E C O N O M I C S B U L L E T I N

Private Education and Positive Growth with Shrinking Income Inequality: A Note

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Abstract

This short paper introduces the external effects of human capital on education as reported by such as Tamura (1991) and so on to Glomm and Ravikumar (1992) and reexamines economic growth and income inequality in the two education systems: one for public and the other for private education. Glomm and Ravikumar show that income inequality does not shrink if the growth rate is positive in private education. On the other hand, we show that both positive growth and income shrinking are brought about by the externality considered in this paper.

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1 Introduction

As the seminal study by Lucas (1988) shows, human capital accumulation through education and/or training is one of the most important engines of endogenous growth. Along the same line, Gloom and Ravikumar (1992) examine how the education system affects economic growth and income inequality. They show that positive economic growth with shrinking income inequality exists not in private education but in public education.

On the one hand, some papers show that an externality affects human capital accumulation. Tamura (1991) incorporates average human capital into individual human capital formation and shows that it affects not only economic growth but income inequality. Tamura (1991) shows that while income inequality shrinks rapidly, the economic growth rate decreases if the effect of externality is large. Redistribution from high income to low income individuals is harmful for economic growth even if income inequality shrinks.[1]

On the other hand, Gradstein and Justman (1997) also incorporate externality into individual human capital formation and show that the economic growth rate increases if the effects of externality are large. Tamura (1991) and Gradstein and Justman (1997) show that incorporating an externality into models in different ways brings about different results. This paper shows an example of one such different way of incorporating an externality and how economic growth and income inequality in private and public education are determined.[2]

We show that an externality in human capital formation affects not the economic growth rate or income level but income inequality in the long run. Moreover, private education brings about positive growth with shrinking income inequality due to the externality considered in this paper. This paper also shows other results that differ from those of some earlier studies, even some that show results in a model which incorporated an externality.

The rest of the paper is structured as follows. Section 2 sets up the basic model. Section 3 derives

[1] Cardak (2004) also shows that redistribution effects education. Cardak (2004) analyzes the economy of both the private and public education system, and shows the two equilibrium: one for the equilibrium students who select public education are at a low income level, and the other for the equilibrium that students who select private education are at a high level income, and that the former is blessed through the spillover of the latter.

[2] Benabou (1996) also shows a model that incorporated the externality of human capital accumulation. In Benabou (1996), average human capital affects individual human capital formation. Average human capital is considered as average in the group to which individuals belong. If average human capital is defined differently, the results are also different.

the equilibrium of private and public education systems. Section 4 discusses the two education systems. The final section offers some concluding remarks.

2 The Model

Consider an overlapping generations economy in which individuals live for two periods: young and adult. The population size of each generation is constant over time, which is normalized to unity without loss of generality. Each generation consists of a continuum of agents, which are differentiated only by the human capital of their parents. An initial generation's human capital h_t^i (i denotes type of individuals) is assumed to be distributed according to a log normal distribution with average μ_0 and variance σ_0.

All individuals have identical preferences for leisure, consumption and bequest. We assume a utlity function u_t as follows:

$$u_t = \alpha \ln n_t + \beta \ln c_{t+1} + \gamma \ln e_{t+1},\ 0 < \alpha, \beta, \gamma. \tag{1}$$

where n_t denotes leisure time in the young period, c_{t+1}, and e_{t+1} denote consumption and education investment (bequest left to their children) in the adult period.

The young are endowed with one unit of time. They allocate it for leisure n_t and schooling $1 - n_t$. A human capital of type i in period $t+1$ is assumed as follows:

$$h_{t+1}^i = A(1 - n_t^i)^\delta (e_t^i)^\epsilon \left(\left(h_t^i\right)^\eta \bar{h}_t^{1-\eta} \right)^\theta,\ 0 < A,\ 0 < \delta,\ \epsilon,\ \eta,\ \theta < 1, \tag{2}$$

where h_t^i and $\bar{h}_t$ denote a human capital of type i and an average human capital in period t, respectively. A distribution of human capital h_t^i among individuals should also be a log normal distribution with an average and variance of μ_t and σ_t, respectively, so $\ln \bar{h}_t \equiv \mu_t + \frac{\sigma_t^2}{2}$. If $\eta = 1$, (2) is equivalent to Glomm and Ravikumar (1992). On the other hand, if $\eta = 0$, (2) is equivalent to Gradstein and Justman (1997).[3]

3 Equilibrium

This section shows the equilirium in both the public and private education system. First, we show the equilibrium in private education.

[3]In addition, Gradstein and Justman (1997) also assume that the innate ability for achieving human capital formation differs among individuals.

