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Ramsey pricing with long run competition

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Abstract

Ramsey pricing of regulated firm services is often considered impractical because of stringent informational requirements. However in cases where demands for regulated firm services are independent and a marginal cost pricing competitive fringe exists, the simple inverse elasticity rule for Ramsey pricing has been shown to apply. This paper extends this result to show that only limited demand elasticity data for the regulated firm is required to apply the same rule, more generally, when zero profits and constant prices exist in the competitive sector in the long run.

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1. Introduction

It has been well established in the economic literature that information requirements for Ramsey or second best optimal pricing for regulated firms can be substantial except in the simplest cases. This is especially true in mixed oligopoly situations where a regulated firm competes against one or more unregulated firms offering substitute or complementary products or services. In particular, several authors have shown that above marginal cost pricing in the competing sector requires own and cross price demand elasticity data in order to establish Ramsey rates different than would have existed in the absence of such competition.

Sherman (1979) analyzed Postal Service Ramsey rates when rival firms price at or above marginal cost. In a more general context, Ware and Winter (1986) established similar results for the above marginal cost case at a Nash equilibrium solution involving Bertrand pricing. In both scenarios, cross price demand elasticities are needed to estimate Ramsey numbers higher (lower) than would have existed without competition when products are substitutes (complements). More recently, Prieger (1996) offered a Ramsey solution under Stackelberg price leadership assumptions which also require the same cross price demand elasticities and a new “strategic” price elasticity based on the competitive firm’s price reaction function.

By contrast, both Braeutigam (1979) and Sherman show that the simple inverse elasticity rule applies when a competitive fringe prices at marginal cost. In this situation, the distinction Prieger makes between “myopic” Ramsey pricing by the regulator and other forms of welfare improving Ramsey rates vanishes. Hence with a marginal cost pricing fringe, the regulator needs only cost and own price demand elasticity data for the regulated firm in order to establish globally optimal Ramsey prices.

However marginal cost pricing is only a sufficient condition for limiting the required demand data. This note establishes a more general condition showing that the same inverse elasticity rule can apply with above marginal cost pricing when rivals are Cournot competitors and zero long run profits prevail. Cournot competition and zero profits might be considered appropriate where there is relatively little product differentiation within the competitive sector and there are a large number of rivals caused by weak scale effects relative to demand. However the regulated firm’s product is differentiated from rivals as a whole, possibly because of a quality or reliability distinction.

The particular model presented assumes that the regulated firm offers a non-competitive and competitive product where the firm acts as a Stackelberg price leader. Rival firms take any changes in the competitive product rate as given when adjusting their own output according to Cournot assumptions. Free entry and exit conditions exist and therefore a zero profit equilibrium is re-established, once new firms enter or existing firms exit as necessary. Within this framework, I demonstrate that deviations from the simple inverse elasticity rule for Ramsey rates require long run changes to the competitive sector rate. I further show that long run price changes are absent with linear demand and therefore the inverse elasticity rule applies in this instance. Therefore situations where limited (regulated firm only) data requirements for Ramsey pricing are considered appropriate might be broader than once thought.

2. The Model

I assume a two stage game where the regulator first sets prices for the regulated firm's two products subject to a break-even constraint, and then the price and the number of firms in the competitive sector adjusts re-establishing equilibrium. Each rival has the same cost structure and therefore identical (zero) profits given the common rate charged.

First, let demand for the non-competitive product, the regulated firm's competitive product and the competitive sector's product be determined by the functions $D^1(p_1)$, $D^2(p_2, p_r)$ and $D^r(p_2, p_r)$ where p_1, p_2 and p_r are the corresponding rates. The usual own price marginal conditions $D^1_{p1} < 0$, $D^2_{p2} < 0$ and $D^r_{pr} < 0$ and product substitutability conditions $D^2_{pr} = D^r_{p2} > 0$ apply. Then the competitive sector long term profit function with n rivals can be written as:

$$\pi^r(p_2, p_r, n) = D^r(p_2, p_r)(p_r - u_r) - nF_r = 0,$$

where u_r and F_r are each rival's constant unit and fixed cost, respectively. The Cournot solution also requires the first order condition:

$$D^r/n + D^r_{pr}(p_r - u_r) = 0.$$

Therefore assuming a non-zero Jacobian for these two expressions, the functions $p_r(p_2)$ and $n(p_2)$ can be established using the implicit function rule. Similarly, the regulated firm's profit function can be specified as:

$$\pi^d(p_1, p_2, p_r) = D^1(p_1)(p_1 - u_1) + D^2(p_2, p_r)(p_2 - u_2) - F_d,$$

where u_1 and u_2 are the constant unit costs for products one and two and F_d is the firm's fixed cost. Finally substituting $p_r(p_2)$ and $n(p_2)$ into the profit functions, total welfare can be shown as dependent only on the two rates for the regulated firm:

$$W(p_1, p_2, p_r(p_2), n(p_2)) = S(p_1, p_2, p_r(p_2)) + \pi^d(p_1, p_2, p_r(p_2)) + \pi^r(p_2, p_r(p_2), n(p_2)),$$

where $S()$ is consumer surplus with zero wealth effects assumed.

The regulator's task is to maximize W subject to $\pi^d(p_1, p_2, p_r(p_2)) = 0$. Therefore writing the Lagrangian:

$$\mathscr{L} = W(p_1, p_2, p_r(p_2), n(p_2)) + \lambda[\pi^d(p_1, p_2, p_r(p_2))],$$

the first order welfare maximizing conditions with respect to p_1 and p_2 can then be shown as:

$$\partial\mathscr{L}/\partial p_1 = - D^1 + (1 + \lambda)\pi^d_{p1} = 0$$

$$= \lambda D^1 + (1 + \lambda)(p_1 - u_1)dD^1/dp_1 = 0,$$

and

$$\partial\mathcal{L}/\partial p_2 = - D^2 - D^r(dp_r/dp_2) + \pi^r{}_{p2} + \pi^r{}_{pr}(dp_r/dp_2) + \pi^r{}_n(dn/dp_2) +$$

$$(1 + \lambda)(\pi^d{}_{p2} + \pi^d{}_{pr}(dp_r/dp_2)) = 0$$

$$= \lambda D^2 + (1 + \lambda)[(p_2 - u_2)(D^2{}_{p2} + D^2{}_{pr}dp_r/dp_2)] - D^r(dp_r/dp_2) = 0,$$

since $\pi^r{}_{p2} + \pi^r{}_{pr}(dp_r/dp_2) + \pi^r{}_n(dn/dp_2) = 0$. The two first order conditions can then be transformed to derive the optimal price mark-ups on the regulated products. First note that the total effect on product two demand from a change in p_2 is the sum of the direct effect and the indirect effect transmitted through an induced change in p_r: $dD^2/dp_2 = D^2{}_{p2} + D^2{}_{pr}dp_r/dp_2$. The following expressions corresponding to the above can then be derived by manipulation:

$$e_1(p_1 - u_1)/p_1 = -\lambda/(1 + \lambda) \qquad (1)$$

$$e_2(p_2 - u_2)/p_2 = -\lambda/(1 + \lambda) + (R_r/R_2)z/(1 + \lambda), \qquad (2)$$

where $e_1 = (dD^1/dp_1)(p_1/D^1)$, $e_2 = (dD^2/dp_2)(p_2/D^2)$, $z = (dp_r/dp_2)p_2/p_r$ and $R_r/R_2 = D^rp_r/D^2p_2$. Finally subtracting *(1)* from *(2)* gives:

$$e_2(p_2 - u_2)/p_2 - e_1(p_1 - u_1)/p_1 = (R_r/R_2)z/(1 + \lambda). \qquad (3)$$

The two terms on the LHS are the Ramsey numbers for the regulated products. This last expression shows that if long term profits are zero in the competitive sector, then the regulator deviates from the simple inverse elasticity rule $e_2(p_2 - u_2)/p_2 = e_1(p_1 - u_1)/p_1$ only when p_2 affects p_r or $z \neq 0$. Otherwise, the Ramsey number for the substitutable product should be set higher (lower) in absolute value than the number for the non-substitutable product if z is negative (positive). However it is important to note that if $z = 0$, then the total effect on product two demand from an own price change reduces to the direct effect only or $dD^2/dp_2 = D^2{}_{p2}$. This implies that the regulator needs to use only the regulated firm's own price demand elasticities to determine optimal rates for p_1 and p_2 according to the simple inverse elasticity rule.

