E c o n o m i c s   B u l l e t i n

Analysing the forward premium anomaly using a Logistic Smooth Transition Regression model.

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Abstract

Several researchers have suggested that exchange rates may be characterized by nonlinear behaviour. This paper examines these nonlinearities and asymetries and estimates a Logistic Transition Regression (LSTR) of Fama Regression with the Risk Adjusted Forward Premia as transition variable. Results confirm the existence of nonlinear dynamics in the relationship between spot exchange rate differential and the forward premium for all the currencies of the sample and for all maturities (three and six-month maturities). Results confirm the insight into the presence of speculation barriers and transaction costs in the foreign exchange rate market that would explain, at least partially, the forward premium anomaly.

Citation: Amri, Sofiane, (2008) "Analysing the forward premium anomaly using a Logistic Smooth Transition Regression model.." *Economics Bulletin,* Vol. 6, No. 26 pp. 1-18
Submitted: April 22, 2008. **Accepted:** July 2, 2008.
URL: http://economicsbulletin.vanderbilt.edu/2008/volume6/EB-08F30042A.pdf

Introduction

The Unbiased Forward Rate Hypothesis (UFRH) stipulates that the forward exchange rate reflects fully the future change of spot exchange rate. Expressed in differences form, UFRH can be represented as follows:

$$\Delta s_{t+n} = \alpha + \beta\left(f_{n,t} - s_t\right) + u_{t+n} \qquad (1)$$

With « Δ » as the first diffrence operator, « s_{t+n} » and « s_t » are the logarithm for the spot exchange rate at the times « t+n » and « t », respectively. « $f_{n,t}$ » is the logarithm of the forward exchange rate for n-period. « u_{t+1} » is an error term with zero conditional mean..

Several authors have tested the UFRH using conventional linear methods under the hypothesis of an efficient foreign exchange market and the Covered Interest Parity relation (CIP) holding for different currencies and for large horizons and maturities **(**Bansal and Dahlquist (2000), Barnhart, McKnown and Wallace (2002), Sarno, Clarida and Leon (2006)**)**. All these papers revealed a inconsistent slope closer to « -1 » than to «1 » as is suggeted by the economic theory.
Explanations of this anomaly are in general related to a time variable risk premium, "peso" problem and rational anticipation. Several papers have attempted to explain the forward anomaly adopting a nonlinear framework using new empirical techniques but the results are still weak and cannot the forward bias completely. Other researches have tried to analyze the deviations from Uncovered Interest Parity (UIP) by modeling the excessive returns and have proposed the hypothesis of the existence of speculation barriers as explanation of the forward puzzle.

The barriers to speculation hypothesis was proposed by Lyons (2001) and is based on the idea that the financial institutions' decision to activate their speculative strategies in the foreign exchange market is dependent on their capacity to clear excessive returns per unit risk (or Sharpe ratio) being greater than the gains they could make by other investment strategies.
So, there is a zone or a band of inaction within which the forward bias doesn't attract the necessary speculative funds in order to bring back the exchange rates toward equilibrium levels.
In this paper it is proposed to analyse another aspect of the forward bias; namely, the existence of asymmetries and nonlinearities in the relation between the spot and forward exchange rates.
The goal is to present a nonlinear model that describes exchange rate dynamics in order to give an explanation to the forward anomaly.
This will be done by means of a smooth transition regression model with a logistic transition function (a Logistic Smooth Transition Regression (LSTR) model) and speed of adjustment toward the equilibrium (the UIP) that depends on the size of the forward premium.
This nonlinear framework presents several features that provide a better description of exchange rate dynamics and help give a new explanation to the asymmetries and nonlinear deviations often raised in previews studies.

The results of the estimation of the LSTR model of UIP yield results that differ according to currency, the horizon of the contract forward (three months or six months) and the transition function.
As in previous papers, the existence of three regimes is observed. The first is the lowest regime

corresponding to a forward premium below the threshold level and where the forward anomaly holds. The second regime is a transition regime characterized by smaller forward premiums and less persistent deviations from UIP than those observed in the previous regime. It is in the third regime where the UIP is most likely to hold.
The remainder of the paper is organized as follows. In the first section, an introduction to the theoretical model from which the nonlinear regression is presented. In the second section, the estimation procedure and different tests are developed. In section 3, the nonlinear dynamics between the spot exchange rate differential and the forward premia is demonstrated using appropriated estimation techniques. The existence of three regimes with different implications for the forward puzzle is also shown. In the end, some concluding remarks are presented.

1. The theorical model :

LSTR models have been introduced first by Granger and Terasvirta (1993) and Teräsvirta (1994). This LSTR model, like STAR models allows a flexible transition and periodic adjustment whose speed is determined by transition variables. The choice of a logistic transition function allows for asymmetries in the adjustment process.

The LSTR model of the Fama regression can be expressed as follows:

$$\Delta s_{t+1} = \left[\alpha_1 + \beta_1\left(f_{t,1} - s_t\right)\right] + \left[\alpha_2 + \beta_2\left(f_{t,1} - s_t\right)\right] F(z_t, \gamma, c) + u_{t+1} \tag{2}$$

« u_{t+1} » is a stationary (I(0)) disturbance term with zero mean and « F (.) » is the transition function that determines the speed of adjustment.

The transition function in the LSTR model is logistic and it can be written as follows:

$$F\left(z_t, \gamma, c\right) = \left(1 + \exp\left(-\gamma\left(z_t - c\right)/\sigma_{z_t}\right)\right)^{-1} \tag{3}$$

« z_t » is the transition variable, and « σ_{z_t} » its standard deviation, « γ » is a parameter and « c » is a localization parameter.

