Multi-product strategies and relative preferences for quality

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Abstract

In this paper we investigate whether a monopoly in a vertically differentiated market may have the incentive to adopt a multi-product strategy if the consumers are concerned by the other consumers' choices. We use a variant of the Mussa and Rosen model where the utility of a consumer is positively or negatively affected by her relative standing i.e how high or low is the quality she chooses with respect to the other consumers' choices. We prove that a multi-product strategy may be adopted by the monopoly if the consumers' social distinction desire is strong enough.

1 Introduction

It is now well recognized that the pleasure of consuming one good may be affected by the consumption choice of other consumers. Let us consider a consumer planning to buy a car or a mobile phone. If she buys the highest quality offered in the market then in addition to the traditional satisfaction derived from the intrinsic characteristics of the product, she derives a positive satisfaction from consuming the best quality offered in the market. Inversely, if she does not buy the highest quality then the consumer loses some satisfaction because she does not consume the best quality in the market when other consumers do. Such behavior is referred to in the literature as search for "prestige" or "status" or "social distinction". The quality chosen by a consumer signals her social rank and the consumers' relative preferences for quality affect their satisfaction. A consumer may be interested by a good because of her relative standing when consuming this good.

In this paper, we consider a vertically differentiated market. We investigate whether relative preferences for quality may favor multi-product strategies. More precisely, we examine whether it is profitable for a monopoly to offer more than one quality of her product when the consumers' relative preferences for quality affect either positively or negatively their utility depending on their quality choice relative to the choice of the other consumers.

Relative preferences were first formally modeled by Duesenberry (1949) who described several examples of interdependence in consumer consumption behavior. Akerlof (1997) proved that the satisfaction of a consumer increases with the difference between her personal status and others' status. Alexopoulus and Sapp (2006) as well as Riechmann (2006) found that firms with relative preferences can affect the outcomes predicted by the standard Cournot and Stackelberg models. In Ben Elhadj (2007), it was proved that when taking into account the consumers' relative preferences for quality, the strategic behavior of a duopoly in a vertically differentiated market is affected.

In the case of a monopoly in a vertically differentiated market, two factors may explain a multi-product strategy: quality costs and income disparities[1] .

Acharyya(1998) demonstrated that if the cost function is sufficiently convex in quality the monopoly offers a separating menu: a different quality for each type of consumers. Acharyya (1998) also proved that the standard parametrization of costs and preferences observed in the literature: linear utilities with respect to price and quality and constant marginal cost of quality, can not generate a separating menu. Kim and Kim (1996) showed that due to cost spillover-effects, a monopoly operating in a vertically differentiated market facing a discrete consumers distribution can reduce total costs of production and increase her profit by jointly producing two products.

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[1]Bonnisseau and Lahmandi-Ayed (2006) proved that deterring entry is not a factor that can favor the adoption of a multi-product strategy.

Acharyya (2005) proved, in a model where the consumers' distribution is discrete, that the constraining effect of income by itself make quality discrimination profitable for a monopoly. Bonnisseau and Ghazzai(2005) determined conditions under which the monopoly adopts a multi-product strategy when consumers are continuously distributed with respect to their intensities of preference for quality and incomes. Quality discrimination is profitable because of income disparities.

Gabszewicz et al. (1986) addressed the same issue considering another vertical differentiation model where consumers differ by their willingness to pay and where this willingness to pay has in the same time a constraining effect (a consumer cannot purchase at a price exceeding her willingness to pay) and a utility effect (richer consumers derive higher utility from consumption than do poorer consumers). They showed that it is profitable for the monopolist to produce the maximal number of products when the market is sufficiently large. These results together with those of Acharyya (2005) highlight the role of the difference in income in the emergence of multiproduct strategies.

We prove in this paper that introducing consumers' relative preferences may lead to multi-product strategies under some conditions on the consumers' distribution and their social desire distinction. Thus, in addition to quality costs and income disparities, relative preferences for quality explain quality discrimination.

The rest of the paper is organized as follows. In section 2, we describe the model. In section 3, we give necessary and sufficient conditions for the monopoly to adopt a multi-product strategy. We conclude in section 4.

2   The model

We consider a vertical differentiation model with relative preferences for quality. The market is monopolized by one firm which can produce different qualities of the same good chosen from the segment $[0, \overline{q}]$.

The production is costless. We choose a costless production to prove that the monopoly may adopt a multi-product strategy only because of the relative preferences for quality. With costless production and when consumers are not income constrained, a multi-product strategy is never profitable for the monopoly whatever is the distribution of the consumers as shown by Acharyya (1998).

