E C O N O M I C S   B U L L E T I N

Overestimation in the Traditional GARCH Model During Jump Periods

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Abstract

The traditional continuous and smooth models, like the GARCH model, may fail to capture extreme returns volatility. Therefore, this study applies the bivariate poisson (CBP)-GARCH model to study jump dynamics in price volatility of crude oil and heating oil during the past 20 years. The empirical results indicate that the variance and covariance of the GARCH and CBP-GARCH models were found to be similar in low jump intensity periods and to diverge during jump events. Significant overestimations occur during high jump time periods in the GARCH model because of assumptions of continuity, and easily leading to excessive hedging and overly measuring risk. Nevertheless, in the CBP-GARCH model, the specific shocks are assumed to be independent of normal volatility and to reduce the persistence of abnormal volatility. Therefore, the CBP-GARCH model is appropriate and necessary in high volatility markets.

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1. INTRODUCTION

Crude oil not only is the world's most actively traded commodity, but also leads to the largest-volume futures contract trading for a physical commodity on the New York Mercantile Exchange (NYMEX). Given their excellent liquidity and price transparency, these futures contracts are used as a key international pricing benchmark. Notably, heating oil is a product refined from crude oil. With the lifting of U.S. price controls on heating oil in the mid 1970's, the NYMEX began developing a heating oil futures contract and, in 1978, introduced the world's first successful energy futures contract. Heating oil futures has become one of the premier distillate contracts in futures trading. In 1983, crude oil futures contracts were introduced. In its early years, the NYMEX Division heating oil contract mainly attracted wholesalers and large consumers of heating oil in the New York Harbor area. Today, a wide variety of businesses, including oil refiners, wholesale marketers, heating oil retailers, trucking companies, airlines, and marine transport operators, as well as other major consumers of fuel oil, have embraced this contract as a risk management vehicle and pricing mechanism. The dominant price component is the cost of crude oil; hence, heating oil prices should be closely linked to the cost of crude oil prices[1]. Observing spot prices over the past 20 years (Fig. 1) reveals similar trends in crude oil and heating oil spot prices, but there are occasional surges in heating oil spot prices, such as at the end of 1989 and in early 2000. These surges result from rapid supply and demand shifts caused by weather, refinery shutdowns, or political instability. As is seen in Panel B of Fig. 1, jumps in returns are occasional. These phenomena can be described by the jump model that focuses on the abnormal volatility. However, almost no studies have investigated jumps in crude oil and heating oil in energy markets. Accordingly, this study employs the correlated bivariate poisson - generalized autoregressive conditional heteroscedasticity model (CBP-GARCH model) to demonstrate the need to consider such jumps. The GARCH model remains the widely used model for researching the volatility behavior of energy assets (Lin and Tamvakis, 2001; Ewing et al., 2002; Sadorsky, 2002; Hammoudeh et al., 2003). However, traditional continuous and smooth models such as the GARCH may fail to capture extreme returns volatility. Accurate estimation of the variance and covariance of the assets can improve performance in forecasting, hedging, risk management, etc.

Volatility estimation and forecasting have been the main task in financial markets during the past two decades, and they are fundamental to most areas of finance -- for example, asset pricing, portfolio selection, volatility relationship, hedging, risk, etc. Most studies assume that time series data follow a smooth and

[1] According to EIA's (Energy Information Agency) Petroleum Marketing Monthly (2001), the final price to consumers of home heating oil can be broken down in percentage terms as follows: 42% crude oil purchase cost, 12% refining costs, and 46% distribution and marketing costs.

continuous volatility process, and GARCH is now widely used in this field (see the survey in Poon and Granger, 2003; Bauwens et al., 2006). However, the existence of jumps implies that diffusion models are misspecified statistically. Jorion (1988) contended that time-varying volatility and occasional jumps are possibly the two most notable features of daily financial time series. Moreover, Park (2002) mentioned that the standardized residuals of the GARCH model still have excess kurtosis, albeit less than for the raw financial returns (also see Bollerslev, 1987; Baillie and Bollerslev, 1989; Hsieh, 1989). Furthermore, Chan (2003) noted that although multivariate GARCH models adequately account for heteroskedasticity, they do not fully capture the leptokurtosis in unconditional distributions that is frequently observed in financial data. Consequently, financial econometrics further investigates volatility with jumps (e.g., Chang and Kim, 2001; Pan, 2002; Eraker, Johannes and Polson, 2003; Chan and Maheu, 2002; Johannes, 2004; Maheu and McCurdy, 2003). Most jump models have been successfully applied to analyze foreign exchange and stock market returns, and these models can improve performance in capturing price behavior in physical commodities. In metal markets, Chan and Young (2006) found that the jump model closely fits copper spot and futures data. Furthermore, this study introduced the jump GARCH model to the energy market and investigated the price behavior of crude oil and heating oil, the most liquid trading assets in NYMEX. The traditional GARCH model is clearly misapplied to energy assets with high volatility, particularly in situations involving large jumps. The problems with estimation that result from the traditional GARCH model in such scenarios are eliminated by the bivariate jump GARCH model, because this model successfully reduces the limitations associated with the continuous volatility process and the univariate jump assumption from previous studies.

Jumps represent a response to unusual news events as part of the latent news process and have the potential to capture both smooth and sudden price volatility movements (Chan and Young, 2006). Press (1967), who introduced an independent jump process in which jump arrival was governed by a Poisson distribution, was the first to apply the Poisson jump model to financial markets. Although jumps cannot be observed, an ex-post filter can always be constructed to infer their probability. Tucker and Pond (1988), Akgiray and Booth (1988) and Hsieh (1989) all found that the Poisson jump model provides an effective statistical characterization of daily exchange rates. The basic jump models have been further extended in various directions, and combining them with the ARCH/GARCH model is an essential application (Jorion, 1988; Vlaar and Palm, 1993). Several studies emphasize that the time-varying jump fits closely with reality (Betas, 1991; Eraker et al., 2002; Das, 2003; Chan and Maheu, 2003; Maheu and McCurdy, 2004). However, all of the above models are limited by the use of the univariate setting for capturing the price volatility of specific assets. Most researchers now accept that financial volatilities move together over time across assets and markets. Recognizing this feature via a multivariate modeling framework yields more relevant empirical models than working with separate univariate models. Therefore, Chan (2003) devised a bivariate jump model that combined the Correlated Bivariate Poisson (CBP) function and the GARCH model to analyze jump dynamics. The CBP-GARCH model was applied to energy markets to examine price volatility for crude oil and heating oil.

