E C O N O M I C S   B U L L E T I N

A note on quality choice with an extended Mussa and Rosen's model

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Abstract

We suggest a model derived from the well-known Mussa and Rosen's model, in which two populations of consumers of opposite tastes co-exist: they rank in exactly the reverse order variants sold at the same price. This model may account for linked and contradictory characteristics in products (as for instance nutritional quality and taste), with consumers attaching more importance to one or to the other aspect. The subgame perfect equilibrium is fully characterized for a costless duopoly choosing qualities then prices.

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1 Introduction

In vertical differentiation models, all consumers are supposed to rank in the same order products sold at the same price, meaning that they all desire quality even if they are not equally willing to pay for it. These models are appropriate to account for competition between firms when no ambiguity exists on the preference of consumers for a given characteristic. Horizontal differentiation models have the opposite feature: when variants are sold at the same price, each consumer has a different preferred variant. Locational differentiation is well represented through this second type of models.

However some situations can be modelled adequately by no one of these well-known types of models. In this paper, we are interested in situations where there exist two linked and contradictory aspects in a product, with consumers disagreeing on the most important one. The paper suggests a model accounting for such situations and characterizes fully the subgame perfect equilibrium of the quality-then-price game for a costless duopoly.

Consider for example environmental quality and "physical" quality. Very often these two characteristics are linked and contradictory. We mean that to have a good physical quality, one must have a bad environmental quality. For instance to have a beautiful leather, chemical products, very harmful to the environment, have to be used. It is reasonable to suppose that some consumers attach importance to the physical quality of leather and that some of them attach importance to the environmental dimension. The same opposition holds between the engine power in a car on the one hand and the fuel consumption and the environmental dimension on the other hand. A practical example is PRIUS, a car model of Toyota with a hybrid engine. Despite a price close to the prices of more classical models in the same category (Avensis turbodiesel for instance in the same brand[1]), consumers continue to buy the classical models. Another interesting example with the same feature is food when we consider taste and nutritional quality. Unfortunately, what is considered tasteful (for instance pastry) is in general not good for health and consumers certainly disagree on the most important among the two aspects.

In this paper we suggest a model derived from the well-known Mussa and Rosen's (1978) one to account for such situations, in which two populations of opposite preferences co-exist. When variants are sold at the same price the two populations rank them exactly in the reverse order. A costless duopoly is supposed to compete for such a market choosing first qualities then prices. The subgame perfect equilibrium is fully characterized. We prove that depending on the relative importance of each population and the potential for differentiation on each type of quality segment, three outcomes are possible: either both firms choose to serve the same population ignoring the other, or, each firm is specialized in one population.

The paper is organized as follows. Section 2 describes the model. Section 3 calculates price equilibrium for given qualities. Section 4 deals with quality choice and Section 5 concludes.

[1]The comparison made on www.Caradisiac.com between the two models showing that both have very comparable characteristics (except for the characteristics stemming from the engine type), allows to place them in the same category.

2 The model

The used model is an extension of the well-known model of Mussa and Rosen (1978). Consider two competing firms $i = 1, 2$, each choosing the quality of its product q_i and its price p_i. The indirect utility function of a consumer of type θ is given by:

$$U_\theta(q_i, p_i) = \begin{cases} \theta q_i - p_i & \text{if she buys one unit of product of quality } q_i \text{ at price } p_i \\ 0 & \text{if she buys nothing} \end{cases}$$

Each consumer is supposed to buy one unit of product from the firm that ensures to her the best utility except if the alternative of no purchase is better than both. Consumers are supposed to be uniformly distributed with a density normalized to 1, on $[\underline{\theta}, \overline{\theta}]$. Qualities are chosen in $[\underline{q}, \overline{q}]$.

We suppose that $\underline{q} < 0$ and $\underline{\theta} < 0$. These are the only differences with the original model. This implies that products sold at the same price are ranked from top to bottom by consumers $\theta > 0$ and in the reverse order by consumers $\theta < 0$. (Consumer $\theta = 0$ is indifferent). Hence the model is not a vertical differentiation one since there is no unanimity in the ranking of variants sold at the same price.

The new hypotheses imply that quality is not desirable by everybody. This is the way we choose to account for the existence of contradictory aspects in products. To fix ideas, imagine that positive qualities correspond to products good for health and that negative ones correspond to tasteful products. Positive θ are the consumers who attach more importance to nutritional quality while negative θ are those who attach more importance to taste.

For simplicity production is supposed to be costless. Firms engage in a two-step game in which they first choose qualities then prices.

3 Price equilibrium

We proceed by backward induction. In this section we solve the price step.