The logistic transition function varies between zero and unity « 0 < F < 1 ». It depends on the transition variable « z_t », so that « $\lim\limits_{z\to\infty} F\left(z_t, \gamma, c\right) = 0$ » and « $F(z_t, \gamma, c)\big|_{z_t = 0.5} = 0.5$ » and « $\lim\limits_{z\to\infty} F\left(z_t, \gamma, c\right) = 1$ ».

As « $\gamma \to \infty$ », the transition function becomes a step function and the LSTR model a threshold model.

When « $\gamma = 0$ » the LSTR model becomes a simple linear regression model with the following parameters: « $\alpha = \alpha_1 + 0.5\alpha_2$ » and « $\beta = \beta_1 + 0.5\beta_2$ ».

Values taken by the transition variable and the transition parameter « γ » determine the speed of adjustment toward equilibrium. In fact, the transition variable « γ » determines the slope of the transition function, hence the transition speed between the extreme regimes regardless of the value of « z_t ». The parameter « c » could be considered as the threshold between two regimes: the first regime corresponds to «$F(z_t, \gamma, c) = 0$ »and the second to « $F(z_t, \gamma, c) = 1$ ».

These features of the LSTR model seem to be very useful and are in relation with the stylized facts reported in the literature (Bansal (1997) and Sarno and al (2005)) notably the fact that the adjustment toward UIP depends on the size of the deviation from the equilibrium.

1.1 The LSTR model

In the proposed model, the transition variable « z_t » is the Risk Adjusted Forward Premia (RAFP) and is equal to « $z_t = (f_t - s_t)/\sigma_{z_t}$ ».
The literature reports two types of transition functions, the logistic function and the exponential function.

The logistic function presents interesting features for the current study since it is more general and more flexible which is very useful in describing the exchange rate dynamics, whereas the exponential function imposes strong restrictions (Baillie R,. T and Kilic, R. (2006)).

In this section, the methodology used in order to determine the best model for every currency's temporal series is proposed.
Indeed, the analysis starts with a linear model (a VAR model) as a starting point. Next, the transition variable is defined in order to test for the presence of nonlinearities and justify the use or non-use of a STR model. Then, the apropriate initial values are found and the estimation of the LSTR model is conducted.
Finally, the last step is the evaluation LSTR model with the use of various tests of misspecification (remaining nonlinearity, error autocorrelation,...)

2. Estimation of the LSTR model:

2.1 Data Sources :

The data sample is composed of weekly observations of US Dollar spot and three and six-month forward exchange rates for the following currencies: Sterling Pound for the period January 1982 – January 2007, Swedish Crown, Euro and Canadian Dollar for the period May 1990- January 2007, and the Swiss franc for the period January 1972 – January 2007.These weekly data are constructed from daily exchange rate observations obtained from Datastream database.

2.2 Estimation of the LSTR model of Fama Regression with three-month forward exchange rates:

In this section, it is proposed to perform the estimation of the LSTR model of Fama regression.
To resolve this model, the the literature (Sarno, Valente and Leon (2004), Baillie and Kilic

(2005),and Krätzig (2005)) suggests the use of the method of nonlinear least square proposed by Teräsvirta (1994). The initial values of the transition function parameters « c » and « γ » are obtained using a sweep research.
Also, as in prevoius papers, the transition variable is normalized by dividing by its standard deviation.
The empirical results are summarized in the table 1 and the analysis will be accompanied with diagrams relative to transition function, transition variable and the LSTR model.
The results of the estimation of the LSTR model with the Three-month Risk adjusted Forward Premia as transition variable gives results that differ according to the currency object of the analysis. Indeed, a negative intercept « α_1 » close to zero is obtained for all currencies of the data sample (except the Sterling Pound) and a consistent slope « β_1 » of the linear part for all currencies (except the Sterling Pound, the Canadian Dollar and the Swedish Corona). It may also be noted that « β_1 » is close to the unity for all currencies except the Canadian Dollar and the Swedish Corona.
The estimation of transition function parameters clearly shows the existence of nonlinearities with a consistent « c » parameter for all currencies of the sample.

Results show that parameters related to the nonlinear part, notably the slope « β_2 », are in general inconsistent (except for the Euro and the Canadian Dollar) or even negative (for the Sterling Pound and the Swiss Franc). These mitigated results confirm the presence of nonlinear dynamics in the exchange rate movements and suggest reconsidering the transition function and / or reviewing the choice of the transition variable (see Sarno, Valente and Leon. 2006). Therefore, in order to analyze these nonlinear dynamics between the spot exchange rate differential and the forward premium, the diagrams relative to the transition function, transition variable and the LSTR model are analyzed.

In fact, the analysis of these diagrams shows the presence of symmetry in the relationship between the forward premium and the transition function for the Sterling Pound. This suggests that the nonlinear dynamics detected in the relationship between the spot exchange three-month forward exchange rates would be better described by an exponential transition function.
Figure 1 shows a smooth transition from one regime to another for most currencies of the sample except the Swiss Franc. It may be noted also, the existence of a crisp transition with jumps from the upper regime to the lower for the Swiss Franc.

Analyzing (Figure 1) the required risk premium level relative to the upper regime of the transition function maybe determined. In fact required risk premium level is -0.7 % for the Swiss Franc, 1 % for the Swedish Corona, 2% for the Canadian Dollar and 2,5% for the Euro. The lower premium risk level for detecting the forward anomaly is equal to -8% for the Canadian Dollar, -0.7 % for the Swiss Franc, 0.5 % for the Euro and % for the Swedish Corona.
These empirical results, although they can be improved using different transition functions and transition variable, confirm the presence speculation barriers.

