{P}$$

subject to

$$c_{1,t} + s_t = w_t(1-\theta) - \tau_t - (e-\beta)n_t , \tag{C1}$$

$$c_{2,t+1} = (1+r_{t+1})s_t + \theta w_{t+1} \cdot [(1-\omega)\bar{n}_t + \omega n_t], \tag{C2}$$

where $0 < \chi < 1$ is the subjective discount factor and $0 < \phi < 1$ captures the importance in the welfare function of consuming while young relative to the utility of children.

[4] We assume agents to have perfect foresight with respect to the level of the future public pension benefit.

Therefore, the first order conditions for an interior solution are given by:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1}{\chi} = 1 + r_{t+1}, \tag{FOC1}$$

$$\frac{\phi}{n_t} = \frac{1}{c_{1,t}} \cdot \left(e - \beta - \omega\theta \frac{w_{t+1}}{1 + r_{t+1}} \right), \tag{FOC2}$$

Eq. (FOC1) equates the marginal rate of substitution between working period and retirement period consumption to their relative prices, whereas Eq. (FOC2) equates the marginal utility of having a child with the involved marginal cost in terms of forgone utility of consumption.

Exploiting (B1)-(B2), (FOC1)-(FOC2) and (C1)-(C2) gives:

$$n_t = \frac{\phi w_t (1 - \theta)}{(1 + \chi + \phi)(e - \beta) + \phi \beta - [(1 + \chi)\omega + \phi]\theta \frac{w_{t+1}}{1 + r_{t+1}}}, \tag{1}$$

$$s_t = \frac{w_t (1 - \theta)\left[\chi (e - \beta) - (\chi \omega + \phi)\theta \frac{w_{t+1}}{1 + r_{t+1}} \right]}{(1 + \chi + \phi)(e - \beta) + \phi\beta - [(1 + \chi)\omega + \phi]\theta \frac{w_{t+1}}{1 + r_{t+1}}}. \tag{2}$$

Now, let $\theta_1 \equiv \frac{\chi(e - \beta)}{\chi \omega + \phi} \cdot \frac{1 + r_{t+1}}{w_{t+1}}$. Then from Eqs. (1) and (2) it can be seen that a necessary and sufficient condition for ensuring a finite positive solution for n_t and s_t is $0 < \theta < \theta_1$.

From Eq. (1) it is easy to see that the partial equilibrium effect of an increase in the child factor ω on the fertility rate is always positive.

2.3 Firms

As regards the production sector, we suppose firms are identical and act competitively. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[5] where Y_t, K_t and $L_t = N_t$ are output, capital and the time-t labour input respectively, $A > 0$ represents a scale parameter and $\alpha \in (0,1)$ is the capital's weight in technology. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function becomes $y_t = Ak_t^{\alpha}$. Assuming total depreciation of capital at the end of each period and knowing that final output is treated at unit price, profits maximisation leads to:

$$r_t = \alpha A k_t^{\alpha - 1} - 1, \tag{3}$$

$$w_t = (1 - \alpha) A k_t^{\alpha}. \tag{4}$$

2.4 Equilibrium

Given the government budget (B1) and (B2), and knowing also that $N_{t+1} = n_t N_t$, the market-clearing condition in goods as well as in capital markets is expressed by the equality $n_t k_{t+1} = s_t$, that is the stock of capital in period $t+1$ equals the amount of resources saved in period t discounted by the number of individuals in the same period. Using (1) and (2), equilibrium therefore implies:

$$k_{t+1} = \frac{\chi}{\phi}(e - \beta) - \frac{\chi \omega + \phi}{\phi} \theta \frac{w_{t+1}}{1 + r_{t+1}}. \tag{5}$$

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[5] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.

Exploiting (3) and (4) and assuming individuals are perfect foresighted, the dynamics of capital boils down to the following long-run stock of capital per-capita:

$$k^*(\omega) = \frac{\chi(e-\beta)\alpha}{\alpha\phi + \theta(1-\alpha)(\chi\omega + \phi)}. \tag{6}$$

3 Comparative static analysis

We now examine the impact of the child factor on the long-run fertility rate. Can the introduction of a FR-PAYG system – which relates pension benefits with the number of children raised – serve as an effective policy instrument to increase population growth? This simple question gives rise to very interesting findings in our basic OLG model.

Exploiting Eqs. (1), (3), (4) and (6), we get:

$$n^*(\omega) = \frac{(1-\alpha)A(1-\theta)[\alpha\phi + \theta(1-\alpha)(\chi\omega+\phi)]\cdot\left[\frac{\chi(e-\beta)\alpha}{\alpha\phi+\theta(1-\alpha)(\chi\omega+\phi)}\right]^{\alpha}}{\alpha[(1+\chi+\phi)(e-\beta)+\phi\beta]+\theta(1-\alpha)[(1+\chi\omega+\phi)e-\beta]}. \tag{7}$$

Therefore, from Eq. (7) the following proposition holds:

Proposition 1. *The introduction of a positive child factor* ω *increases the long-run fertility rate if and if* $\alpha < \alpha_2$.