This study has three aims. First, this investigation aims to accurately model volatility for high-volatility energy market assets. This study provides a complete analysis of the price volatility of crude oil and heating oil over the past 20 years and examines whether the jump model has better performance. Second, this study attempts to use bivariate jump models to accurately estimate the volatilities of two closely related assets. This model is applied in this way not only because of volatility spillovers between markets and assets, but also because of the importance of the covariance between series. Therefore, following Chan (2003), this study discusses the volatility characteristics of crude oil and heating oil using the correlated bivariate jump model. We further relax the strong restrictions on jump size and intensity in the simple CBP-GARCH model. Both the asymmetric time-varying assumption in jump size and the autoregressive term in jump intensity are added in the model[2]. Third, this study attaches importance to the problem of overestimation and investigates whether it occurs in the traditional GARCH model when considering jump events. Overestimation of variance and covariance will bias further applications of the model, such as hedging, value of risk, portfolio constructing, etc. Hedging is particularly important during high-volatility periods. Overestimation will cause excessive hedging together with increased costs and reduced hedge effectiveness. Moreover, overestimation of volatility increases the value at risk and leads to the loss of potential profit. The remainder of this paper is organized as follows. Section 2 presents the methodology of the GARCH and CBP-GARCH models. Section 3 then explains the data and descriptive statistics. Next, Section 4 describes the empirical results. Finally, the last section presents conclusions.

2. METHODOLOGY

GARCH model

The GARCH model has been widely used in volatility estimation since being introduced by Bollerslev (1986). The standard VEC bivariate model proposed by

[2] Thanks for the anonymous reviewers' suggestion. These extensions provide significant improvement in volatility forecasting over the simple CBP-GARCH model.

Bollerslev, Engle and Wooldridge (1988) is briefly described by

$$R_t = \mu + \varepsilon_t \tag{1}$$

where R_t is a 2×1 vector of returns, μ is a drift coefficient, and the error term of the returns ε_t follows the normal distribution with mean 0 and variance H_t. The conditional variance equation is

$$H_t = C + A\varepsilon_{t-1}\varepsilon'_{t-1} + BH_{t-1} \tag{2}$$

where C is a 3×1 column vector and A and B are 3×3 matrices. Furthermore, assuming normally distributed errors in the estimation process implies the following simple log-likelihood function,

$$L(\theta) = -\frac{1}{2}\sum_{t=1}^{T}(\ln|H_t| + \varepsilon'_t H_t^{-1}\varepsilon_t) \tag{3}$$

where θ represents the vector of parameters to be estimated and T is the number of observations. Since the log-likelihood function in this case is non-linear, we use numerical maximization techniques to estimate the model.

CBP-GARCH model

The CBP-GARCH model is a combination of the GARCH (Bollerslev, 1986) and the Poisson Correlated function (M'Kendrick, 1926; Campbell, 1934). The model is defined as follows:

$$R_t = \hat{R} + \varepsilon_t + J_t, \tag{4}$$

where R_t is a 2×1 vector of returns consisting of a mean equation $\hat{R}$, a random disturbance ε_t, and a jump component J_t. The random disturbance follows a bivariate normal distribution with zero mean and variance covariance matrix $\widetilde{H}_t$.

In a bivariate framework, the jump component J_t has a bivariate normal distribution with zero mean and variance covariance matrix Δ_t. The normal disturbance and the jump components are assumed to be independent, defined as:

$$J_t = \begin{bmatrix} \sum_{i=1}^{n_{1t}} Y_{1t,i} - E_{t-1}(\sum_{i=1}^{n_{1t}} Y_{1t,i}) \\ \sum_{j=1}^{n_{2t}} Y_{2t,j} - E_{t-1}(\sum_{j=1}^{n_{2t}} Y_{2t,j}) \end{bmatrix} \tag{5}$$

Here, Y_i is a random variable called jump size. The sum of the Y_i means that the return may experience "n" number of jumps depending on the news content that enters the market within any single time period t. Each of these jump sizes is governed by a normal distribution with mean θ and variance δ^2. In other words, the jump size for the two spots (crude oil and heating oil) can be characterized as:

$$Y_{1t,i} \sim N(\theta_{1t}, \delta_1^2) \text{ and } Y_{2t,j} \sim N(\theta_{2t}, \delta_2^2). \tag{6}$$

The mean of the distribution is allowed to vary asymmetrically over time as a function of the size and sign of recent returns in each market:

$$\theta_{1t} = \theta_{10} + \theta_1^- R_{1t-1}(1 - I(R_{1t-1})) + \theta_1^+ R_{1t-1} I(R_{1t-1})$$

$$\theta_{2t} = \theta_{20} + \theta_2^- R_{2t-1}(1 - I(R_{2t-1})) + \theta_2^+ R_{2t-1} I(R_{2t-1}) \tag{7}$$

where $I(x) = 1$ if $x > 0$ and 0 otherwise.

In equation (5), two discrete counting variables n_{1t} and n_{2t} control the arrival of jumps and they are constructed by three independent Poisson variables, namely, n_{1t}^*, n_{2t}^*, and n_{3t}^*. Each one of these variables has a probability density function given by

$$P(n_{it}^* = j | \Phi_{t-1}) = \frac{e^{-\lambda_i} \lambda_i^j}{j!}. \tag{8}$$

The expected value and the variance of n_{it}^* are both equal to λ_i, which is also referred to as the jump intensity. The correlated jump intensity counters (M'Kendrick, 1926; Campbell, 1934) are defined as

$$n_{1t} = n_{1t}^* + n_{3t}^* \text{ and } n_{2t} = n_{2t}^* + n_{3t}^*. \tag{9}$$

By construction, each of these counting variables (n_{1t} and n_{2t}) is capable of generating independent jumps (n_{1t}^* and n_{2t}^*) and correlated jumps (n_{3t}^*). The latter contribute jumps to both series.