There are two types of consumers characterized by their intensity of preference for quality. We consider n_1 consumers with low preference for qualities characterized by θ_1 and n_2 consumers with high preference for quality characterized by θ_2 with $\theta_2 > \theta_1 > 0$. Each consumer is supposed to buy at most one quality from the qualities offered by the monopoly. The quality purchased is the one that ensures to her the highest utility except if the consumer is better when making no purchase.
Because we only consider two types of consumers, the monopoly will offer at most two

qualities q_1 and q_2 with $q_2 > q_1$. The indirect utility of a consumer is then given by:

$$U_i = \begin{cases} -p_i + \theta_i q_i + \beta(q_i - q_j) & \text{if she buys quality } q_i \\ 0 & \text{if she buys neither quality} \end{cases}$$

with $i = 1, 2$; $j = 1, 2$ and $i \neq j$.

The utility function of a consumer is given by the standard utility function in a vertically differentiated market introduced by Mussa and Rosen (1978) to which we add a term $(\beta(q_i - q_j))$ relative to the relative preferences for quality of consumers [2]. If $\beta(q_i - q_j)$ is positive $(q_i > q_j)$, the utility of a consumer is positively affected because she is consuming a higher quality than the other quality available in the market. If $\beta(q_i - q_j)$ is negative$(q_i < q_j)$, the utility of a consumer is negatively affected because she is consuming a lower quality than the other quality available in the market. β may be interpreted as the social distinction desire. β is positive and the larger is β, the more important is the impact of relative preferences in the satisfaction of consumers.

By her choice of prices and qualities: p_1, p_2, q_1 and q_2, the monopoly objective is to maximize her profit. The monopoly can choose a separating menu by offering different price-quality combinations to different consumers'types or the monopoly can choose to offer only one quality. When offering only one quality, the monopoly can either cover the whole market or cater for only the high-type consumers. In the next section, we provide the conditions under which the monopoly has a multi-product strategy i.e the monopoly offers a separating menu to the consumers.

3 The Monopoly Strategy

In this section, we assume first that the monopoly is offering a separating menu to the consumers. We find the necessary conditions for the monopoly to adopt a multi-product strategy. Then, we compare the monopoly's profit when she offers two qualities to her profit when she offers only one quality.
The monopoly's problem, when offering a separating menu, is the following:

$$\max \Pi = n_1 p_1 + n_2 p_2 \text{ (s.t)}$$
$$\begin{cases} 0 \leq p_1 \leq \theta_1 q_1 + \beta(q_1 - q_2) & (1) \\ 0 \leq p_2 \leq \theta_2 q_2 + \beta(q_2 - q_1) & (2) \\ \theta_1 \leq \frac{p_2 - p_1}{q_2 - q_1} - 2\beta & (3) \\ \theta_2 \geq \frac{p_2 - p_1}{q_2 - q_1} - 2\beta & (4) \end{cases}$$

Condition (1) implies that type-θ_1 consumers prefer buying the quality q_1 than making no purchase.
Condition (2) implies that type-θ_2 consumers prefer buying the quality q_2 than making

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no purchase.

Condition (3) means that type-θ_1 consumers prefer q_1 to q_2.

Condition (4) means that type-θ_2 consumers prefer q_2 to q_1.

With the four conditions cited above, we ensure that the market is covered and that the monopoly is offering a separating menu. The quality q_1 is purchased by type-θ_1 consumers and the quality q_2 is purchased by type-θ_2 consumers. Thus, the market is ordered as in the standard Mussa and Rosen Model. Consumers with low intensity of preference for quality buy the low quality. Consumers with high intensity of preference for quality buy the high quality [3].

Solving the monopoly's problem is equivalent to consider a two-step game where the monopoly chooses her qualities first and then fixes the prices. By Lemma 1, we give a first necessary condition that must be satisfied by the qualities for the existence of a separating menu. The low quality q_1 must be high enough with respect to the high quality q_2 to be purchased by the consumers.

Lemma 1 *A necessary condition to offer a separating menu is* $q_1 \geq \frac{\beta}{\beta+\theta_1}q_2$.

Proof. Immediate. If $q_1 < \frac{\beta}{\beta+\theta_1}q_2$, condition (1) in the profit maximization problem is never satisfied and the profit maximization problem has no solution. ∎

If the condition stated in Lemma 1 is not satisfied, type-θ_1 consumers are better making no purchase. The quality offered to them is too low with respect to the quality offered to type-θ_2 consumers. The existence of relative preferences prevent them from buying the lowest quality even if offered at a null price.

Assuming that $q_1 \geq \frac{\beta}{\beta+\theta_1}q_2$, we represent in Figure 1 by the shaded area all the prices p_1 and p_2 that satisfy conditions (1), (2), (3) and (4).