3. Long Run Impacts on the Competitive Sector

This section shows that linear demand is sufficient to establish that the competitive sector price is unaffected in the long term by regulated rates. More generally, it also describes the condition determining the direction of impact of p_2 on p_r. First note that under Cournot competition, each rival j perceives its profit function as:

$$\pi^r{}_j = (D^r(p_2, p_r) - \sum V_i)(p_r - u_r) - F_r,$$

where $i = 1$ *to* n, $i \neq j$, and the sum of all other firm volumes in the industry $\sum V_i$ is assumed fixed. Therefore each firm varies p_r to achieve:

$$\partial\pi^r_j/\partial p_r = D^r - \sum V_i + D^r_{pr}(p_r - u_r) = 0$$

or

$$D^r/n + D^r_{pr}(p_r - u_r) = 0, \qquad (4)$$

since all V_i are equal. Holding all V_i constant, each firm's second order condition is:

$$2D^r_{pr} + D^r_{prpr}(p_r - u_r) < 0. \qquad (5)$$

Also equilibrium with costless entry and exit requires the following zero long run profit condition:

$$(D^r/n)(p_r - u_r) - F_r = 0. \qquad (6)$$

Marginal effects on p_r from changes in p_2 are then determined by totally differentiating *(4)* and *(6)* with respect to p_2. Therefore:

$$[D^r_{pr}(1 + 1/n) + D^r_{prpr}(p_r - u_r)]dp_r/dp_2 – [D^r/n^2]dn/dp_2 = -D^r_{p2}/n - D^r_{prp2}(p_r - u_r)$$

$$[D^r/n + (D^r_{pr}/n)(p_r - u_r)]dp_r/dp_2 – [D^r(p_r - u_r)/n^2]dn/dp_2 = -(D^r_{p2}/n)(p_r - u_r).$$

The determinant of the Jacobian formed by the matrix of bracketed terms on the LHS is then:

$$Z = -[D^r_{pr}(1 + 1/n) + D^r_{prpr}(p_r - u_r)]D^r(p_r - u_r)/n^2 + [D^r/n + (D^r_{pr}/n)(p_r - u_r)]D^r/n^2$$

$$= -(D^r_{pr}(p_r - u_r) - D^r/n + D^r_{prpr}(p_r - u_r)^2)D^r/n^2.$$

Using *(4)* to substitute for D^r/n then signs Z as:

$$-[2D^r_{pr}+ D^r_{prpr}(p_r - u_r)](p_r - u_r)D^r/n^2 > 0,$$

from *(5)*.

Then by Cramer's rule, the marginal effect on price is determined directly from:

$$dp_r/dp_2 = [(D^r_{p2}/n + D^r_{prp2}(p_r - u_r))D^r(p_r - u_r)/n^2 - (D^r_{p2}/n)D^r(p_r - u_r)/n^2]/Z$$

$$= D^r_{prp2}D^r(p_r - u_r)^2/Zn^2,$$

and substituting for Z:

$$dp_r/dp_2 = -D^r_{prp2}(p_r - u_r)/[2D^r_{pr}+ D^r_{prpr}(p_r - u_r)],$$

in which case $sign[dp_r/dp_2] = sign[D^r_{prp2}]$. Also note that the effect on competitive firm volume is directly evidenced by *(6)*. In particular, D^r/n decreases, stays the same or increases with respect to increases in p_2 as dp_r/dp_2 is positive, zero or negative.

The key result here is that with linear demand $D^r_{prp2} = 0$ so that $dp_r/dp_2 = 0$ and then the simple inverse elasticity rule applies in the long run using only own price elasticities for the regulated firm. Therefore the distinction between Prieger's myopic result and optimal Ramsey pricing disappears in this instance. At a more general level, if $dp_r/dp_2 \neq 0$ then cross price demand data is required to set optimal second best rates. For example if demand elasticities are constant, it can be easily verified that $dp_r/dp_2 < 0$ and therefore $e_2(p_2 - u_2)/p_2 - e_1(p_1 - u_1)/p_1 < 0$ by *(3)*. In this case, the price mark-up for the substitutable product should be set higher relative to the non-substitutable product than implied by the simple inverse elasticity rule. This last result is consistent with general findings from the literature when the number of competitors is assumed fixed.

4. References

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