Using the change of variables method and integrating out n_{3t}^* yields the joint probability density for n_{1t} and n_{2t}, given as:

$$P(n_{1t} = i, n_{2t} = j | \Phi_{t-1}) = \sum_{k=0}^{\min(i,j)} e^{-(\lambda_1 + \lambda_2 + \lambda_3)} \frac{\lambda_1^{j-k} \lambda_2^{j-k} \lambda_3^k}{(i-k)!(j-k)!k!}. \tag{10}$$

The expected number of jumps is equal to

$$E(n_{it}) = \lambda_i + \lambda_3. \tag{11}$$

The definition of the time varying jump intensities is given by Chan (2003), that is,

$$\lambda_{1t} = \lambda_1 + \eta_1^2 r_{1t-1}^2 + \gamma_1 \lambda_{1t-1}$$

$$\lambda_{2t} = \lambda_2 + \eta_2^2 r_{2t-1}^2 + \gamma_2 \lambda_{2t-1}$$

$$\lambda_{3t} = \lambda_3 + \eta_3^2 r_{1t-1}^2 + \eta_4^2 r_{2t-1}^2 + \gamma_3 \lambda_{3t-1}, \tag{12}$$

where r_{it-1} is the rate of return for asset i at time $(t-1)$ and r_{it-1}^2 is an approximation of the last period's volatility. The jump intensities are assumed to be related to market conditions, which are related in volatility. In the same way, the covariance is governed by variations in the last period's volatilities from both series. The parametric structure not only introduces additional jump dynamics into the model but also allows a time-varying correlation between the counting variables n_{1t} and n_{2t}. The correlation is calculated as follows:

$$\mathrm{Corr}(n_{1t}, n_{2t}) = \frac{\lambda_{3t}}{\sqrt{(\lambda_{1t} + \lambda_{3t})(\lambda_{2t} + \lambda_{3t})}}. \tag{13}$$

Combining the GARCH model with the CBP function, the probability density function for R_t given i and j jumps in spot 1 and spot 2 is defined by

$$f(R_t \mid n_{1t} = i, n_{2t} = j, \Phi_{t-1}) = \frac{1}{(2\pi)^{N/2}} \left| H_{ij,t} \right|^{-1/2} \exp\left[-u'_{ij,t} H_{ij,t}^{-1} u_{ij,t}\right], \tag{14}$$

where $u_{ij,t}$ is the usual error term with the jump component $J_{ij,t}$ representing the effect of i and j jumps:

$$u_{ij,t} = R_t - \hat{R} - J_{ij,t} = \begin{bmatrix} r_{1t} - \hat{r}_1 - i\theta_{1t} + (\lambda_{1t} + \lambda_{3t})\theta_{1t} \\ r_{2t} - \hat{r}_2 - j\theta_{2t} + (\lambda_{2t} + \lambda_{3t})\theta_{2t} \end{bmatrix}. \tag{15}$$

The variance covariance matrix $H_{ij,t}$ can be separated into two parts: the variance covariance matrix for the normal random disturbance $\widetilde{H}_t$ and for the jump components $\Delta_{ij,t}$.

First, the variance and covariance matrix for the normal random distribution can be defined as in equation (2), in which the term $\widetilde{\varepsilon}_{t-1}$ refers to the sum of a disturbance and a jump component. Second, the variance covariance matrix for the jump components is

$$\Delta_{ij,t} = \begin{bmatrix} i\delta_1^2 & \rho_{12}\sqrt{ij}\,\delta_1\delta_2 \\ \rho_{12}\sqrt{ij}\,\delta_1\delta_2 & j\delta_2^2 \end{bmatrix}, \tag{16}$$

where ρ_{12} is the correlation coefficient between Y_{1t} and Y_{2t}. The variance covariance matrix for the CBP-GARCH model is then a sum of $\widetilde{H}_t$ and $\Delta_{ij,t}$.

Finally, the conditional density of returns is defined by

$$P(R_t | \Phi_{t-1}) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} f(R_t | n_{1t} = i, n_{2t} = j, \Phi_{t-1}) P(n_{1t} = i, n_{2t} = j, \Phi_{t-1}) . \qquad (17)$$

The log likelihood function is simply the sum of the log conditional densities:

$$\ln L = \sum_{t=1}^{N} \ln P(R_t | \Phi_{t-1}) . \qquad (18)$$

Numerical maximization techniques are used to estimate the model.

DATA and DESCRIPTIVE STATISTICS

This study analyzed the price volatility of the WTI crude oil spot price and the New York Harbor No. 2 heating oil spot price using the CBP-GARCH model. The sample period ran from June 2, 1986 to July 31, 2007 and contained 5,319 observations. All data was obtained from the U.S. Department of Energy (DOE). Table 1 lists the descriptive statistics for spot returns[3]. Crude oil and heating oil displayed similar returns, while the standard deviation of heating oil slightly exceeded that of crude oil. However, the time series plots in Fig. 1 reveal that the higher variance of heating oil resulted from the occasional large price changes. Historically, heating oil prices are generally higher during the winter months when demand is stronger, but specific events in recent years have led to what seems to be a weakening of this seasonal effect on average (Fig. 2). More notably, jumps of heating oil during the sample period occur at the end of 1989 and in the early parts of 2000, 2003 and 2005. The biggest heating oil crisis occurred in February 2000[4] in response to a reduction in supply; cold weather was responsible for driving up demand at the end of 1989; and a combination of low inventories, high winter demand and the specter of war looming caused the high prices in early 2003. In late August and September 2005, the heating oil price was at a near record high, up to 217.67, because of hurricanes Katrina and Rita. As for crude oil, the largest jump events are related to the Gulf War. The first war was in 1990 and 1991, and the second one began in 2003. Lastly, the upward trends of both crude oil and heating oil drop due to the crude inventory having jumped far more than expected and the unusually mild winter weather. Furthermore, both the crude oil and heating oil returns exhibit negative skewness and are leptokurtic. The skewness of crude oil and heating oil is -1.0402 and -1.7944, respectively, and the excess kurtosis is 19.7780 and 40.4658, respectively, with all the values being significant at the 1% level. Table 1 also lists the covariance/correlation

[3] Both the series are stationary in the Dickey-Fuller test and the Phillips-Perron test.

[4] According to the DOE, consumers paid an average of $1.21 per gallon throughout the winter in 1999; however, during late January to early February 2000, the prices quickly went from $1.21 to $1.99 per gallon, an increase of 64%. The DOE established the Northeast Heating Oil Reserve in July 2000 to guard against potential shortfalls and price spikes.

matrix. The static correlation coefficient and covariance are 0.6552 and 4.2989, respectively, and indicate a strong and positive relationship between crude oil and heating oil during the past 20 years.