3.1 Private Education

In private education, each individual decides schooling time $1 - n_t^i$, consumption c_{t+1}^i and education investment e_{t+1}^i to maximize their utility under the budget constraint $c_{t+1}^i + e_{t+1}^i = h_{t+1}^i$. The optimization yields as follows:

$$1 - n_t^i = \frac{\delta(\beta+\gamma)}{\alpha+\delta(\beta+\gamma)}, \tag{3}$$

$$e_{t+1}^i = \frac{\gamma}{\beta+\gamma} h_{t+1}^i, \tag{4}$$

$$c_{t+1}^i = \frac{\beta}{\beta+\gamma} h_{t+1}^i. \tag{5}$$

Let us investigate the dynamics of human capital which has an average μ_t and variance σ_t^2. Considering (2)-(4), we obtain the following equation:

$$\ln h_{t+1}^i = X + (\epsilon + \theta\eta) \ln h_t^i + \theta(1-\eta) \ln \bar{h}_t, \tag{6}$$

where $X \equiv \ln A + \delta \ln \frac{\delta(\beta+\gamma)}{\alpha+\delta(\beta+\gamma)} + \epsilon \ln \frac{\gamma}{\beta+\gamma}$. Considering (6) and $\ln \bar{h}_t = \mu_t + \frac{\sigma_t^2}{2}$, we obtain the following equation:

$$\mu_{t+1} = X + (\epsilon+\theta)\mu_t + \frac{\theta(1-\eta)\sigma_t^2}{2} \leftrightarrow \frac{\mu_{t+1}}{\mu_t} = \frac{X}{\mu_t} + (\epsilon+\theta) + \frac{\theta(1-\eta)}{2}\frac{\sigma_t^2}{\mu_t}, \tag{7}$$

$$\sigma_{t+1}^2 = (\epsilon+\theta\eta)^2\sigma_t^2 \leftrightarrow \frac{\sigma_{t+1}^2}{\sigma_t^2} = (\epsilon+\theta\eta)^2. \tag{8}$$

ϵ, θ and η determines whether positive growth is brought about or not, and whether or not income inequality shrinks. Each regime is determined according to the parameters shown as Fig.1.

[Insert Fig.1 around here.]

Positive growth and increasing income inequality in regime L, positive growth and shrinking income inequality in regime M, and no growth and shrinking income inequality in regime N are brought about. There is no regime L in Glomm and Ravikumar (1992). This regime exists due to the externality η. The growth rate in the long run is given by $\lim_{t\to\infty} \frac{\mu_{t+1}}{\mu_t} = \lim_{t\to\infty} \frac{X}{\mu_t} + \epsilon + \theta + \frac{\epsilon+\theta(1-\eta)}{2} \lim_{t\to\infty} \frac{\sigma_t^2}{\mu_t}$. If $\lim_{t\to\infty} \frac{\sigma_t^2}{\mu_t} = 0$ and $\epsilon + \delta > 1$, the growth rate in the long run $\frac{\mu_{t+1}}{\mu_t} - 1$ is $\epsilon + \delta - 1$, which is constant. Otherwise, the growth rate increases over time due to income inequality. Incidentally, if $\epsilon + \delta < 1$, no growth gives μ^{*Pri} which is μ_t in the steady state as follows:

$$\mu^{*Pri} = \frac{X}{1-(\epsilon+\theta)}. \tag{9}$$

3.2 Public Education

In public education, we must consider two optimization problems: one for households and the other for government. An individual of type i faces a budget constraint $c^i_{t+1} = (1-\tau_{t+1})h^i_{t+1}$, where τ_{t+1} denotes the tax rate. Public education investment E_{t+1} is given by

$$E_{t+1} = \tau_{t+1}\bar{h}_{t+1}. \tag{10}$$

Government decides the level of E_{t+1} (or τ_{t+1}). Individuals decide schooling time $1-n^i_t$ to maximize their utility, so that schooling time is shown as follows:

$$1 - n^i_t = \frac{\beta\delta}{\alpha+\beta\delta}. \tag{11}$$

Next, government decides the tax rate to maximize the following indirect utility function v_t:

$$v_t = \alpha \ln \frac{\alpha}{\alpha+\beta\delta} + \beta \ln(1-\tau)h^i_{t+1} + \gamma \ln \tau \bar{h}_{t+1}. \tag{12}$$

