E C O N O M I C S B U L L E T I N

Capacity Choice in a Mixed Duopoly under Price Competition

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Abstract

This paper shows that when firms compete on prices in a mixed duopoly, the public firm chooses over-capacity when products are substitutes and under-capacity when products are complements. The private firm always chooses under-capacity. This result is in contrast with that obtained in the literature assuming quantity competition.

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1 Introduction

The issue of the capacity chosen by firms in a mixed oligopoly has been analyzed by Wen and Sasaki (2001), Nishimori and Ogawa (2004) and Lu and Poddar (2005). These papers show that, in a mixed duopoly market, under Cournot competition with homogeneous products the public firm strategically chooses under-capacity while the private firm chooses excess capacity. This result is in sharp contrast with the conventional wisdom that holding excess capacity plays an essential role as a strategic device in the pure oligopoly market (see, for example, Dixit, 1980; Brander and Spencer, 1983 and Horiba and Tsutsui, 2000). Ogawa (2006) extends the above papers by assuming heterogeneous products in a mixed duopoly in which firms compete on quantities. He shows that the public firm chooses over-capacity when products are complements and under-capacity when products are substitutes.

In this paper we analyze the issue of the capacity chosen by firms in a mixed duopoly by considering that firms compete on prices.[1] Thus we analyze whether the type of competition in the product market affects the results obtained by the papers cited above. We obtain that when products are substitutes, the private firm chooses under-capacity and the public firm chooses over-capacity. When products are complements, both firms choose under-capacity. Therefore, the result obtained in this paper is in contrast with that obtained in the relevant literature by assuming that firms compete on quantities.

2 The model

We consider an economy comprising one private firm and one public firm, denoted by 1 and 2, respectively. On the consumption side, there is a continuum of consumers of the same type whose utility function is linear. The representative consumer maximizes $U(q_1,$

[1] There are few papers analyzing the case in which firms decide prices. See, for example, Anderson et *al*. (1997), Tasnádi (2006), Ogawa and Kato (2006) and Bárcena-Ruiz (2007).

$q_2)-p_1q_1-p_2q_2$, where $q_i \geq 0$ is the amount of the good i and p_i is its price (i = 1, 2). The function $U(q_1, q_2)$ is assumed to be quadratic, strictly concave and symmetric in q_1 and q_2:

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}((q_1)^2 + 2bq_1q_2 + (q_2)^2),$$

where parameter b measures the degree to which goods are substitutes. If $b \in (0, 1)$ products are substitutes while if $b \in (-1, 0)$ products are complements. Demand functions are then given by:

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2}, \; i \neq j; \;\; i, j = 1, 2. \tag{1}$$

The two firms have the same technology, represented by the cost function: $C(q_i, x_i)$, where q_i and x_i are the production and capacity, respectively, of firm i. Following Vives (1986), Nishimori and Ogawa (2004) and Lu and Podar (2005), we specify the cost function as:

$$C(q_i, x_i) = m\, q_i + (q_i - x_i)^2, \; i = 1, 2.$$

This cost function shows that excess capacity or under-capacity would prove inefficient. In fact, when quantity equals capacity the long-run average cost is minimized.[2]

The profit function of firm i is given by:

$$\pi_i = p_i\, q_i - mq_i - (q_i - x_i)^2, \; i = 1, 2, \tag{2}$$

where q_i is given by (1). We measure social welfare as the sum of consumer surplus (denoted by *CS*) and producer surplus (denoted by *PS*). Therefore, social welfare is given by:

[2] We assume $a > m$ to assure that the production level of both firms is positive.

$$W = CS + PS, \tag{3}$$

where $PS = \pi_1 + \pi_2$ and consumer surplus is given by:

$$CS = U(q_1, q_2) - p_1 q_1 - p_2 q_2 = \frac{(p_1)^2 - 2bp_1p_2 + (p_2)^2 + 2a(1-b)(a - p_1 - p_2)}{2(1-b^2)}. \tag{4}$$

The objective of this paper is to analyze the strategic choice of capacity made by firms in a mixed duopoly under price competition. To that end we propose a two-stage game with the following timing. In the first stage firms choose their capacity simultaneously and independently. In the second stage, after observing the capacity choice, firms choose their prices simultaneously and independently. We solve the game by backward induction from the last stage to obtain a subgame perfect Nash Equilibrium.

3 Results

In the second stage of the game, given the production capacities chosen by firms in the first stage, the private firm chooses the price, p_1, that maximizes its profit while the public firm chooses the price, p_2, that maximizes social welfare. Solving these problems we obtain the reaction functions in prices:

$$\begin{aligned} p_2 &= \frac{2a(1-b)^2 - (1-b^2)(2x_2 - 2bx_1 - (1-b)m) + b(5-b^2)p_1}{3+b^2}, \\ p_1 &= \frac{a(1-b)(3-b^2) - (1-b^2)(2x_1 - m) + b(3-b^2)p_2}{2(2-b^2)}. \end{aligned} \tag{5}$$

As firms compete on prices in the product market, when products are substitutes, $b \in (0,1)$, if one firm raises its price the other firm will react by raising its price too. By

contrast, when products are complements, $b\in$(-1,0), if one firm raises its price the other firm will react by lowering its price. From (5) we obtain:

$$p_2 = \frac{a(8-b-3b^2-b^3+b^4)-4(2-b^2)x_2-2b(1+b^2)x_1+m(4+b-2b^2+b^3)}{12-5b^2+b^4},$$

$$p_1 = \frac{a(9-3b-3b^2+b^3)-2b(3-b^2)x_2-2(3-2b^2+b^4)x_1+m(1+b)^2(3-3b+b^2)}{12-5b^2+b^4}. \quad (6)$$