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[3] $\frac{p_2-p_1}{q_2-q_1} - 2\beta$ in conditions (3) and (4) is the intensity of preference for quality of a consumer indifferent between buying q_1 or q_2.
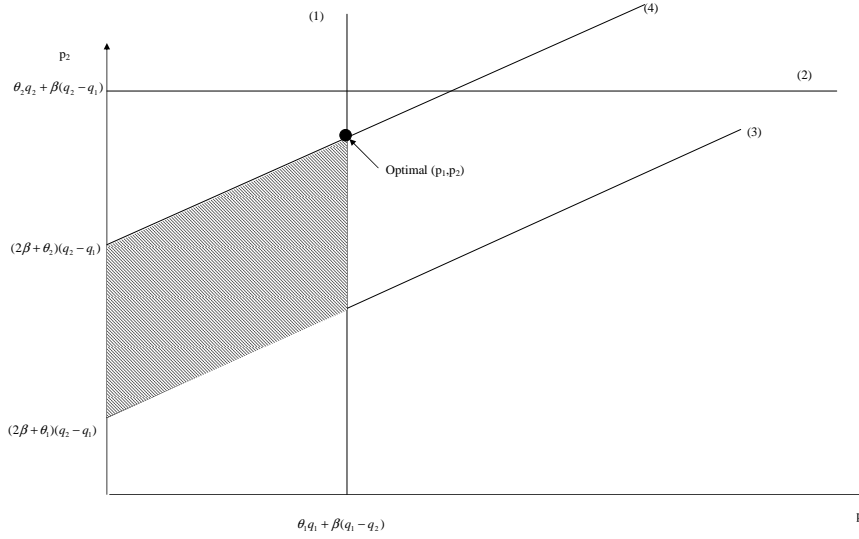
Figure 1: Prices satisfying conditions (1) to (4)

As the monopoly's profit is linear with respect to p_1 and p_2, we easily deduce from Figure 1 the optimal prices as functions of q_1 and q_2. They correspond to the maximal prices p_1 and p_2 belonging to the shaded area. Thus, we have:

$$p_1(q_1, q_2) = \theta_1 q_1 + \beta(q_1 - q_2)$$
$$p_2(q_1, q_2) = \theta_1 q_1 + (\beta + \theta_2)(q_2 - q_1)$$

In maximizing the monopoly's profit, we can notice that consumers with low preference for quality have no surplus. All type-θ_1 consumers are indifferent between purchasing q_1 and not purchasing. Condition (1) is binding for these consumers. Condition (4) is binding for type-θ_2 consumers which are indifferent from purchasing q_2 or q_1. Type-θ_2 consumers have a positive surplus as they are charged a price lower than their reservation price $\theta_2 q_2 + \beta(q_2 - q_1)$. [4] This ensures that type-θ_2 consumers will not mimic type-θ_1 consumers and will buy exactly the quality targeted at them.

Replacing in the monopoly's profit p_1 and p_2 by their optimal values, we obtain the following reduced form of the profit:
$$\Pi(q_1, q_2) = (n_1\theta_1 + n_1\beta - n_2\beta - n_2(\theta_2 - \theta_1))q_1 + (-n_1\beta + n_2\theta_2 + n_2\beta)q_2.$$

It is straightforward to check that the profit is increasing in q_2 if $\frac{n_1}{n_2} \leq 1 + \frac{\theta_2}{\beta}$ and decreasing in q_1 if $\frac{n_1}{n_2} \leq \frac{\beta + \theta_2 - \theta_1}{\beta + \theta_1}$. Thus, a second necessary condition for a separating menu to exist is given by Lemma 2.

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[4]Let us compare the reservation price of type-θ_2 consumers $p_2 = \theta_2 q_2 + \beta(q_2 - q_1)$ when the monopoly offer only one quality of type-θ_2 consumers to the optimal price $p_2 = \theta_1 q_1 + (\beta + \theta_2)(q_2 - q_1)$ when the monopoly offer two qualities. $[\theta_2 q_2 + \beta(q_2 - q_1)] - [\theta_1 q_1 + (\beta + \theta_2)(q_2 - q_1)] = (\theta_2 - \theta_1)q_1 > 0$

Lemma 2 *A necessary condition to offer a separating menu is $\frac{n_1}{n_2} \leq \frac{\beta + \theta_2 - \theta_1}{\beta + \theta_1}$.*

Proof. The profit is increasing in q_2 if $\frac{n_1}{n_2} \leq 1 + \frac{\theta_2}{\beta}$ and decreasing in q_1 if $\frac{n_1}{n_2} \leq \frac{\beta + \theta_2 - \theta_1}{\beta + \theta_1}$. We can easily check that $\frac{\beta + \theta_2 - \theta_1}{\beta + \theta_1} < 1 + \frac{\theta_2}{\beta}$. ∎

We deduce from Lemmas 1 and 2 that if a separating menu is offered, the optimal prices and qualities are:

$$p_1^* = 0; \; q_1^* = \frac{\beta}{\beta + \theta_1}\overline{q}$$
$$p_2^* = (2\beta + \theta_2)(\frac{\theta_1}{\beta + \theta_1})\overline{q}; \; q_2^* = \overline{q}.$$

Offering a positive quality q_1 at a null price means that the monopoly does not have any additional profit from type-θ_1 consumers. However, offering two qualities instead of one enables *a priori* the monopoly to set a higher price for the quality q_2 than when she offers only one quality as the willingness to pay of type-θ_2 consumers is higher. In fact, type-θ_2 consumers have an additional utility resulting from their consumption of the best quality offered in the market.