2.3 Estimation of the LSTR model of Fama regression with six-month forward exchange rates

The estimation of the LSTR model with the six-month Risk Forward Adjusted Premia as a transition variable shows mitigated results. First, a consistent near to zero intercept of the linear

part « α_1 » for all currencies of the sample and a positive and consistent slope coefficient « β_1 » only for the Sterling Pound and the Swiss Franc can be noticed. Second, the estimation of the nonlinear part of the LSTR model shows the clear presence of nonlinearities in the relationship between the forward premium and the spot exchange rate differentials with a consistent parameter « c ».
The empirical results summarized in the table 2 show that parameters related to the nonlinear part, notably the slope « β_2 », are inconsistent for all currencies except the Swiss Franc.
These mitigated results confirm the presence of nonlinear dynamics between the spot and the forward exchange rates, and are in favor of the reformulation of the logistic transition function, choosing a different transition variable.
The transition function diagrams over the transition function (the six-month Risk Adjusted Forward Premium) clearly show the existence of an asymmetric nonlinear relationship for all currencies of the sample. The analysis of the diagram analysis confirms the insight into the presence of a smooth transition between regimes for all the currencies of the sample in the case of six-month Risk Adjusted forward premia.

The analysis of these diagrams allows us to determine the required level of Risk Adjusted Forward Premia relative to the upper regime of the transition function. This level differs from one currency to another and is -0.3% for the Sterling Pound, -0.1% for the Swiss Franc, 1% for the Canadian Dollar, 2% for the Euro and the Swedish Corona.
A lower threshold value of the Risk Adjusted Forward Premia below which the forward anomaly maybe detected was also determined. This value is -1.5% for the Sterling Pound and the Swiss franc, 0.06% for the canadian Dollar, 0.5% for the Euro and 1% for the Swedish Corona.

CONCLUSION

The estimation of the Logistic Smooth Transition Rregression model of Fama Regression with the Risk Adjusted Forward Premia as transition variable confirms findings from previous studies and shows clearly the existence of nonlinear dynamics in the relationship between spot exchange rate differential and the forward premium and this is for all the currencies of the sample and for all maturities (three and six-month maturities).
Results show also the existence of three regimes: an inner regime defined by a band where the probability to detect the forward anomaly is strong, and two outer regimes characterized by a forward premium high enough (in absolute value) to attract speculators. So, in outer regimes, the Uncovered Interest Parity is very likely to hold.
These results confirm the insight into the presence of speculation barriers and transaction costs in the foreign exchange rate market that would explain, at least partially, the forward anomaly. There results can be further improved, especially for the Sterling Pound and the Swiss Franc, by modifying the proposed smooth transition regression and adopting an exponential transition function for example; in order to better describe the exchange rate dynamics and take into account the symmetric relationship revealed in the Sterling Pound's diagram. The currency transition function diagrams reveal also the existence of brutal jumps from the lower to the upper regime especially when the transition variable is three-month Risk Adjusted Forward Premia.

Table 1

Estimation of the LSTR Model

$$\Delta s_{t+1} = \left[\alpha_1 + \beta_1\left(f_{t,3} - s_t\right)\right] + \left[\alpha_2 + \beta_2\left(f_{t,3} - s_t\right)\right] F\left(z_t, \gamma, c\right) + u_{t+1}$$

(With three-month forward exchange rates)

	Sterling Pound	Swiss Franc	Euro	Canadian Dollar	Swedish Corona
α_1	0.04827 (0.0272)	-0.00722 (0.0004)	-0.00388 (0.0007)	-0.13640 (0.0204)	-0.00328 (0.0009)
β_1	0.96218 (0.3336)	0.98764 (0.0107)	0.98018 (0.0092)	-1.85703 (0.2584)	-2.86579 (0.1782)
α_2	-0.04294 (0.0279)	0.00460 (0.0005)	0.02967 (0.0021)	0.13150 (0.0214)	0.03777 (0.0055)
β_2	-0.31118 (0.3070)	-0.03691 (0.0123)	0.11806 (0.0216)	1.59342 (0.2549)	1.75848 (0.3512)
γ	2.08035 (0.7486)	29929.584 (872.945)	2.314 (0.3577)	2.91290 (0.5576)	23.37114 (9.0429)
C	-0.04719 (0.0078)	-0.00717 (0.0000)	0.01090 (0.0009)	-0.04283 (0.0035)	0.00884 (0.0002)
AIC	-9.0954	-9.3410	-7.7637	-7.0485	-5.9822
SC	-9.0717	-9.3188	-7.7547	7.0257	-5.9732
HQ	-9.0865	-9.3327	-7.7605	-7.0400	-5.9790
Adjusted R^2	0.4617	0.9601	0.8373	0.8602	0.0838
Standard Deviation of Transition variable	0.0213	0.0047	0.0106	0.0308	0.0085
Standard Deviation of Residuals	0.0106	0.0093	0.0206	0.0294	0.0502

Key : Standard deviation errors are in parentheses below the corresponding parameter estimates. AIC, SC and HQ are Akaike Information Criterion, Schwarz Criterion and Hannan-Quinn Criterion respectively.

Table 2

Estimation of the LSTR Model

$$\Delta s_{t+1} = \left[\alpha_1 + \beta_1\left(f_{t,6} - s_t\right)\right] + \left[\alpha_2 + \beta_2\left(f_{t,6} - s_t\right)\right] F\left(z_t, \gamma, c\right) + u_{t+1}$$

(with six-month forward exchange rates)