Lemma 1 *For given qualities, equilibrium prices and profits at equilibrium are provided in the following table.*

	Equilibrium prices	*Profits at equilibrium*
$0 < q_1 \leq q_2 \leq \overline{q}$	$p_1^* = \frac{\overline{\theta} q_1 (q_2 - q_1)}{4q_2 - q_1}$	$\pi_1^* = \frac{\overline{\theta}^2 q_1 q_2 (q_2 - q_1)}{(4q_2 - q_1)^2}$
	$p_2^* = \frac{2\overline{\theta} q_2 (q_2 - q_1)}{4q_2 - q_1}$	$\pi_2^* = \frac{4\overline{\theta}^2 q_2^2 (q_2 - q_1)}{(4q_2 - q_1)^2}$
$\underline{q} \leq q_1 \leq q_2 < 0$	$p_1^* = \frac{2\underline{\theta} q_1 (q_2 - q_1)}{q_2 - 4q_1}$	$\pi_1^* = \frac{4q_1^2 \underline{\theta}^2 (q_2 - q_1)}{(q_2 - 4q_1)^2}$
	$p_2^* = \frac{\underline{\theta} q_2 (q_2 - q_1)}{q_2 - 4q_1}$	$\pi_2^* = \frac{\underline{\theta}^2 q_1 q_2 (q_2 - q_1)}{(q_2 - 4q_1)^2}$
$\underline{q} \leq q_1 \leq 0 \leq q_2 \leq \overline{q}$	$p_1^* = -\frac{q_1 \underline{\theta}}{2}$	$\pi_1^* = -\frac{q_1 \underline{\theta}^2}{4}$
	$p_2^* = \frac{q_2 \overline{\theta}}{2}$	$\pi_2^* = \frac{q_2 \overline{\theta}^2}{4}$

Lemma 1 provides for each couple of given qualities the price equilibrium and the profits at equilibrium. The major part of calculations exist in the literature (Chin and Shoi 1992). In the proof provided in Appendix, calculations are given briefly focusing on differences and remarks relevant for the specified model.

4 Quality choice

We are now ready to analyze quality choice. To do so, we study the profit function of Firm i at price equilibrium w.r.t. its quality q_i, for a given quality of its competitor q_j. This analysis leads first to Lemma 2 then to Proposition 1. Proofs are given in Appendix.

Lemma 2 *The best reply of Firm i to a given q_j of its competitor is necessarily either $\underline{q}$, $\overline{q}$ or $(4/7)q_j$.*

Lemma 2 limits the relevant set of qualities possible to be a best reply to some quality of the competitor, which limits in the same time the equilibrium candidates. After noting that the couples where qualities are equal cannot be equilibria, there are three equilibrium candidates (and their mirrors) in quality terms: $(\underline{q}, (4/7)\underline{q})$, $(\underline{q}, \overline{q})$ and $(\overline{q}, (4/7)\overline{q})$. The next proposition specifies necessary and sufficient conditions for each candidate to be an equilibrium.

Proposition 1 *At the subgame perfect equilibrium, three outcomes are possible. They are summarized in the table below.*

Quality choice	*Necessary and sufficient condition(s)*
$(\underline{q}, (4/7)\underline{q})$	$\lvert\underline{q}/\overline{q}\rvert \geq 12\frac{\overline{\theta}^2}{\underline{\theta}^2}$
$(\underline{q}, \overline{q})$	$(1/12)\frac{\overline{\theta}^2}{\underline{\theta}^2} \leq \lvert\underline{q}/\overline{q}\rvert \leq 12\frac{\overline{\theta}^2}{\underline{\theta}^2}$
$(\overline{q}, (4/7)\overline{q})$	$\lvert\underline{q}/\overline{q}\rvert \leq (1/12)\frac{\overline{\theta}^2}{\underline{\theta}^2}$

Qualitatively, Proposition 1 implies that three regions may be relevantly distinguished as $|\underline{q}/\overline{q}|$ varies relatively to $\frac{\overline{\theta}^2}{\underline{\theta}^2}$. For sufficiently high $|\underline{q}/\overline{q}|$, both firms produce negative qualities and serve partially only negative θ; positive θ are neglected by both firms. The reverse phenomenon is observed for sufficiently low $|\underline{q}/\overline{q}|$: only positive θ are served and negative θ are neglected. For intermediate $|\underline{q}/\overline{q}|$, one firm produces the lowest quality possible, serving (partially) negative θ and the other produces $\overline{q}$ serving positive θ; thus a sort of "specialization" of each firm on each type of consumers' segment, occurs.