EMPIRICAL RESULTS

Estimation results of GARCH and CBP-GARCH model

The empirical results of the GARCH and CBP-GARCH models are listed in Table 2, and volatility and covariance are illustrated in Fig. 3. As for the GARCH model, all the parameters in the conditional variance equation are significant under the 1% level. This indicates that the volatility of both crude and heating oil (h_{11} and h_{22}) are directly affected by past innovation and volatility. Higher levels of historical conditional volatility are associated with increased current conditional volatility. Conditional covariance (h_{12}) also exhibits the characteristics of volatility clustering. Next, in the CBP-GARCH model, all the parameters in the GARCH volatility terms are significant under the 1% level, as in the GARCH model. Furthermore, the jump components of jump size and intensity are discussed below. The jump size means are significantly negative at the 1% level (θ_{10} and θ_{20}), and the asymmetric effects only exist in the heating oil market. The jump sizes of heating oil are significantly positive in relation to last period's positive returns (θ_2^+). This indicates that the mean of the jump size is increasing while the last period's heating oil prices go up. However, the negative information of the down spot prices does not affect the jump size mean.

The variance of the jump size (δ) is 4.5647 and 4.1609 for crude oil and heating oil, respectively, and both are significant under the 1% level, indicating that jump variance is higher for crude oil. The jump correlation is up to 0.9164 between crude oil and heating oil, revealing that the bivariate jump setting is essential in this study. Both jump intensities (λ_{1t} and λ_{2t}) are significantly related to the historical volatility (η) and the autoregressive terms (γ). The former relationship is stronger for crude oil (η_1), and the latter one is stronger for heating oil (γ_2). Moreover, the characteristics of the jump intensity covariance (λ_{3t}) are the same with the specific jump intensity of crude oil and heating oil. The parameters, η_3, η_4 and γ_3, are all significant under the 1% level. Figure 4 shows the jump intensities. Figure 5 also plots the average monthly jump intensity; a clear seasonal effect can be observed for heating oil. On average, the jump intensity of heating oil is highest in February, followed by January, March and then December[5]. The jump intensity is lowest in July, June and May. Additionally, the jump intensity of crude oil is more stable and higher

5 By observing the past 20 years, the jump intensity is not at high level except in March 2000. Therefore, this paper re-computes the average values without the abnormal March 2000, and the average jump intensity in March is lower than December.

than that of heating oil, except in February. Otherwise, the correlation between the number of jumps is as shown in Fig. 6. The average correlation over the last 20 years is 0.6808, and the correlation coefficient is lower when the range between crude oil and heating oil is widening, particularly when heating oil prices diverge from crude oil prices.

Overestimation in the GARCH model during the jump periods

Table 3 lists the descriptive statistics of the variance. The average variances for crude oil and heating oil are 6.5171 and 7.0393 under the GARCH model, compared to 5.0153 and 5.7260 under the CBP-GARCH model. Moreover, the covariance is 4.4250 using the GARCH model and 3.6437 using the CBP-GARCH model. These analytical results demonstrate that the CBP-GARCH model is characterized by smooth estimation results and a relatively narrow variation range. Furthermore, in regard to Figure 4, which illustrates jump intensity, four periods with extremely high jump intensities were selected to analyze in depth, including two jump events in heating oil which were previously described (Panel A to B in Fig. 7) and two jump events in crude oil resulting from the two Gulf Wars (Panel C and D in Fig. 7). The variances, found to be similar using the GARCH and CBP-GARCH models in peacetime[6], diverged during these specific periods. The variances are higher for the GARCH model than the CBP-GARCH model[7]. Take Panel B as an example: shrinking supply rapidly drove up heating oil prices during Feb. 2000, and the two models clearly displayed different estimations during that period. Furthermore, when the jumps in the price of crude oil during the two Gulf wars are examined, estimations of variance in the GARCH model are higher as well.

The findings in this study are also supported by the evidence listed in Table 4 and Table 5. Both tables report the forecasting errors of the measured volatility that comparing with the standard deviation of price differences[8] using mean absolute percentage error (MAPE)[9]. The forecasting errors are grouped by two different rules. Table 4 lists the daily forecasting errors both in the peacetime and high jump time periods, that defined as the lowest 25% and highest 25% fractiles of jump probabilities respectively, and table 5 lists the monthly average forecasting errors owing to strong seasonal effects. First, all the MAPEs are lower in the CBP-GARCH model than in the GARCH model, and all the difference values are significantly

[6] Peacetime in this paper indicates low jump probability period, during which the series data are smooth and continuous without sudden events such as the Gulf Wars.

[7] All the characteristics of covariance are similar with variance. To save space in this paper, we do not report the covariance figures.

[8] Regnier (2007) indicates that the standard deviation of log price differences is the best general measure of volatility and an indicator of changes in volatility over time. Therefore, volatility in this paper is measured as standard deviation over a 3-year period of log differences.

[9] MAPE is the average of the absolute errors expressed in percentage terms.

positive in the sign test. In Table 4, the differences are 0.0364 and 0.0476 in peacetime and 0.7224 and 0.5586 in the high jump time period for crude oil and heating oil, respectively. In other words, the measurement errors of the CBP-GARCH models are lower than those of the GARCH model by at least 55% in the high jump period. The greater the difference, the better the performance of the CBP-GARCH model.

We have similar results in Table 5. Combined with the monthly jump intensity in Figure 5, the percentage error is much lower for the CBP-GARCH model than the GARCH model during the months of higher jump intensity, whereas the difference values in Table 5 are higher. Take heating oil as an example: the highest and lowest difference values are 0.5919 and 0.0239 in February and July respectively, which is in contrast to the highest and lowest jump intensity. That is, the measurement errors of the CBP-GARCH models are lower than those of the GARCH model, with 59% and 2%, respectively, in February and July. The same obvious results appear in crude oil; the difference drops along with the drop in jump intensities. In the highest jump intensity month, January, the CBP-GARCH model outperforms the GARCH model, and the measurement error is 45% lower in the CBP-GARCH model. The measurement error is only 6% lower during the lowest jump intensity month, July. All the results imply that the CBP-GARCH model performs much better in high jump intensity months, while this performance decreases as the jump intensity decreases.

This study argues that, because of the assumption of continuity, variance may be overestimated in the traditional GARCH model during high volatility periods. That is, overall shocks cannot be distinguished as normal or abnormal shocks, thus moving the volatility to a high level in the next period. Nevertheless, the CBP-GARCH model assumes that the specific shock takes the form of a jump, independent of normal volatility, and reduces the persistence of abnormal volatility. Accordingly, the variances in the GARCH model exceed those in the CBP-GARCH model when facing specific events or the assets with high volatility. Further applications can therefore be easily biased based on the overestimation of variance and covariance.