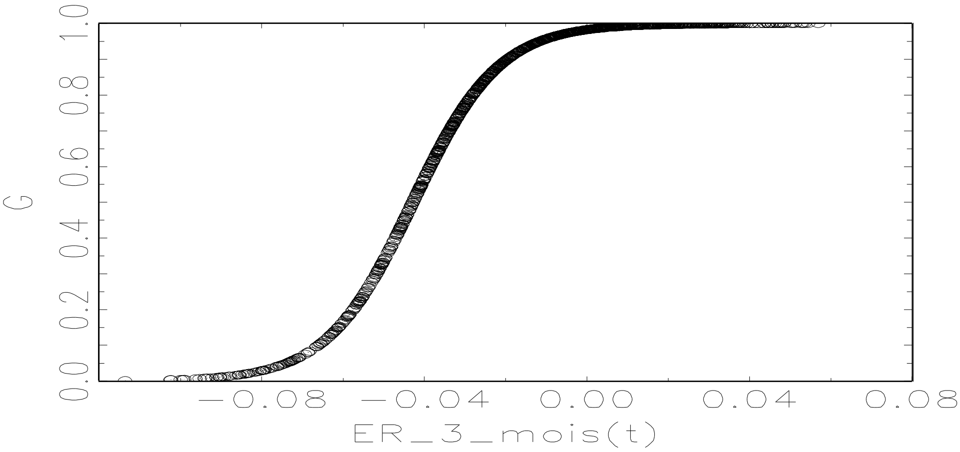
	Sterling Pound	Swiss Franc	Euro	Canadian Dollar	Swedish Corona
α_1	0.00247 (0.0008)	-0.00753 (0.0003)	-0.01563 (0.0013)	-0.02857 (0.0013)	-0.00741 (0.0013)
β_1	0.14727 (0.0156)	0.97509 (0.0029)	-2.70459 (0.1347)	-0.50469 (0.029)	-3.15286 (0.1369)
α_2	0.01440 (0.0052)	-0.00790 (0.0006)	0.12931 (0.0085)	-0.07238 (0.0084)	0.21920 (0.0101)
β_2	-0.23340 (0.154)	-0.04401 (0.0044)	0.07390 (0.2214)	-0.13164 (0.088)	-2.38891 (0.3089)
γ	9.27508 (5.3201)	4.51750 (1.7050)	8.44737 (1.1993)	1.29315 (2.2579)	12.69701 (1.4710)
C	0.01135 (0.0030)	-0.00800 (0.0007)	0.01706 (0.0005)	0.00754 (0.0001)	0.01751 (0.0002)
AIC	-8.7157	-1.0313e+01	-5.6330	-6.6781	-5.4105
SC	-8.6919	-1.0291e+01	-5.6240	-6.6539	-5.4015
HQ	-8.7068	-1.0305e+01	-5.6298	-6.6690	-5.4073
Adjusted R^2	0.2130	0.9963	0.1550	0.2809	0.2339
Standard Deviation of Transition variable	0.0331	0.0072	0.0189	0.0054	0.0158
Standard Deviation of Residuals	0.0128	0.0057	0.0598	0.0354	0.0668

Key : As for table 1.

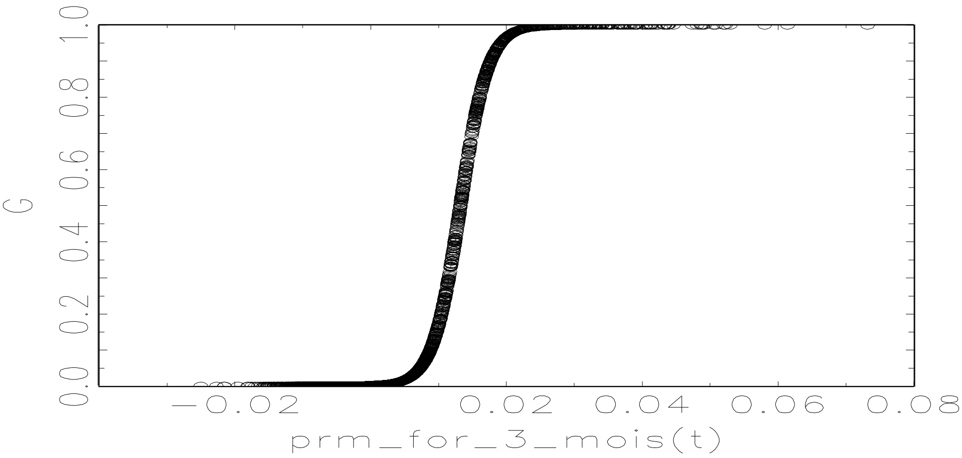
FIGURE I

Translation function over transition variable

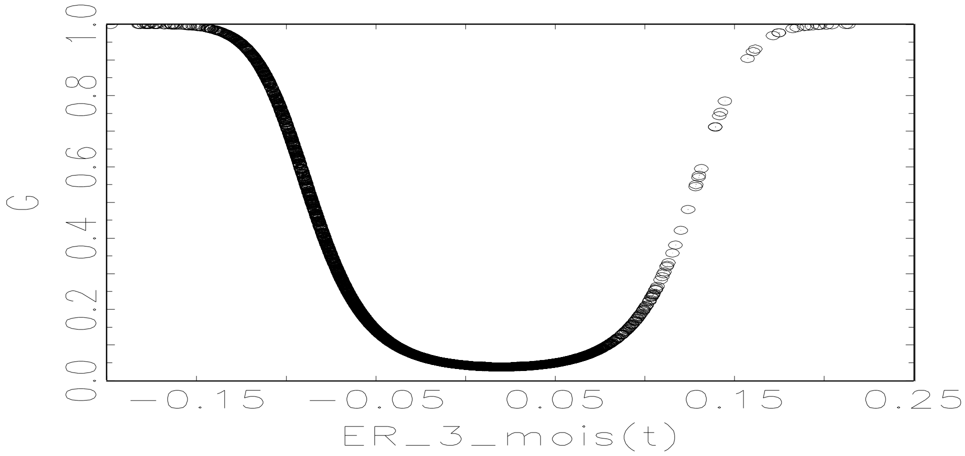
(Risk Adjusted Forward Premia)