This result stems from a tradeoff of firms between operating on an interesting demand segment with the competitor and minimizing price competition by being alone on a segment. This depends on the relative potential for differentiation on each type of quality segment measured by $|\underline{q}/\overline{q}|$ and the relative size of each consumers' segment measured by $\frac{\overline{\theta}^2}{\underline{\theta}^2}$.

For high values of $|\underline{q}/\overline{q}|$ (Case 1), the potential for differentiation on the negative quality segment is high relative to the positive one. Thus the relative potential for differentiation offered on the negative qualities is sufficient for both firms to operate on the same segment. The reverse phenomenon is observed when $|\underline{q}/\overline{q}|$ is low (Case 3), in which case firms choose both to serve the positive segment of consumers as the relative potential for differentiation is sufficient on the positive segment of qualities. In the intermediate case (Case 2), neither the positive segment nor the negative one offers a sufficient potential for differentiation and firms choose to operate on distinct segments of qualities serving each a distinct segment of consumers. This may be seen differently when results are written in terms of the relative size of the consumers' segments.

1. When $\frac{\overline{\theta}^2}{\underline{\theta}^2} < -\frac{\underline{q}}{12\overline{q}}$, firms produce qualities $(\underline{q}, \frac{4}{7}\underline{q})$ serving only negative θ.

2. When $-\frac{\underline{q}}{12\overline{q}} < \frac{\overline{\theta}^2}{\underline{\theta}^2} < -\frac{12\underline{q}}{\overline{q}}$ firms produce $(\underline{q}, \overline{q})$, one of them serving negative θ and the other positive ones.

3. Finally when $\frac{\overline{\theta}^2}{\underline{\theta}^2} > -\frac{12\underline{q}}{\overline{q}}$, firms produce $(\overline{q}, \frac{4}{7}\overline{q})$ serving only positive θ.

When $\frac{\overline{\theta}^2}{\underline{\theta}^2}$, the relative size of the positive segment of consumers, is too small (Case 1), the negative segment is relatively so important that firms find it better to serve both the negative θ behaving as if the positive one did not exist. A symmetric phenomenon happens when $\frac{\overline{\theta}^2}{\underline{\theta}^2}$ is too large (Case 3) meaning that the positive segment is so important that both firms decide to serve this segment. In both cases, operating on an interesting consumers' segment outweighs the benefit to be alone on a consumers' segment. It is only for intermediate values of $\frac{\overline{\theta}^2}{\underline{\theta}^2}$ (Case 2) that firms specialize each in a different segment of consumers and maximize product differentiation. In this case, the positive and negative segments have comparable sizes, making better to be on different segments.

5 Conclusion

Allowing in the model of Mussa and Rosen, simply to the lower-bounds of the consumers' segment and the quality segment to be negative, provides a simple and tractable model that accounts for a variety of interesting stylized facts in which two contradictory aspects exist. Three outcomes are possible: either both firms serve the same population ignoring the other or each one is specialized in one population.

We thus better understand why firms may choose to "ignore" some population. By a rough intuitive approach, one may think that they should always specialize in a different population to maximize differentiation and be a local monopoly. This reasoning is nevertheless valid only for populations of comparable sizes. When one population is very large relative to the other, it becomes more profitable for both to serve it and ignore the small population. The analysis shows that the apparaisal of the relative size of populations should be done relative to the relative potential for differentiation on each quality segment. As a by-product the existence in a given sector of firms all of the same type (serving the same type of consumers) does not necessarily mean that all consumers are of that same type but imply that the size of the other population is relatively small and/or that the relative potential for differentiation on the segment quality corresponding to the ignored population is small.

Above the obtained results, the model offers a possibility to deepen the question of quality choice in a new direction not explored so far. According to the adopted interpretation, the model would be used differently. Interpreted in terms of environmental and physical qualities, the question of setting up an eco-label or some other regulating policy may be addressed.

Besides the examples cited in the introduction, the model may be interpreted in terms of innovation or R&D. Suppose that positive qualities represent innovating products and negative ones represent the classical ones. Thus positive θ would be the "open-minded" consumers attracted by novelties while the negative ones the conservative consumers favouring the well-known products. According to the importance of the open-minded consumers relative to the conservative ones and the relative potential for differentiation offered on each type of quality segment, firms will either offer only innovating products, only classical products or both types of products. The impact of the classical economic tools or organization modes of R&D may be addressed in this special setting.