CONCLUSIONS

This study examines the price volatility of crude oil and heating oil during the past 20 years using the CBP-GARCH model. Both features of jump and bivariate are considered in the CBP-GARCH model for fitting the data accurately, especially during jump periods. The empirical results indicate that the CBP-GARCH model outperforms the GARCH model; however, the performance decreases as the jump intensity decreases. The measurement errors of the CBP-GARCH models are lower than those of the GARCH model, with 55% and 72% for heating oil and crude oil, respectively, in a high jump period. These measurement errors are 4% and 3% lower in peacetime. In monthly results, the measurement errors of the CBP-GARCH models are lower than those of the GARCH model in the highest jump intensity month, with 59% and 45% for heating oil and crude oil, respectively. These measurement errors are 2% and 6% lower in the lowest jump intensity month. That is, the CBP-GARCH model can capture the volatility reasonably accurately, especially during high jump time periods, during either occurrences of specific jump events or high jump intensity months (usually in winter months). Furthermore, the variance and covariance of the GARCH and CBP-GARCH models were found to be similar in peacetime, but divergent when jump events such as the Gulf Wars occurred. Due to the assumption of continuity in the traditional GARCH model, both the variance and covariance of the GARCH model are overestimations. That is, the overall shocks cannot be distinguished as normal or abnormal shocks, and they move the volatility to a high level in the next period. Further applications can be easily biased based on this overestimation. Nevertheless, the CBP-GARCH model provides more information for regulating the defects in the GARCH model. In the CBP-GARCH model, the specific shocks are assumed to be independent of normal volatility and to reduce the persistence of abnormal volatility. Therefore, the CBP-GARCH model is appropriate and necessary in high volatility markets. The overestimation of variance and covariance will bias further applications of the GARCH model and can lead to, for example, excessive hedging. For this reason, this paper is useful to traders, speculators and other participants in markets seeking to reduce transaction costs and maximize profits.

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Table

Table 1. Descriptive statistics of return

	Mean	Std. deviation	Min.	Max.	Skewness	Excess kurtosis
Crude oil	0.0326	2.4662	-40.6396	19.1506	-1.0402***	19.7780***
Heating oil	0.0311	2.6610	-47.0117	22.9538	-1.7944***	40.4658***
Covariance/Correlation Matrix						
	Crude oil		Heating oil			
Crude oil	6.0810		0.6552			
Heating oil	4.2989		7.0797			

Notes: *** represents significance under 1% level. The covariance/correlation matrix has the covariance on and below the diagonal and the correlation above it.

Table 2. Empirical results of GARCH and CBP-GARCH model

	GARCH model	CBP-GARCH model
Mean equation		
μ_1	-0.0019	0.0149
μ_2	0.0386	0.0503 ***
Variance equation		
c_{11}	0.1983 ***	0.1042 ***
c_{12}	0.1696 ***	0.1075 ***
c_{22}	0.1806 ***	0.1473 ***
a_{11}	0.1432 ***	0.0380 ***
a_{12}	0.1169 ***	0.0448 ***
a_{22}	0.1135 ***	0.0569 ***
b_{11}	0.8359 ***	0.9165 ***
b_{12}	0.8481 ***	0.9038 ***
b_{22}	0.8601 ***	0.8889 ***
Jump size		
θ_{10}		-0.9077 ***
θ_1^-		-0.0790
θ_1^+		0.1108
θ_{20}		-0.5631 ***
θ_2^-		-0.0459
θ_2^+		0.3465 ***
δ_1		4.5647 ***
δ_2		4.1609 ***
ρ		0.9164 ***
Jump intensity		
λ_1		0.0008
λ_2		0.0008
λ_3		0.0003 **
η_1		0.0487 ***
η_2		0.0211 ***
η_3		0.0134 ***
η_4		0.0054 ***
γ_1		0.5655 ***
γ_2		0.8725 ***
γ_3		0.9710 ***
Log-likelihood value	-21228.0279	-20738.8757

Notes: *, **, *** represent significance under 10%, 5% and 1% levels, respectively. Please coordinate equations (1) and (2) with the part of mean and variance equations in the table, equations (6), (7), and (16) with jump size, and equation (12) with jump intensity.

Table 3. Descriptive statistics of measured volatilities

	Mean	Std. deviation	Min.	Max.
Crude oil				
GARCH	6.5171	9.4734	1.4683	254.8724
CBP-GARCH	5.0139	3.9005	2.4486	73.5950
Heating oil				
GARCH	7.0393	16.6943	1.5167	466.6135
CBP-GARCH	5.7260	9.7425	2.2646	251.5470
Covariance				
GARCH	4.4250	6.7689	-0.5266	213.8136
CBP-GARCH	3.6437	3.4125	1.3508	86.0255
Correlation				
GARCH	0.7296	0.1442	-0.0616	0.9028
CBP-GARCH	0.7224	0.1041	0.1198	0.9363

Notes: *, **, *** represent significance under 10%, 5% and 1% levels, respectively.

Table 4. Forecasting errors in peace and high jump periods

	Crude oil			*Heating oil*		
	(1) GARCH	(2) CBP-GARCH	(1)-(2) Difference	(1) GARCH	(2) CBP-GARCH	(1)-(2) Difference
The peace time	0.4973	0.4609	0.0364*	0.4597	0.4121	0.0476*
The high jump time	1.2135	0.4911	0.7224*	1.4870	0.9284	0.5586*

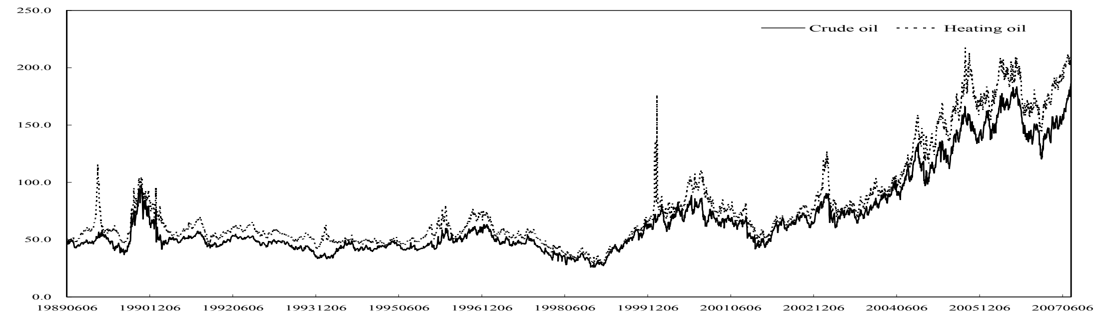
Notes: * represents significance under 5% levels in the sign test. The lowest 25% and highest 25% fractiles of jump probabilities are defined as the peacetime and high jump time periods, respectively.