The quality offered to type-θ_1 consumers is the lowest quality that satisfies Lemma 1. Type-θ_1 are not willing to purchase a lower quality than this one.

Let us now compare the monopoly's profit when she offers a separating menu to her profit when she offers only one quality. The monopoly can choose between two strategies:

1. Offer one quality for the two types of consumers.

2. Offer one quality for the type-θ_2 consumers.

When only one quality is offered the relative preferences for quality do not exist anymore. The utility function is given then by the standard utility function [5].

If the monopoly offers one quality and covers all the market, the optimal price and quality are $p^* = \theta_1 \overline{q}$ and $q^* = \overline{q}$. The maximal price the monopoly can fix is the reservation price of type-θ_1 consumers $p = \theta_1 q_1$. The monopoly's profit is maximized when she produces the highest possible quality. From straightforward calculations, we can check that if the condition of Lemma 2 is satisfied, it is more profitable for the monopoly to offer a separating menu. Thus, if the condition $\frac{n_1}{n_2} \leq \frac{\beta + \theta_2 - \theta_1}{\beta + \theta_1}$ is satisfied, the monopoly's strategy consisting in offering one quality to all consumers is always dominated by the multi-product strategy.

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[5]When the monopoly offers only one quality, the utility of a consumer is given by:

$$U_i = \begin{cases} -p_i + \theta_i q_i & \text{if she buys quality } q_i \\ 0 & \text{if she buys neither quality} \end{cases}$$
with $i = 1, 2$.

If the monopoly offers one quality for the type-θ_2 consumers, the optimal price and quality are $p^* = \theta_2 \bar{q}$ and $q^* = \bar{q}$. The maximal price the monopoly can fix is the reservation price of type-θ_2 consumers $p = \theta_2 q_2$. The monopoly's profit is maximized when she produces the highest possible quality. From straightforward calculations, we can check that if the condition $\theta_2 < 2\theta_1$ is satisfied, it is more profitable for the monopoly to offer a separating menu. In fact, when offering a separating menu, the monopoly has no additional profit from serving type-θ_1 consumers but the price she sets is higher than when she only caters for type-θ_2 consumers if $\theta_2 < 2\theta_1$. Therefore,

Proposition 1 *The optimal strategy for the monopoly is to offer a separating menu if and only if* $\beta \geq \frac{n_1\theta_1 - n_2(\theta_2 - \theta_1)}{n_2 - n_1}$ *and* $\theta_2 < 2\theta_1$.

Proof. Immediate. $\frac{n_1}{n_2} \leq \frac{\beta + \theta_2 - \theta_1}{\beta + \theta_1}$ and $\theta_2 < 2\theta_1$ imply $\frac{n_1}{n_2} \leq 1 + \frac{\theta_2 - 2\theta_1}{\beta + \theta_1} < 1$. Thus, $n_1 < n_2$ and $\beta \geq \frac{n_1\theta_1 - n_2(\theta_2 - \theta_1)}{n_2 - n_1}$. $\blacksquare$

Under some conditions on the consumers' intensity of preference for quality and their social distinction desire, we prove that the optimal monopoly strategy is to offer two qualities. In fact, the consumers' social distinction desire must be strong enough to justify a multi-product strategy. When the monopoly offers a separating menu, she has no additional profit from type-θ_1 consumers. The only reason for offering a separating menu is that the monopoly can charge a higher price to type-θ_2 consumers than when she offers the same quality for all the consumers. The additional profit she obtains from type-θ_2 consumers (when offering a separating menu) compensates the profit she has from type-θ_1 consumers when she offers the same quality for all if the consumers' social distinction desire is strong enough.

We can also notice that if the intensity of preference for quality of type-θ_2 consumers is too high with respect to the intensity of preference for quality of type-θ_1 consumers $(\theta_2 \geq 2\theta_1)$. The monopoly is better catering only for type-θ_2 consumers. The price she can fix for the high quality in this case $(p_2 = \theta_2\bar{q})$ is greater than the price fixed when offering a separating menu.

4 Conclusion

In this paper, we have analyzed the optimal strategy for a monopoly when relative preferences for quality exist. Even when the production is costless and even if the consumers are not income constrained, a multi-product strategy is profitable for the monopoly when some conditions on the consumers' intensity of preference for quality and their social distinction desire are satisfied.

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