Proof. The proof straightforwardly derives by:

$$\frac{\partial n^*(\omega)}{\partial \omega}\Big|_{\omega=0} = \frac{n^*(0)(1-\alpha)\chi\theta\cdot\left[-\mathrm{H}_1\alpha^2 + \mathrm{H}_2\alpha + \mathrm{H}_3\right]}{\phi\cdot[\alpha+\theta(1-\alpha)]\cdot\{\alpha[(1+\chi+\phi)(e-\beta)+\phi\beta]+\theta(1-\alpha)[(1+\phi)e-\beta]\}}, \tag{8}$$

where $\mathrm{H}_1 \equiv (1+\chi-\theta)(e-\beta)+\phi e(1-\theta) > 0$, $\mathrm{H}_2 \equiv (1+\chi)(e-\beta)-\theta[2(e-\beta)+\phi e]$ and $\mathrm{H}_3 \equiv \theta(e-\beta) > 0$. Therefore, $\frac{\partial n^*(\omega)}{\partial \omega}\Big|_{\omega=0} = 0$ if and only if:

$$-\mathrm{H}_1\alpha^2 + \mathrm{H}_2\alpha + \mathrm{H}_3 = 0. \tag{9}$$

Since $\Delta \equiv \mathrm{H}_2{}^2 + 4\mathrm{H}_1\mathrm{H}_3 > 0$, then by applying the Descartes' rule of sign we find that, independently of the sing of H_2, there always exist two real roots $\alpha_1 \equiv \frac{\mathrm{H}_2 - \sqrt{\mathrm{H}_2{}^2 + 4\mathrm{H}_1\mathrm{H}_3}}{2\mathrm{H}_1} < 0$ and $\alpha_2 \equiv \frac{\mathrm{H}_2 + \sqrt{\mathrm{H}_2{}^2 + 4\mathrm{H}_1\mathrm{H}_3}}{2\mathrm{H}_1} > 0$ which solve Eq. (9).[6] Since $\alpha_1 < 0$ it is automatically ruled out.

Thus,

$$\begin{cases} \frac{\partial n^*(\omega)}{\partial \omega}\Big|_{\omega=0} > 0 \quad iff \quad \alpha < \alpha_2 \\ \frac{\partial n^*(\omega)}{\partial \omega}\Big|_{\omega=0} < 0 \quad iff \quad \alpha > \alpha_2 \end{cases},$$

Q.E.D.

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[6] Notice that $\alpha_2 < 1$. This condition in fact implies $\sqrt{\mathrm{H}_2{}^2 + 4\mathrm{H}_1\mathrm{H}_3} < 2\mathrm{H}_1 - \mathrm{H}_2$, where $2\mathrm{H}_1 - \mathrm{H}_2 > 0$. Therefore, we can write $\mathrm{H}_2{}^2 + 4\mathrm{H}_1\mathrm{H}_3 < (2\mathrm{H}_1 - \mathrm{H}_2)^2$ and thus $\mathrm{H}_3 < \mathrm{H}_1 - \mathrm{H}_2$. Applying the definitions of H_1, H_2 and H_3, the latter inequality gives $\phi e > 0$. Therefore, α_2 cannot be higher than unity.

Proposition 1 says that, if the capital's weight in technology is low enough, even if pension benefits are linked with the individual's desired number of children, an increase in the child factor does always reduce the long-run fertility rate.
The economic intuition is the following: while the impact effect, when prices are exogenous, of an increase in the child factor on fertility is, as expected, always positive, the long-run result, when the policy effects on both wage and interest rates is taken into account (i.e., a higher ω reduces wages and increases the interest rate), may be negative. Moreover it is easy to see that the undesirable effect on fertility occurs for very realistic parameter values, as shown the following example.

3.1 A numerical illustration

An example, chosen only for illustrative purposes, of the result stated in Proposition 1 is summarised in Table 1. We take the following parameter values: $A=10$ (simply a scale parameter in production), $\chi = 0.30$ (as in De La Croix and Michel, 2002, p. 50), $\phi = 0.40$,[7] $e=1$ (the fixed cost of child-rearing, assumed to be around the 35 per-cent of the competitive wage), $\beta = 0.85$ (the fixed child subsidy) and $\theta = 0.30$ (the contribution rate). This parameter set generates $\alpha_2 = 0.3065$. As regards the capital's weight in production we consider the conventional value $\alpha = 0.33$. In this case, Table 1 illustrates the reduction in fertility rates determined by an increasing child factor in the pension system.[8]

Table 1. Effects of changes in the child factor.