Table 5. Forecasting errors sorted by month

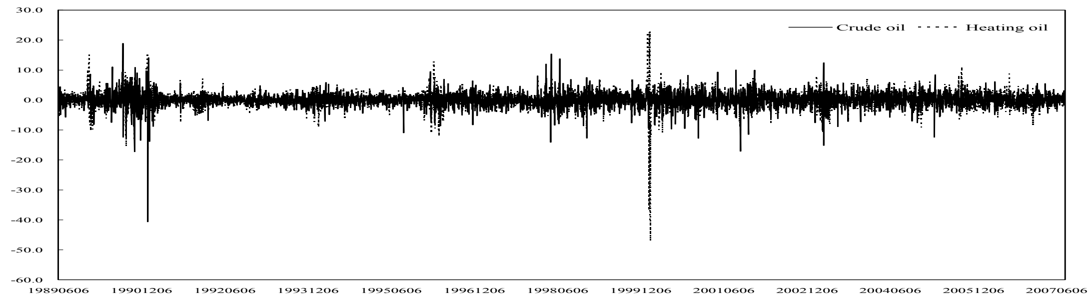
	Crude oil				*Heating oil*		
	(1) GARCH	(2) CBP- GARCH	(1)-(2) Difference		(1) GARCH	(2) CBP- GARCH	(1)-(2) Difference
Jan.	0.8821	0.4312	0.4509*	Jan.	1.3286	0.8412	0.4874*
Feb.	0.5649	0.4255	0.1393	Feb.	1.6681	1.0762	0.5919*
Mar.	0.7002	0.4368	0.2634*	Mar.	0.7560	0.5679	0.1881*
Apr.	0.7947	0.4902	0.3045*	Apr.	0.6063	0.4914	0.1149*
May	0.6087	0.4347	0.1740*	May	0.4505	0.3896	0.0609*
Jun.	0.5672	0.4019	0.1654*	Jun.	0.4053	0.3702	0.0351*
Jul.	0.4287	0.3654	0.0633*	Jul.	0.3993	0.3754	0.0239*
Aug.	0.7078	0.4696	0.2382*	Aug.	0.6456	0.5026	0.1430*
Sep.	0.5743	0.4399	0.1343*	Sep.	0.5755	0.4506	0.1250*
Oct.	0.5714	0.3803	0.1911*	Oct.	0.5063	0.4085	0.0978*
Nov.	0.5286	0.3672	0.1614*	Nov.	0.3736	0.3356	0.0380
Dec.	0.5516	0.3240	0.2276*	Dec.	0.4327	0.3525	0.0803*

Notes: * represents significance under 5% levels in the sign test.

Figures



Part A. Time series plots of spot prices (cents per gallon)



Part B. Returns

Figure 1. Time series plots and returns of crude oil and heating oil

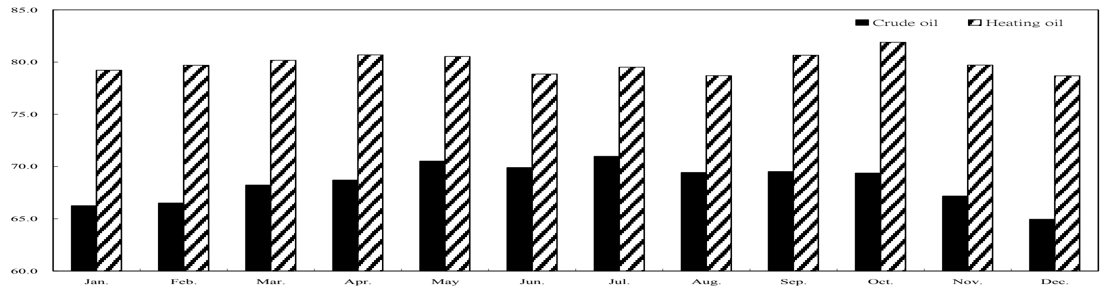
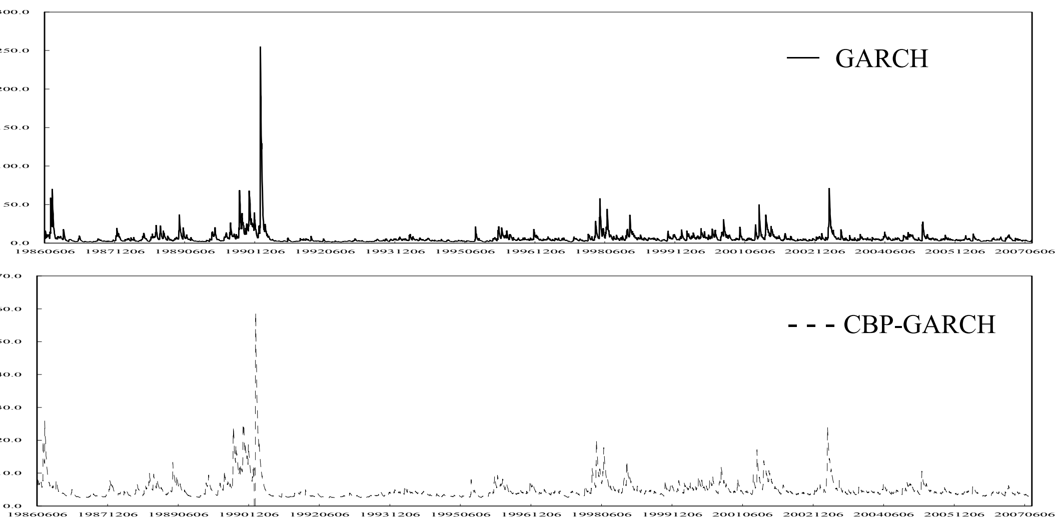
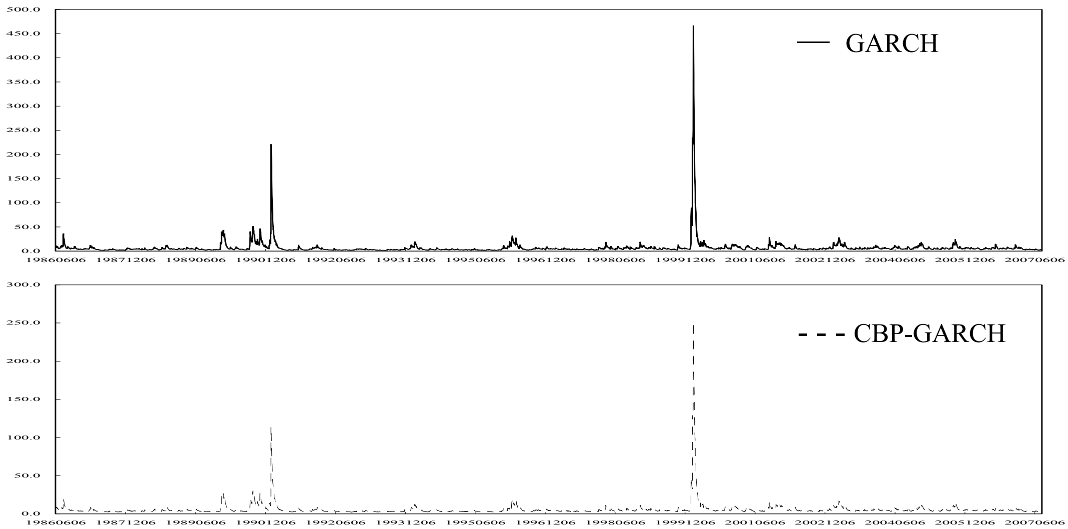