Having now in mind taste and nutritional quality, if the segment of positive θ is not sufficiently large relative to the negative one, no product good for health will be sold on the market. Consumers attaching importance to health will be ignored and will have to comply to the "majority rule". Now imagine that preferences may change depending on what is consumed or what is proposed on the market and in advertising, i.e. the more you eat pastry or the more you see advertising spots on pastry, the more you appreciate it. The segment of positive θ should in this case shrink everyday and reinforce the firms' choice. Thus consumers may be locked in an initial situation favorable to taste and unfavorable to nutritional quality. This intuition may be tested through a dynamical model, in which the consumers' segment at one period depends on the offer of products in the preceding period.

Appendix

Proof of Lemma 1.

First case: $0 < q_1 < q_2 \leq \overline{q}$. In this case, only positive θ are served. The no purchase alternative is better for negative θ. Calculations are the same as Choi and Shin (1992). Briefly, when $\frac{p_2-p_1}{q_2-q_1} > \frac{p_1}{q_1}$ profits are given by:

$\pi_1 = p_1 \left(\frac{p_2-p_1}{q_2-q_1} - \frac{p_1}{q_1}\right)$ and $\pi_2 = p_2 \left(\overline{\theta} - \frac{p_2-p_1}{q_2-q_1}\right)$.

First order conditions and further calculations lead to the equilibrium prices and the profits given in the lemma.

Second case: $\underline{q} \leq q_1 < q_2 < 0$. Here only negative θ are served. Similar calculations lead to the equilibrium prices and profits[2]. Note that both prices and both profits at equilibrium are positive as $q_i < 0$, $\underline{\theta} < 0$, and $4q_1 < q_1 < q_2 < 0$.

Third case: $\underline{q} \leq q_1 < 0 < q_2 \leq \overline{q}$. In this case, Firm 1 serves negative θ while Firm 2

[2]The lowest and the highest quality firms not playing symmetric roles, calculations cannot be avoided in this case.

serves positive ones. Profit functions are: $\pi_1 = p_1\left(\frac{p_1}{q_1} - \underline{\theta}\right)$ and $\pi_2 = p_2\left(\overline{\theta} - \frac{p_2}{q_2}\right)$.

Each firm is a monopoly on the served segment of consumers. Equilibrium prices are thus $\begin{cases} p_1 = \frac{-q_1\underline{\theta}}{2} \\ p_2 = \frac{q_2\overline{\theta}}{2} \end{cases}$, and profits at price equilibrium are $\begin{cases} \pi_1 = -\frac{q_1\underline{\theta}^2}{4} \\ \pi_2 = \frac{q_2\overline{\theta}^2}{4} \end{cases}$.

Note also that both prices and both profits are positive (recall that $q_1 < 0$).

Two special cases must be considered apart. Suppose that $q_i = 0$. In this case consumers buy product i only when $p_i = 0$. Thus $\pi_i = 0$. Two subcases are possible. When $q_1 < q_2 = 0$ at price equilibrium profits are given by $\pi_1 = -\frac{q_1\underline{\theta}^2}{4}$ and $\pi_2 = 0$. When $q_1 = 0 < q_2$, at price equilibrium profits are $\pi_1 = 0$ and $\pi_2 = \frac{q_2\overline{\theta}^2}{4}$. Both subcases amount to make $q_i = 0$ in the formulae obtained in the third case.

Consider finally the special case: $q_1 = q_2$. A profit destructive competition (à la Bertrand) occurs when qualities are equal, which amounts to make $q_1 = q_2$ in the formulae obtained in the three first cases. ∎

Proof of Lemma 2. Three cases have to be distinguished.

First case: $q_j < 0$.

The profit of Firm i at price equilibrium writes in this case as:

$$\pi_i = \begin{cases} \frac{4q_i^2\underline{\theta}^2(q_i - q_j)}{(q_i - 4q_j)^2} & \text{if} \quad q_i \leq q_j \\ \frac{\underline{\theta}^2 q_i q_j (q_i - q_j)}{(q_i - 4q_j)^2} & \text{if} \quad q_j < q_i \leq 0 \\ \frac{q_i\overline{\theta}^2}{4} & \text{if} \quad 0 < q_i \end{cases}$$

On the interval $[\underline{q}, q_j]$ we calculate the log derivative of the profit function w.r.t. q_i and we prove that the derivative of the profit is always negative on this interval. Hence π_i is decreasing on $[\underline{q}, q_j]$. On the interval $[q_j, 0]$, the derivative of the profit has the same sign as $(-q_j)(4q_j - 7q_i)$. Hence π_i admits a local maximum at $\tilde{q}_i = (4/7)q_j$. Finally on the interval $[0, \overline{q}]$, the profit is always increasing.

Therefore, the best reply of Firm i to any $q_j < 0$ must be either $\underline{q}$, $\tilde{q}_i = (4/7)q_j$ or $\overline{q}$.

Second case: $q_j > 0$.