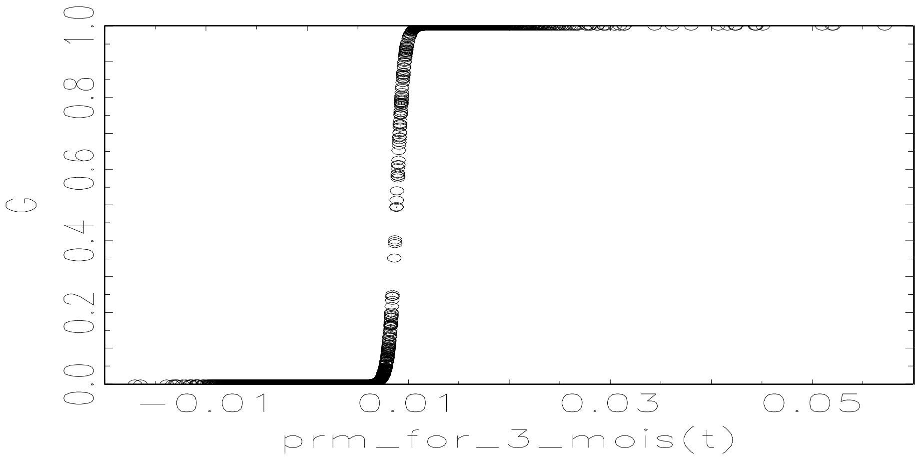
Canadian Dollar



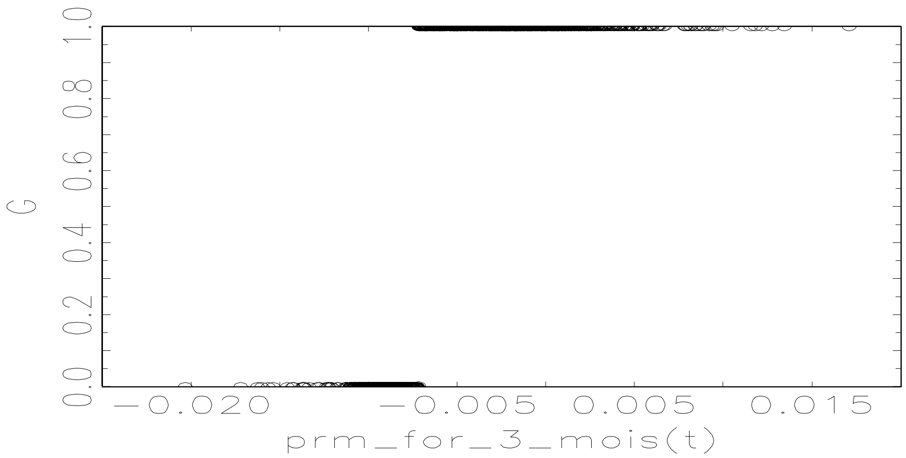
Euro



Sterling Pound



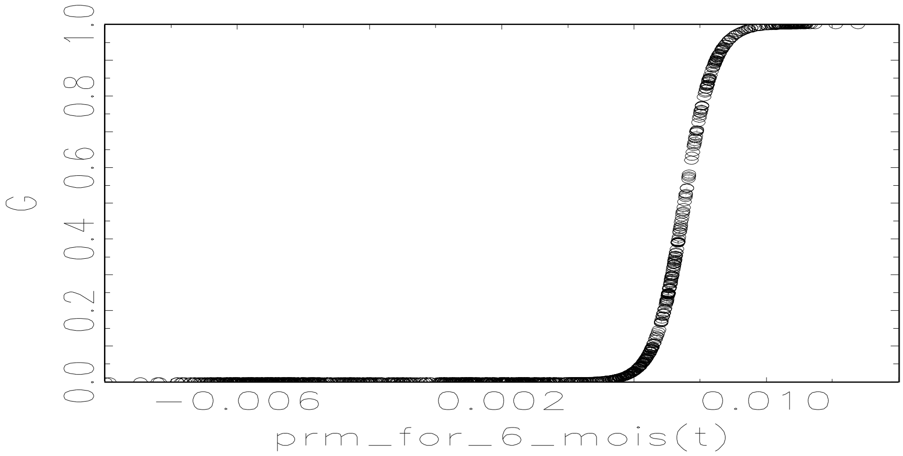
Swedish Corona



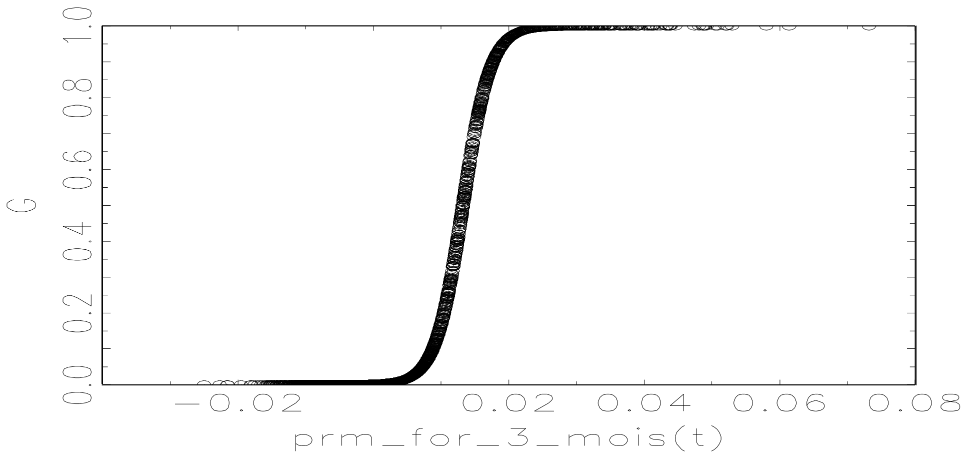
Swiss Franc

FIGURE II

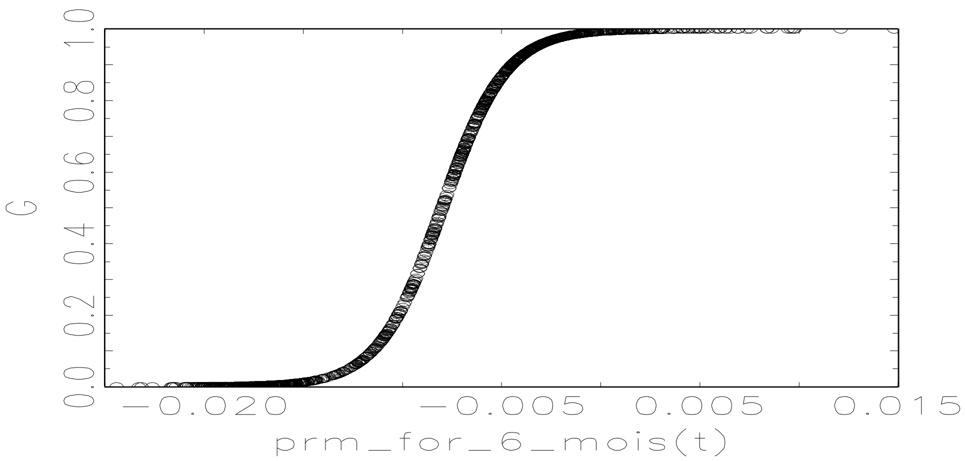
Transition function over transition variable
(Six-month Risk Adjusted Forward Premia)