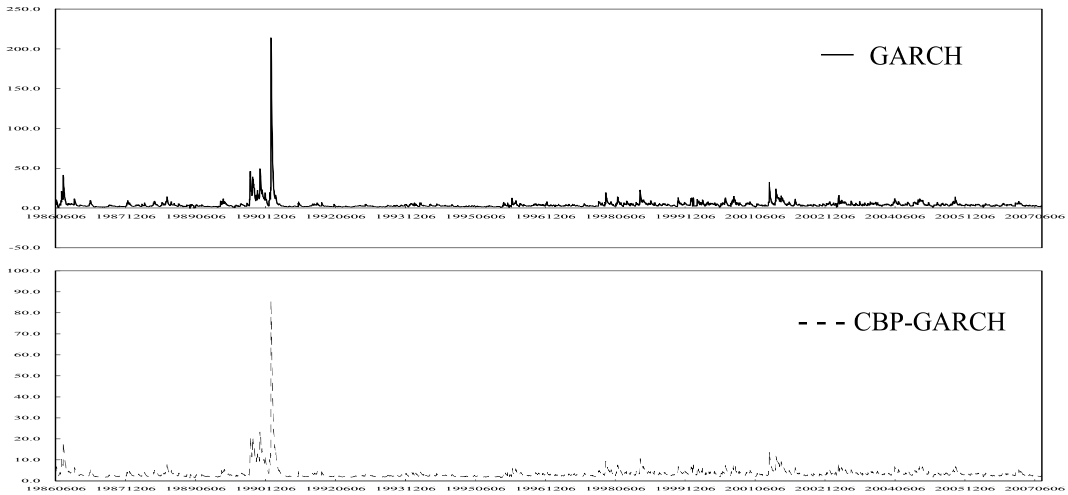
Figure 2. The average monthly spot price from June 1986 to July 2007.



Part A. The conditional variance of crude oil



Part B. The conditional variance of heating oil



Part C. The covariance between crude oil and heating oil
Figure 3. The conditional variance and covariance under GARCH and CBP-GARCH model