In this case, the profit of Firm i at price equilibrium writes as:

$$\pi_i = \begin{cases} \frac{-q_i\underline{\theta}^2}{4} & \text{if} \quad q_i \leq 0 (< q_j) \\ \frac{\overline{\theta}^2 q_i q_j (q_j - q_i)}{(4q_j - q_i)^2} & \text{if} \quad 0 < q_i \leq q_j \\ \frac{4\overline{\theta}^2 q_i^2 (q_i - q_j)}{(4q_i - q_j)^2} & \text{if} \quad (0 <) q_j < q_i \end{cases}$$

As in the first case, we prove that π_i is decreasing on $[\underline{q}, 0]$, that it reaches a local maximum on $[0, q_j]$ at $\tilde{q}_i = (4/7)q_j$ and that it is increasing on $[q_j, \overline{q}]$. Therefore the best reply of Firm i to any q_j of its competitor must be either $\underline{q}$, $\tilde{q}_i = (4/7)q_j$ or $\overline{q}$.

Third case: $q_j = 0$.

The profit of Firm i is then given by:

$$\pi_i = \begin{cases} \frac{-q_i\theta^2}{4} & \text{if} \quad q_i \leq 0 \\ \frac{q_i\overline{\theta}^2}{4} & \text{if} \quad q_i > 0 \end{cases}$$

π_i thus reaches its maximal value either at $\underline{q}$ or at $\overline{q}$.

■

Proof of Proposition 1.

1) $(\underline{q}, (4/7)\underline{q})$ is an equilibrium if and only if each quality is a best response to the other. Taking the possible best replies into account, we must first have $\pi_2(\underline{q}, (4/7)\underline{q}) \geq \pi_2(\underline{q}, \overline{q})$ to ensure that the best reply of Firm 2 to $q_1 = \underline{q}$ is $(4/7)\underline{q}$, which is equivalent after calculations to $\underline{q} \leq -12\overline{q}\frac{\overline{\theta}^2}{\underline{\theta}^2}$.

Second we must have:

$$\begin{cases} \pi_1(\underline{q}, (4/7)\underline{q}) \geq \pi_1(\overline{q}, (4/7)\underline{q}) \\ \pi_1(\underline{q}, (4/7)\underline{q}) \geq \pi_1((4/7)(4/7)\underline{q}, (4/7)\underline{q}), \end{cases} \tag{1}$$

to ensure that the best reply of Firm 1 to quality $(4/7)\underline{q}$ is $\underline{q}$. The first inequality of Inequations 1 is always satisfied. The second inequality is equivalent to

$$\underline{q} \leq -(12/7)\overline{q}\frac{\overline{\theta}^2}{\underline{\theta}^2}.$$

Therefore $(\underline{q}, (4/7)\underline{q})$ is an equilibrium if and only if $\underline{q} \leq -12\overline{q}\frac{\overline{\theta}^2}{\underline{\theta}^2}$.

2) We now examine the couple $(\underline{q}, \overline{q})$. Similarly to the first case, for this couple to be an equilibrium we must have:

$$\begin{cases} \pi_1(\underline{q}, \overline{q}) \geq \pi_1((4/7)\overline{q}, \overline{q}) \\ \pi_2(\underline{q}, \overline{q}) \geq \pi_2(\underline{q}, (4/7)\underline{q}) \end{cases},$$

which reduce to

$$-12\overline{q}\frac{\overline{\theta}^2}{\underline{\theta}^2} \leq \underline{q} \leq -(1/12)\overline{q}\frac{\overline{\theta}^2}{\underline{\theta}^2}.$$

3) Finally for $(\overline{q}, (4/7)\overline{q})$ to be an equilibrium, we must have:

$$\begin{cases} \pi_2(\overline{q}, (4/7)\overline{q}) \geq \pi_2(\overline{q}, \underline{q}) \\ \pi_1(\overline{q}, (4/7)\overline{q}) \geq \pi_1(\underline{q}, (4/7)\overline{q}) \\ \pi_1(\overline{q}, (4/7)\overline{q}) \geq \pi_1((4/7)(4/7)\overline{q}, (4/7)\overline{q}) \end{cases},$$

which reduce to

$$\underline{q} \geq -(1/12)\overline{q}\frac{\overline{\theta}^2}{\underline{\theta}^2}.$$

■

References

[1] Choi, C.J. and H.S. Shin (1992) "A comment on a model of vertical product differentiation", *The Journal of Industrial Economics* **40, 2**, 229-231.

[2] Mussa M., and S. Rosen (1978) "Monopoly and Product Quality", *Journal of Economic Theory* **18**, 301-317.