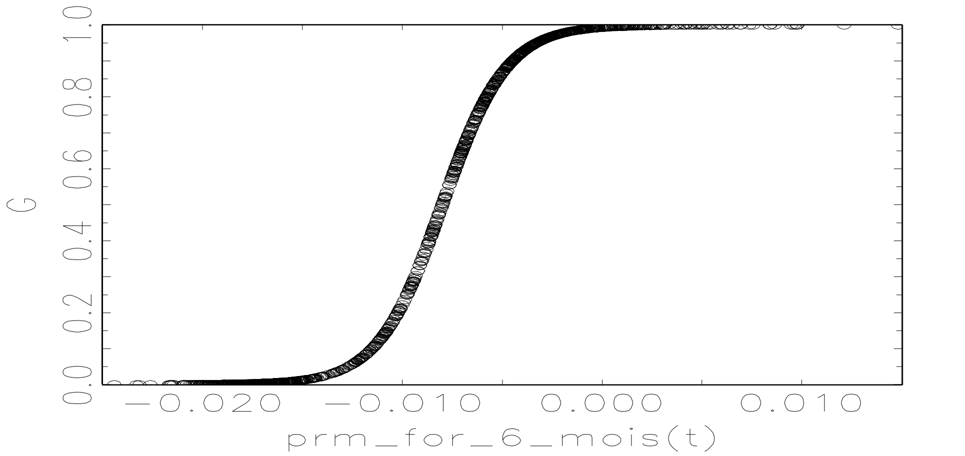
Canadian Dollar



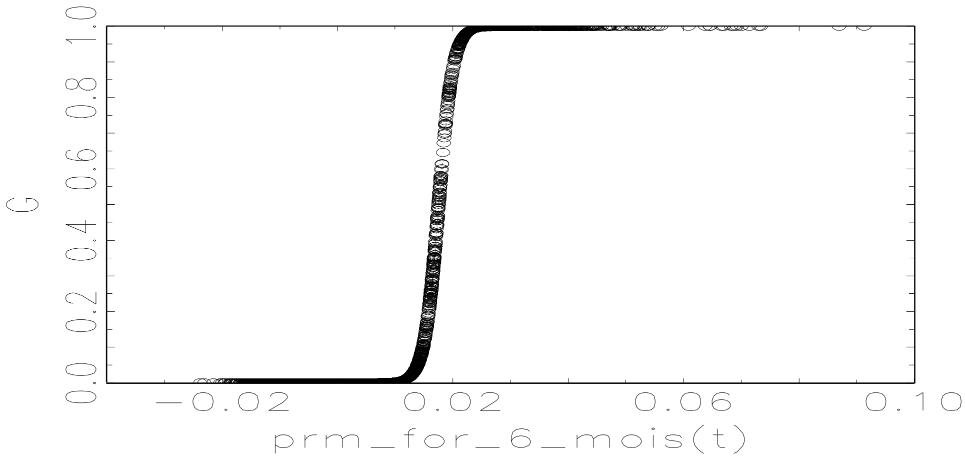
Euro



Sterling Pound



Swiss Franc

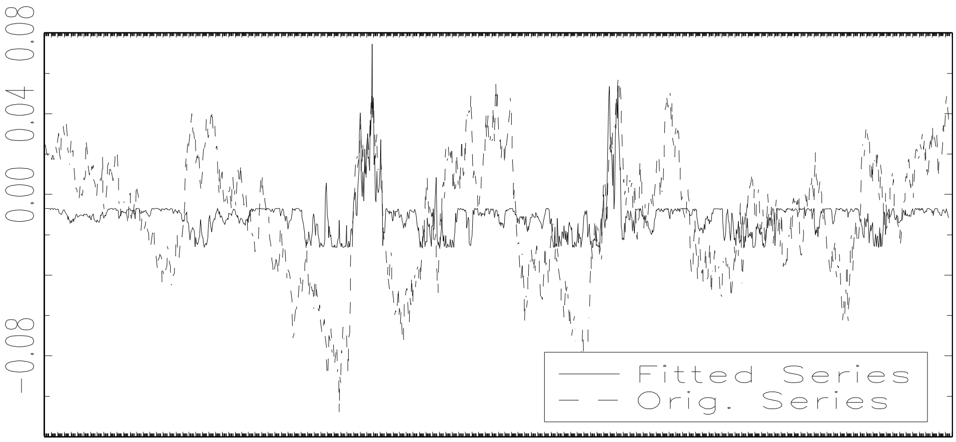


Swedish Corona

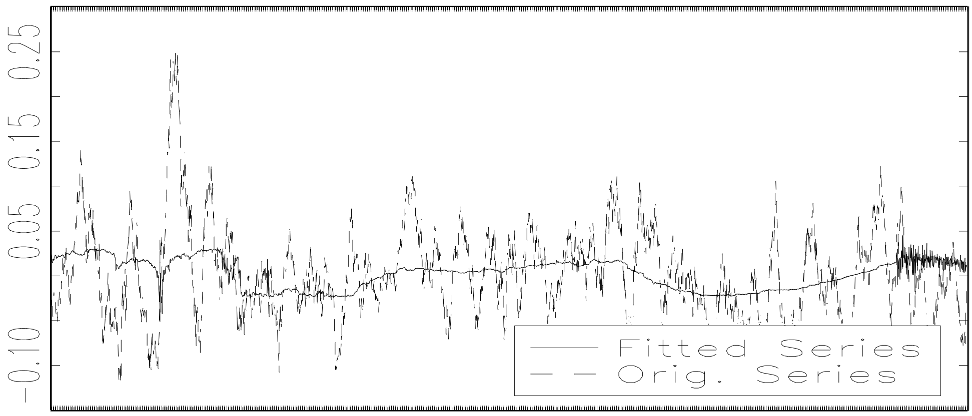
Figure III

Original series vs Fitted series

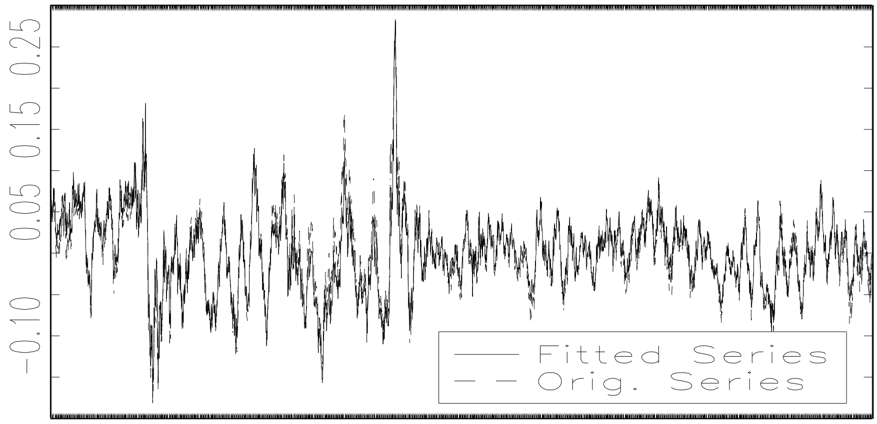