In the first stage, firms decide their capacity level. In this stage, given (6), the private firm chooses the capacity, x_1, that maximizes its profit and the public firm chooses the capacity, x_2, that maximizes social welfare. Solving these problems we obtain:

$$x_2 = \frac{(a-m)(24-15b-23b^2+10b^3+13b^4-4b^5-2b^6+b^7)}{24-38b^2+23b^4-6b^6+b^8},$$

$$x_1 = \frac{6(a-m)(2-2b-b^2+b^3)}{24-38b^2+23b^4-6b^6+b^8}. \quad (7)$$

It has to be noted that the sign of the denominator in (7) is always positive for $b\in$(-1,1). From (1), (6) and (7) we obtain:

$$q_2 = \frac{(a-m)(24-18b-20b^2+14b^3+9b^4-5b^5-b^6+b^7)}{24-38b^2+23b^4-6b^6+b^8},$$

$$q_1 = \frac{(a-m)(1-b)(12-5b^2+b^4)}{24-38b^2+23b^4-6b^6+b^8}. \quad (8)$$

Finally, from (7) and (8) it is easy to obtain that:

$$x_2 - q_2 = \frac{(a-m)b(1-b)^2(3+3b-b^2-b^3)}{24-38b^2+23b^4-6b^6+b^8},$$

(9)

$$x_1 - q_1 = -\frac{(a-m)b^2(1-b+b^2-b^3)}{24-38b^2+23b^4-6b^6+b^8}.$$

From (9), the following result is obtained.

Proposition. Under price competition, when the products are substitutes the public firm chooses over-capacity, $x_2>q_2$, and when the products are complements it chooses under-capacity, $x_2<q_2$. The private firm chooses under-capacity, $x_1<q_1$, in both cases.

The explanation of this result is as follows. Given that the public firm maximizes social welfare it cares about the consumer surplus and, thus, about the output of industry. As a result, the public firm tries to make the private firm produce a great deal in the duopoly market. However, as firms compete on prices, when products are substitutes the private firm tries to increase its price in order to reduce market competition. When products are complements the private firm has incentives to reduce its price.

When products are substitutes, from (6) we obtain that there is a negative relationship between the capacity level of the public firm and the price level of the private firm and, thus, a positive relationship with the output level of the private firm. In this case, the public firm can improve social welfare by increasing its own capacity. Similarly, there is a negative relationship between the capacity level of the private firm and the price level of the public firm and, thus, a positive relationship with the output level of the public firm. In this case, the private firm can reduce market competition by decreasing its own capacity. Hence, the private firm chooses under-capacity and the public firm chooses over-capacity.

When the products are complements, a reduction in the capacity level of the public (private) firm reduces the price of the private (public) firm and thus increases the output level of the private (public) firm. Hence, the over-capacity strategy for the

public firm does not carry over into the case of products which are complements. In this case, both firms choose under-capacity.

4 Conclusions

In this paper we consider product differentiation in a mixed duopoly model in which firms compete on prices to get new results concerning the capacity choice behavior of public firms. The main result obtained in literature when firms compete on quantities and products are heterogeneous is that in the mixed duopoly both the public and private firms strategically choose over-capacity when products are complements. When products are substitutes the public firm chooses under-capacity and the private firm over-capacity. We show in this paper that these results are not robust to changes in the type of competition in the product market. In fact, under price competition the opposite result is obtained: the public firm chooses over-capacity when products are substitutes, and under-capacity when products are complements; the private firm always chooses under-capacity.

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References

Anderson, S., de Palma, A. and J-F. Thies (1997) "Privatization and Efficiency in a Differentiated Industry" *European Economic Review* **41**, 1635-1654.

Bárcena-Ruiz, J.C. (2007) "Endogenous Timing in a Mixed Duopoly: Price Competition", *Journal of Economics* **91**, 263-272.

Brander J.A. and B.J. Spencer (1983) "Strategic Commitment with R&D: the Symmetric Case" *Bell Journal of Economics* **14**, 225-235.

Dixit, A. (1980) “The Role of Investment in Entry Deterrence” *Economic Journal* **90**, 95-106.

Horiba, Y. and S. Tsutsui (2000) “International Duopoly, Tariff Policies and the Case of Free Trade” *Japanese Economic Review* **51**, 207-220.

Lu, Y. and S. Poddar (2005) “Mixed Oligopoly and the Choice of Capacity” *Research in Economics* **59**, 365-374.

Nishimori, A., and H. Ogawa (2004) “Do Firms always Choose Excess Capacity?” *Economics Bulletin* **12**, 1-7.

Ogawa, H. (2006) “Capacity Choice in the Mixed Duopoly with Product Differentiation”, *Economics Bulletin* **12**, 8, 1-6.

Ogawa, A. and K. Kato (2006) “Price Competition in a Mixed Duopoly” *Economics Bulletin* **12**, 4-15.

Tasnádi, A. (2006) “Price vs. Quantity in Oligopoly Games” *International Journal of Industrial Organization* **24**, 541-554.

Vives, X (1986) “Commitment, Flexibility, and Market Outcomes” *International Journal of Industrial Organization* **4**, 217-229.

Wen, M. and D. Sasaki (2001) “Would Excess Capacity in Public Firm be Socially Optimal?” *Economic Record* **77**, 283-290.