The tax rate causing the maximization of v_t is

$$\tau_{t+1} = \frac{\gamma}{\beta+\gamma}. \tag{13}$$

Let us investigate the dynamics of human capital which has an average μ_t and variance σ^2_t. Considering (2), (10), (11) and (13), we obtain the following equation:

$$\ln h^i_{t+1} = Y + \theta\eta \ln h^i_t + \theta\epsilon(1-\eta)\ln \bar{h}_t, \tag{14}$$

where $Y \equiv \ln A + \delta \ln \frac{\beta\delta}{\alpha+\beta\delta} + \epsilon \ln \frac{\gamma}{\beta+\gamma}$. Moreover, we also obtain the following equations:

$$\mu_{t+1} = Y + (\epsilon+\theta)\mu_t + \frac{\epsilon+\theta(1-\eta)}{2}\sigma^2_t \leftrightarrow \frac{\mu_{t+1}}{\mu_t} = \frac{Y}{\mu_t} + (\epsilon+\theta) + \frac{\epsilon+\theta(1-\eta)}{2}\frac{\sigma^2_t}{\mu_t}, \tag{15}$$

$$\sigma^2_{t+1} = \theta^2\eta^2\sigma^2_t \leftrightarrow \frac{\sigma^2_{t+1}}{\sigma^2_t} = \theta^2\eta^2. \tag{16}$$

Income inequality shrinks due to $\theta^2\eta^2 < 1$, therefore, $\frac{\mu_{t+1}}{\mu_t}$ in the long run is $\epsilon+\theta$.[4] If $\epsilon+\theta > 1$, that results in economic growth in the long run. On the other hand, if $\epsilon+\theta < 1$, there is no growth. Then, μ^{*Pri} which is μ_t in the steady state is shown as follows:

$$\mu^{*Pri} = \frac{Y}{1-(\epsilon+\theta)} \tag{17}$$

[4]Note that $\lim_{t\to\infty}\frac{\mu_{t+1}}{\mu_t} = \lim_{t\to\infty}\frac{Y}{\mu_t} + (\epsilon+\theta) + \frac{\epsilon+\theta(1-\eta)}{2}\lim_{t\to\infty}\frac{\sigma^2_t}{\mu_t}$.

4 Comparison between Two Education Systems

First, if positive economic growth is brought about in the long run and income inequality shrinks in private education, the growth rate of μ_t is $\epsilon + \theta - 1$ in both private and public education. Though the growth rate of μ_t does not depend on the parameters of utility fuction, the growth rate of h_t does. The growth rate of h_t in private education is then greater than that in public education. In any case, an externality parameter η does not affect the growth rate of either μ_t or h_t.

Second, if economic growth is not achieved in the long run, then income inequality shrinks and μ_t (or h_t) converges to a certain value. Then, comparing μ^{*Pri} with μ^{*Pub}, $\mu^{*Pub} > \mu^{*Pri}$. In this case, an externality parameter η does not affect μ^{*Pri} or μ^{*Pub}. That gives the following proposition:

Proposition If $\epsilon + \theta > 1$ and $\epsilon + \theta\eta < 1$, positive economic growth and shrinking income inequality result in both private and public education. If $\epsilon + \theta > 1$ and $\epsilon + \theta\eta > 1$, positive economic growth and increasing income inequality result in private education. If $\epsilon + \theta < 1$, neither economic growth nor increasing income inquality are brought about, and an income converges to a constant in both education systems.

5 Conclusions

This note presents a model with an externality of human capital such as that of Tamura (1991) which is incorporated into the Glomm and Ravikumar (1992) model, and shows economic growth and income inequality in both private and public education. It is shown that the externality in the long run does not affect economic growth or the level of income during shrinking income inequality. However, the externality does affect on income inequality in private education, so that what emerges is shown that positive economic growth with shrinking income inequality not shown in Glomm and Ravikumar (1992). Though we consider some ways to incorporate an externality into models such as that of Gradstein and Justman (1997), this note shows that different ways of incorporating into models bring about different results, and offers one example of such a way to incorporate an externality.

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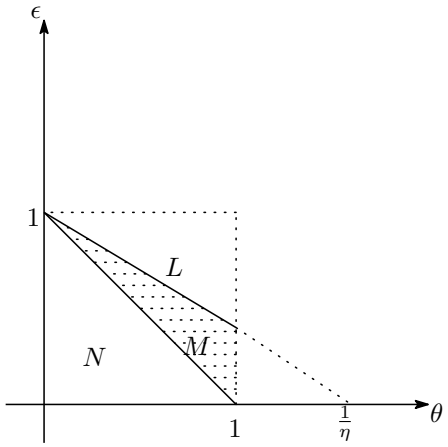

Fig.1: Income Growth, Inequality and Parameters