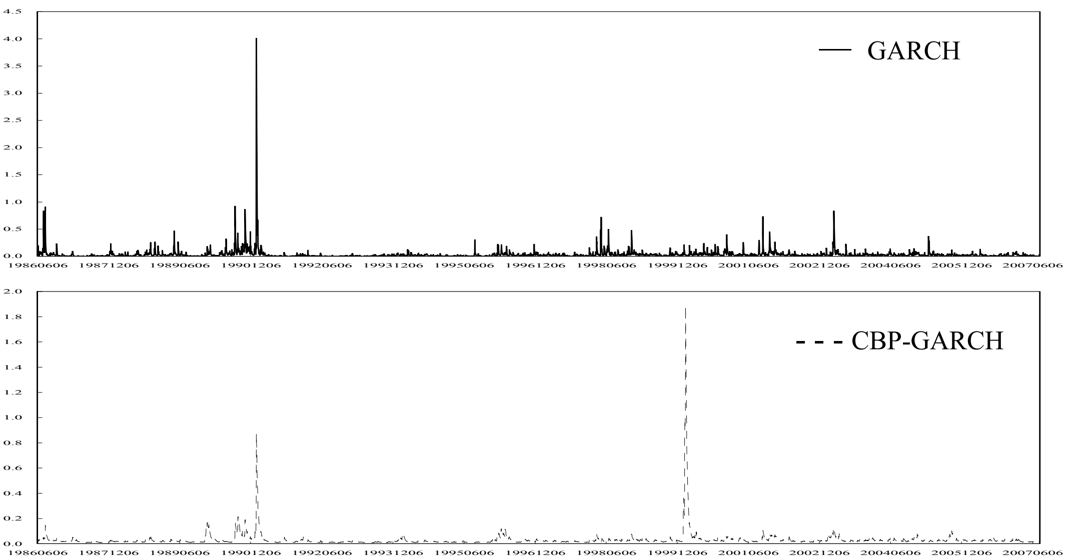


Figure 4. The jump intensity of crude oil and heating oil

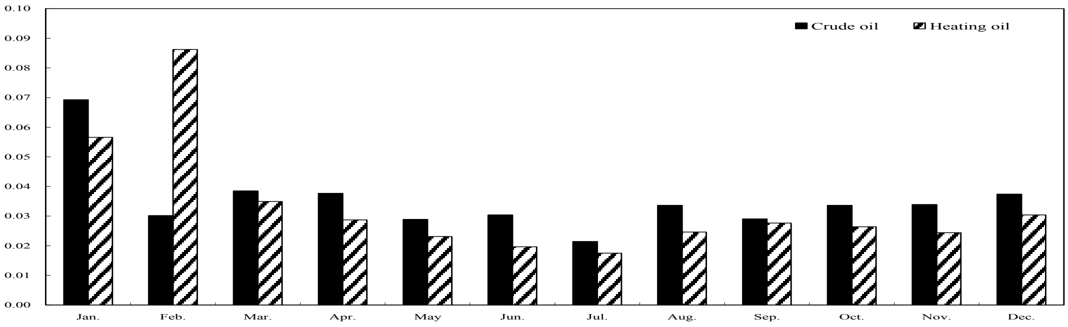


Figure 5. The average monthly jump intensity

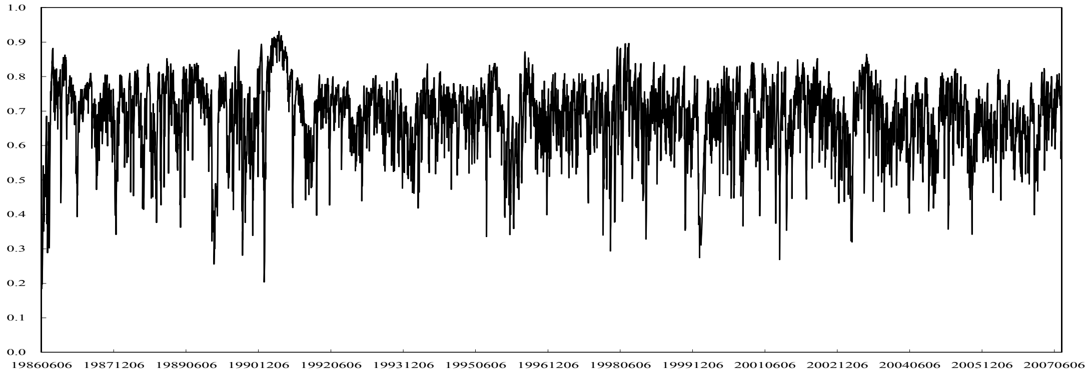
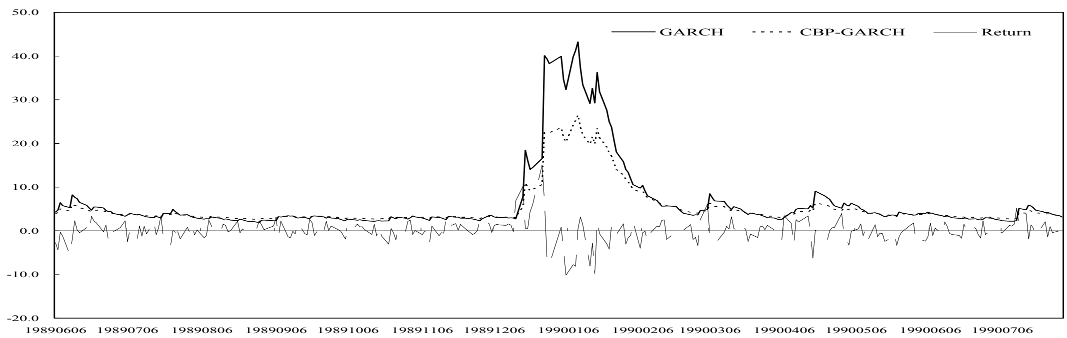
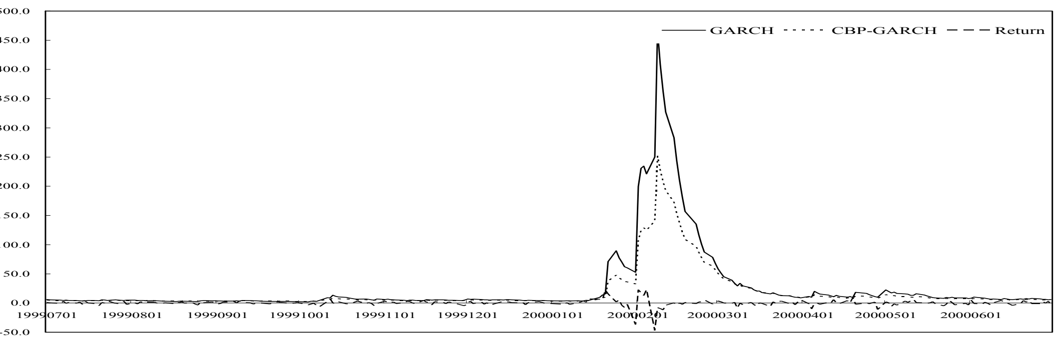


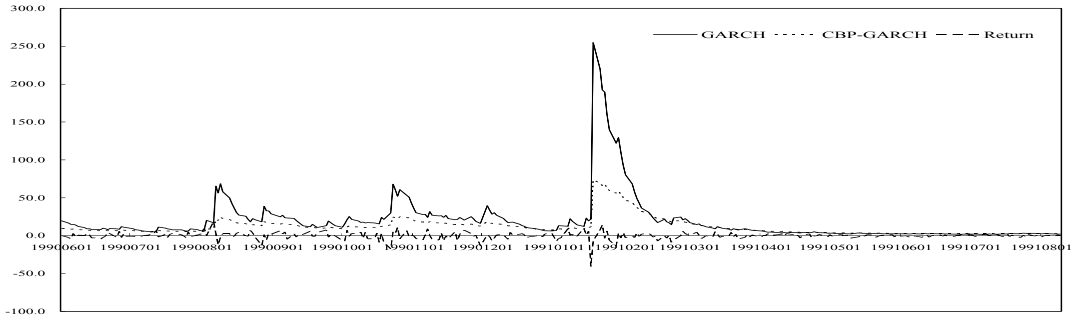
Figure 6. The correlation between the number of jumps of crude oil and heating oil



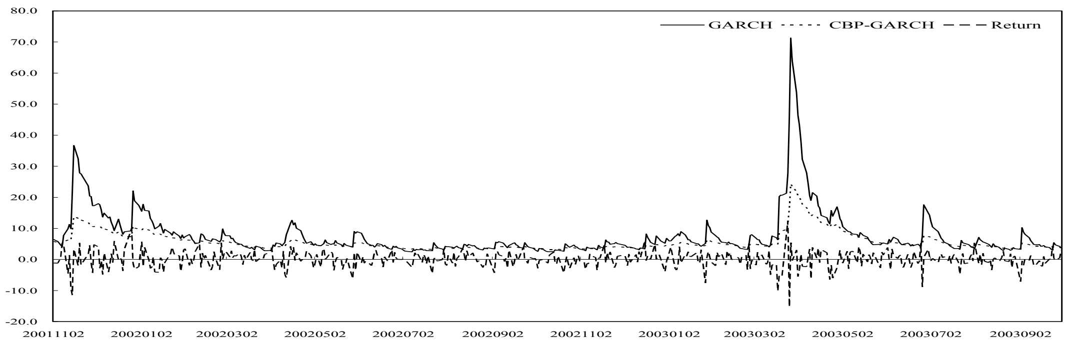
Part A. Heating oil variance (July 1989-June 1990)



Part B. Heating oil variance (July 1999-June 2000)



Panel C. Crude oil variance (Gulf War I)



Panel D. Crude oil variance (Gulf War II)

Figure 7. Variance in each model during the specific periods