Three-month forward exchange rate



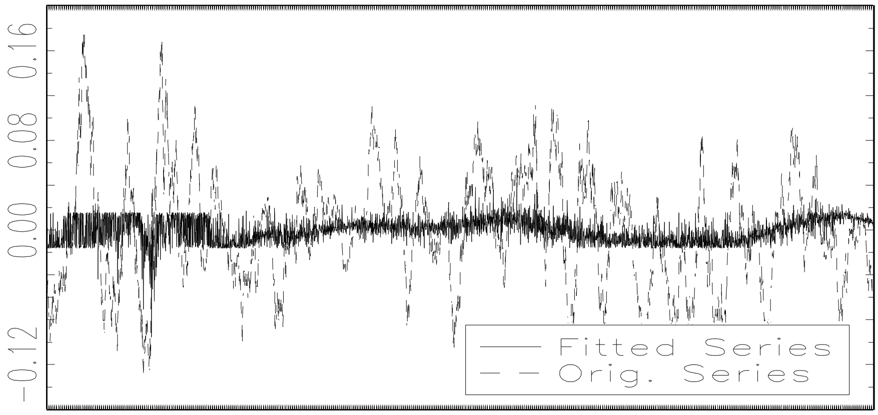
Canadian Dollar



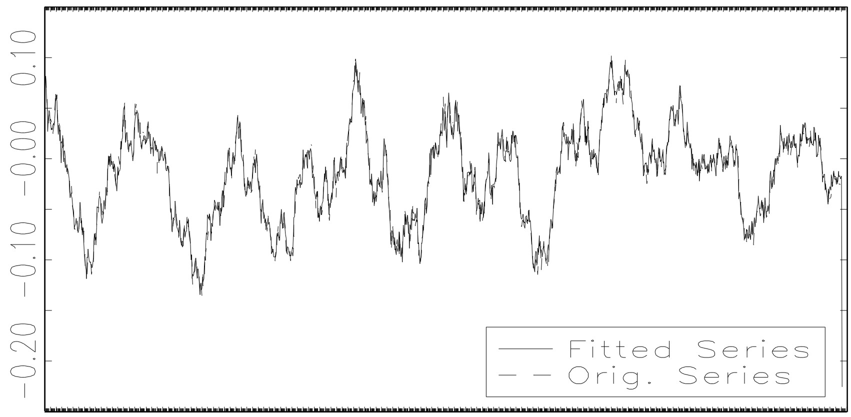
Euro



Sterling Pound



Swedish Corona

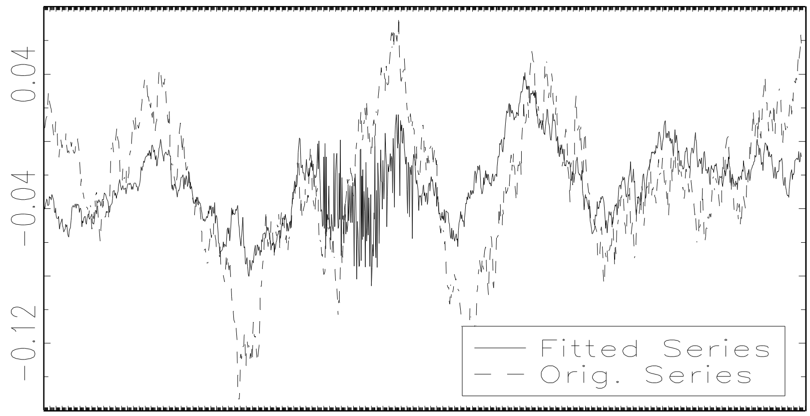


Swiss Franc

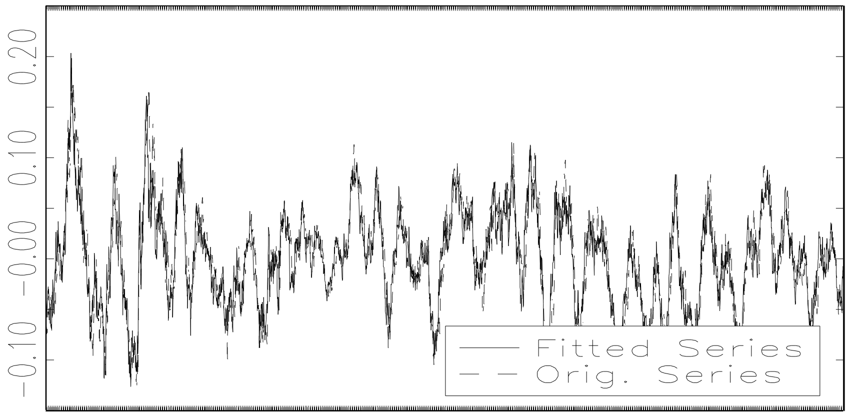
FIGURE IV