ω	$n^*(\omega)$
0	1.3490
0.20	1.3470
0.40	1.3442
0.60	1.3409
0.80	1.3371
1	1.3330

3 Conclusions

Since recent studies have argued that the introduction of a fertility related pension reform could be a remedy against the plague of low fertility, then in this paper we investigate, in an overlapping generations general equilibrium context, whether and how the introduction of such a reform reaches its goal. We show that a FR-PAYG pension, in contrast with the suggestions of the preceding literature, may obtain the paradoxical result to disincentive fertility, especially when the capital share in production is sufficiently high.
In spite of the specific assumptions of the model, the results are suggestive rather than definite, some policy implications should be noted: if the objective is to increase (reduce the drop in) fertility, which seems to be insufficient to keep balanced a publicly provided PAYG pension scheme, our theoretical analysis suggests that, in countries in which the capital's share in production is high enough, such as Italy (see, for instance Jones, 2003), policymakers should be cautious in reforming pension systems according to the FR-PAYG mechanism, as the final result may be to disincentive and thus reduce individuals' fertility behaviour.

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[7] As regards the value of the taste for the number of children in the utility function, we note that Strulik (2004a, 2004b) used $\phi = 0.33$ and $\phi = 0.50$, respectively (re-expressed on the basis of the preference parameters adopted here), and thus we used an intermediate value, $\phi = 0.40$.

[8] This example suggests that countries such as Italy, which is largely plagued by low fertility but having a high capital's share in production (e.g., approximately $\alpha \cong 0.50$), may seriously reduce further population growth if a FR-PAYG pension reform is introduced.

References

[1] Abio, G., G. Mahieu and C. Patxot (2004). "On the optimality of PAYG pension systems in an endogenous fertility setting", *Journal of Pension Economics and Finance*, vol. 3 (01), May, pp. 35-62.

[2] Bental, B. (1989). "The old age security hypothesis and optimal population growth", *Journal of Population Economics*, vol. 1 (4), March, pp. 285-301.

[3] Cigno, A. (1993). "Intergenerational transfers without altruism: Family, market and state", *European Journal of Political Economy*, vol. 9 (4), November, pp. 505-518.

[4] De La Croix, D. and P. Michel (2002). *A theory of economic growth. Dynamics and policy in overlapping generations*, Cambridge University Press.

[5] Diamond, P. A. (1965). "National debt in a neoclassical growth model", *American Economic Review*, vol. 55 (5), December, pp. 1126-1150.

[6] Fenge, R. and V. Meier (2004). "Are family allowances and fertility-related pensions Siamese twins?", CESifo Working Paper, March, no. 1157.

[7] Fenge, R. and V. Meier (2005). "Pensions and fertility incentive", *Canadian Journal of Economics*, vol. 38 (1), February, pp. 28-48.

[8] Galor, O. and D. N. Weil (1996). "The gender gap, fertility, and growth", *American Economic Review*, vol. 86 (3), June, pp. 374-387.

[9] Groezen, B. van, T. Leers and L. Meijdam (2003). "Social security and endogenous fertility: Pensions and child allowances as Siamese twins", *Journal of Public Economics*, vol. 87 (2), February, pp. 233-251.

[10] Jones, C. I. (2003). "Growth, capital shares, and a new perspective on production functions", U.C. Berkeley working Paper, Department of Economics, available at http://elsa.berkeley.edu/~chad.

[11] Kolmar, M. (1997). "Intergenerational redistribution in a small open economy with endogenous fertility", *Journal of Population Economics*, vol. 10 (3), August, pp. 335-356.

[12] Nishimura, K. and J. Zhang (1992). "Pay-as-you-go public pensions with endogenous fertility", *Journal of Public Economics*, vol. 48 (2), July, pp. 239-258.

[13] Nishimura, K. and J. Zhang (1995). "Sustainable plans of social security with endogenous fertility", *Oxford Economic Papers*, vol. 47 (1), January, pp. 182-194.

[14] Sinn, H. W. (2004). "The pay-as-you-go pension system as fertility insurance and an enforcement device", *Journal of Public Economics*, vol. 88 (7-8), July, pp. 1335-1357.

[15] Strulik, H. (2004a). "Child mortality, child labour and economic development", *Economic Journal*, vol. 114 (497), July, pp. 547-568.

[16] Strulik, H. (2004b). "Economic growth and stagnation with endogenous health and fertility", *Journal of Population Economics*, vol. 17 (3), August, pp. 433-453.

[17] Zhang, J. and J. Zhang (1995). "The effects of social security on population and output growth", *Southern Economic Journal*, vol. 62 (2), October, pp. 440-450.