E C O N O M I C S B U L L E T I N

Utility functions, future consumption targets and subsistence thresholds

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Abstract

If the consumer's risk aversion behavior varies intertemporally and if the risk aversion coefficient on future consumption becomes very large, the consumer tends to aim at a fixed future consumption target. A by-product is a reinterpretation of subsistence theories of consumption.

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1. Introduction

Empirical evidence from consumer surveys and the like suggests that consumers often save to meet a fixed future consumption target.[1] If this were so, it would have important consequences for saving behavior and capital markets. Saving would be negatively interest-elastic, the gross substitutability property would not obtain between current and future consumptions, resulting in instability and multiplicity of equilibria, including some with high saving at very low, possibly zero, interest rates[2]. Under certain assumptions, such behavior can generate a rich dynamics, including historical traps, implosions involving dwindling capital stock and increasing interest rates, and optimal one-time interventions [we refer readers to another paper of ours for a detailed treatment of the implications for equilibrium and dynamics]. Further, with future consumption targets fixed, income shocks would be absorbed essentially by current consumption, yielding an alternative explanation of the excess-sensitivity-of-consumption-to-income puzzle.

However, theoretical work has essentially ignored fixed consumption targeting behavior. Perhaps this is because such behavior has been generally regarded as an oddity, irreconcilable with standard utility maximization. The present paper seeks to dispel this notion and show how such behavior can be generated from conventional utility functions.

Carroll (2004) has explored a somewhat similar theme – that of *buffer stock saving*. However he is concerned with consumers who aim at a fixed ratio of wealth to permanent income, which is quite different from the phenomenon we seek to explore. Our consumer targets a fixed consumption level rather than a saving ratio and this target, unlike Carroll's, is independent of wealth, interest rates and permanent income.

The mathematical analysis that we apply, both to this theme and to a second application (subsistence thresholds) is very simple. But it has never been done before and does shed light on empirically observed phenomena (like future consumption targets and subsistence thresholds) that have never quite been explained within the framework of standard utility functions.

2. The Model

Consider an additive two-period utility function

$$U(c_1, c_2) = u(c_1) + \frac{c_2^{1-\theta} - 1}{(1-\theta)(1+\rho)} \tag{1}$$

Here second period felicity is CRRA in c_2 with risk-aversion coefficient θ and time preference rate ρ. First period felicity is any function of first period consumption which does not share the *same* risk aversion coefficient as second period felicity, and such that u'( ) > 0, u'' < 0. Given a wealth constraint and a fixed interest rate r, an interior maximum of $U(c_1,c_2)$ implies

$$\frac{c_2^{-\theta}}{(1+\rho)u'(c_1)} = \frac{1}{1+r} \tag{2}$$

$$\text{or} \quad c_2 = [\frac{(1+\rho)u'(c_1)}{1+r}]^{-1/\theta} \tag{3}$$

[1] Samwick (1998) cites evidence from the Survey of Consumer Finance (1992).

[2] On the aggregative plane, such behavior has been observed, for instance, in China, which saves 50% of its GDP while the Bank of China's deposit rate, after two recent increases, has just reached 2.52% with CPI inflation currently running at 5%. There have of course been other attempts at explaining the Chinese paradox.

In the limit as $\theta \rightarrow \infty$, $c_2 \rightarrow 1$ – provided $u'(c_1)$ is independent of θ. Asymmetrically infinite second-period risk aversion suffices to ensure target saving behavior. There is no loss of generality due to the unit value of c_2 since we can choose units as we please.

Alternatively, in the total utility function (1), consider the second-period felicity component. As $\theta \rightarrow \infty$, $(1-\theta) \rightarrow -\infty$, and

$$\lim_{\theta\rightarrow\infty} \frac{c_2^{1-\theta}-1}{1-\theta} = 0 \text{ for } c_2 \geq 1 \quad (4)$$

$$= -\infty \text{ for } c_2 < 1 \quad (4a)$$

(1), at this limit, reduces to

$$U(c_1,c_2) = u(c_1) \text{ for } c_2 \geq 1 \quad (5)$$

$$= -\infty \text{ for } c_2 < 1 \quad (5a)$$

This explains why consumers target one and only one unit of consumption in the second period. Given the wealth constraint, any additional second period consumption would reduce first period consumption and felicity and, therefore according to (5), total utility as well. Any shortfall in second period consumption below the target level results on the other hand in infinite misery and must therefore be avoided if this is at all possible within the wealth constraint.

Future consumption targeting is thus associated with intense future risk aversion – so intense, in fact, that interest rate, time preference and wealth level cease to matter. Such intense risk aversion may not be uncommon among consumers in economies that have a long history of turbulence and volatility.

3. A by-product: a reinterpretation of subsistence theories of consumption

While the consumption targets considered above admit of neither upward nor downward variation, the concept of a subsistence level of consumption (as used for instance in Steger (2000), Ben-David (1998), Sharif (1986), Stigler (1945)) implies only downward rigidity. A by-product of our results however is such a subsistence model. Consider utility function (1) with $c_1 = c_2 = c$ and $\rho = 0$:

$$U(c) = u(c) + (c^{1-\theta} - 1)/(1-\theta) \quad (1a)$$

We have now switched from a two-period model of consumption choice to a static utility function with two components: an arbitrary one and another of CRRA form. These two components correspond to two possible elements of the total utility derived from consumption: a sociological element (based for instance on concepts like status value) and a biological one. We assume that biological utility is additively separable from sociological utility and that one of the two component functions, possibly the biological, is CRRA. θ is the risk aversion coefficient associated with biological utility and a high value of θ reflects the fact that people are reluctant to gamble with the biological aspects of consumption.

Repeating the argument in equations (4) to (5a) above, this reduces, in the limit $\theta \rightarrow \infty$, to

$$U(c) = u(c) \text{ for } c \geq 1 \quad (6)$$

$$= -\infty \text{ for } c < 1 \quad (6a)$$

A downward-rigid subsistence consumption level $c = 1$ is implied (without loss of generality due to the unit value of c). This is of some interest because there is no evidence of a rigid physiological minimum level of consumption required to sustain life.[3] What we suggest here is

[3] See Sukhatme and Margen (1981), Edmundson and Sukhatme (1990).

a model of subsistence based on risk aversion.[4] Needless to say, it is easy to recast our argument in the framework of multi-period choice.

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[4] The subsistence concept can be endogenized either (1) in terms of a relationship between nutrition and working efficiency – as in Dasgupta (1997) – which yields a given subsistence level for any specific level of technology and complementary inputs – or (2) in terms of a relationship between nutrition and fertility which yields a given subsistence level for any specific mortality rate – as in Dalgaard and Strulik (2007). Such an endogenous subsistence concept is not our focus in this paper.