Original series vs Fitted series

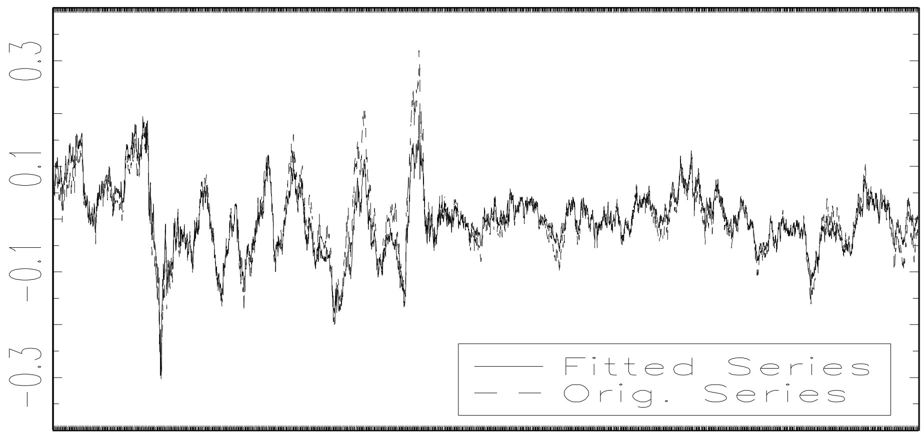
Six-month forward exchange rate



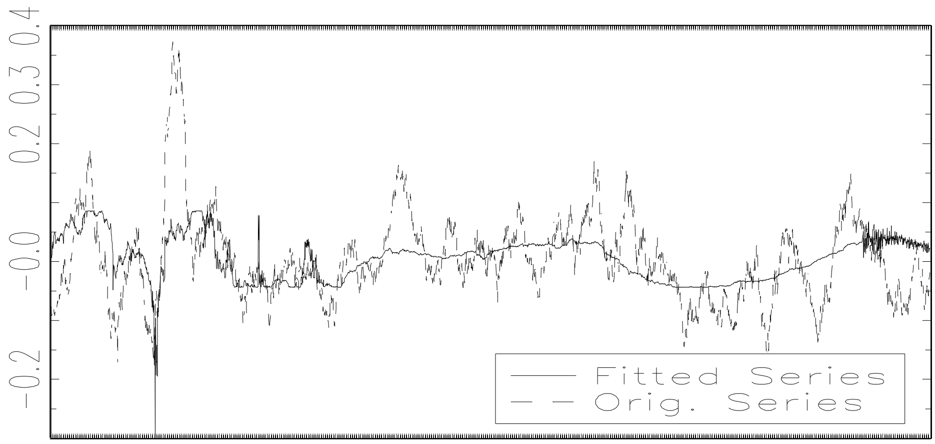
Canadian Dollar



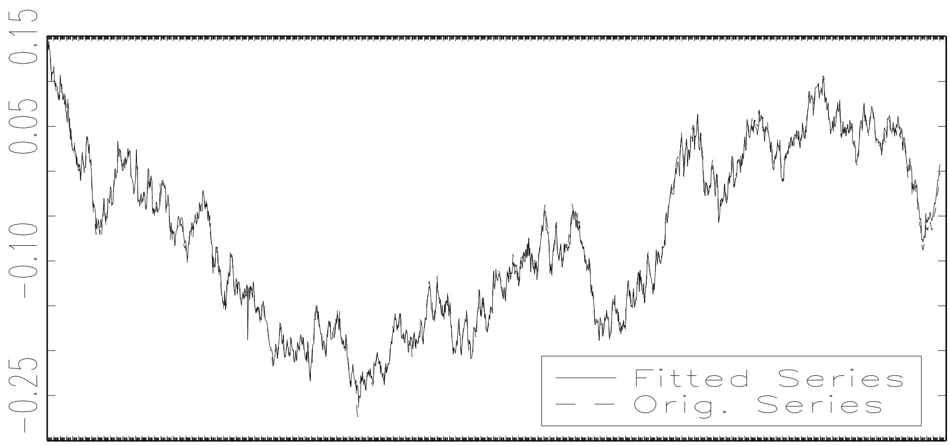
Euro



Sterling Pound



Swedish Corona



Swiss Franc

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