E C O N O M I C S B U L L E T I N

Stackelberg mixed oligopoly with asymmetric subsidies

Vasileios Zikos
Loughborough University

Abstract

In a mixed oligopoly, when the public leader becomes a private leader and the government provides output subsidies, then privatization causes the optimal subsidy, profits and welfare to fall [Economics Letters 83 (2004) 411]. We show instead that if the leader and the followers receive asymmetric, rather than symmetric subsidies, the first-best optimum can be restored. In this case, privatization bears no consequences on the followers' subsidy, output and welfare.

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1 Introduction

Public firms operate in oligopolistic markets and compete with private firms in the provision of products and services. A market, where a public firm, instructed to maximize its contribution to social surplus, interacts with a multitude of profit maximizing firms, is termed as a mixed oligopoly (see e.g., Cremer, Marchand and Thisse [1991], De Fraja and Delbono [1989, 1990], George and La Manna [1996] for an excellent survey). The regulation of mixed oligopolies has received significant attention in the industrial organization literature. This has been coupled (in some cases) with an interest to investigate the consequences of privatization on the optimal policies. The importance of the latter cannot be exaggerated, given the widespread adoption of privatization programmes in several countries such as Italy, France, Israel, Portugal (see Anderson et al., 1997, for a more detailed discussion).

An important conclusion from this strand of literature is that output, profits and welfare at the first-best optimum are identical, irrespective of whether the public firm: (i) moves simultaneously with the private firms, (ii) is a Stackelberg leader, or (iii) is privatized and acts simultaneously with the private firms to maximize profits; so-called "irrelevance result" (see White, 1996; Poyago–Theotoky, 2001; Myles, 2002).

More recently, Fjell and Heywood (2004) showed that this result can be overturned, when assuming that the public leader becomes a private leader after the privatization. In this case, the optimal output subsidy, profits and welfare are all reduced in the post-privatization case. However, this "relevance result" is generated on the assumption of "equal subsidies for different firms",[1] which preserves an inefficient allocation of production costs across firms (i.e., a second-best optimum is attained). Indeed, by considering the more plausible case of *asymmetric* subsidies, we show that the irrelevance result can be recovered (at least partially). In this respect, the optimal subsidies bring about the first-best allocation, since they can tackle the existing market failures. More specifically, we find that the leader receives a smaller subsidy and the followers a larger subsidy than under an equal subsidies policy; and that the output of the leader becomes smaller but the output of the followers larger. It turns out, however, that the latter effect is stronger and indeed may well dominate, thus causing the level of total output to rise. Finally, we obtain that the increase in total production and the associated efficiency enhancing effect of the optimal subsidies (such that output is equalized across firms) will lead to an increase in the social surplus to the first-best level.

2 The model

Our envisaged industry consists of one public firm and n private firms serving a single market of homogeneous good. The inverse demand function is linear $p = a - Q$, where $Q = q_0 + \sum_{i=1}^{n} q_i$ is total output made up from a 'public' and a 'private' component, respectively. All firms are assumed to operate under identical cost functions. These are of the standard form $C_j(q_j) = c + (1/2)kq_j^2$, $j \in \{0, ..., n\}$, $k > 0$, implying an increasing marginal cost.[2] We set $c = 0$ so as to abstein from entry issues, while avoiding any loss of generality.

[1] The leader who enjoys a first-mover advantage, receives a subsidy equal to the followers'.

[2] This assumption is standard in the mixed oligopoly literature and serves the purpose of ruling out the uninteresting case of a public monopoly.

We will compare the post-privatization symmetric subsidies regime with the asymmetric subsidies one. Each private firm's profit function is given by

$$\pi_i = q_i(a - \sum_{j=0}^{n} q_j) - (1/2)kq_i^2 + sq_i, \;\; i \in \{1, ..., n\}, \tag{1}$$

where s is the output subsidy. Similarly, the profit function for the public firm is

$$\pi_0 = q_0(a - \sum_{j=0}^{n} q_j) - (1/2)kq_0^2 + s_0 q_0, \tag{2}$$

with s_0 denoting the public firm's output subsidy. Social welfare is given by

$$W = (1/2)Q^2 + \sum_{j=0}^{n} \pi_j - s_0 q_0 - s \sum_{i=1}^{n} q_i. \tag{3}$$

The standard indirect effect of the subsidy s amounts to saying that the public firm's decision variable will be affected indirectly by the private firms' responsiveness to their own subsidy.

Our three stage game with observable actions is in order. In stage one, the government commits to an output subsidy. That is, market participants receive (i) identical subsidies or (ii) all subsidies are equal except for the public firm's. In stage 2, the public firm chooses its output to maximize profits. The private firms' output selection follows in the last stage, 3.

2.1 The privatized Stackelberg oligopoly with identical subsidies

Solving for the SPNE of this game by backward induction, the reaction function of the i-th firm in the last stage is

$$q_i(q_0) = \frac{a + s - q_0}{1 + k + n}. \tag{4}$$

The privatized leader, taking into account (4), sets its output to maximize own profits for a given subsidy. The solution to this yields

$$q_0(s) = \frac{(1 + k)(a + s)}{(1 + k)(2 + k) + nk}. \tag{5}$$

The optimal policy then follows from the maximization of (3) with respect to s

$$s = \frac{a\Omega}{B}, \tag{6}$$

where $\Omega = (1+ 2n) + k(4 + 7n + 2n^2) + k^2(6 + 9n + 4n^2 + n^3) + k^3(4 + 5n + 2n^2) + k^4(1 + n)$ and $B = (1 + 2n)^2 + k(5 + 17n + 15n^2 + 4n^3) + k^2(10 + 28n + 21n^2 + 6n^3 + n^4) + k^3(10 + 22n + 13n^2 + 3n^3) + k^4(5 + 8n + 3n^2) + k^5(1 + n)$.

Using the solution for the optimal subsidy, we obtain the following SPNE outcomes

$$\begin{aligned} q_0 &= \frac{a(1+k)(n+k+1)E}{B} \\ q_i &= \frac{a\left[kn+(1+k)^2\right]E}{B} \\ p &= \frac{akC}{B} \\ W &= \frac{a^2E}{2B}, \end{aligned} \tag{7}$$

where $E=(1+k)^2+n(1+k)(2+k)+kn^2$ and $C=(n^2+3n+1)+k(4+10n+4n^2)+k^2(6+12n+5n^2+n^3)+k^3(4+6n+2n^2)+k^4(1+n)$.

The government has two targets to control for: underproduction and inefficiency in the allocation of equilibrium cost across firms. Given that the public firm is in the leader position, the optimal output subsidy can address the former distortion alone. Indeed, the fact that all firms are subsidized at the same rate reinforces the first-mover advantage of the leading firm and, in turn, enlarges the gap between price and the public firm's marginal cost.[3] This line of justification was provided by Fjell and Heywood (2004) to explain as to why the irrelevance result of White (1996) and Poyago–Theotoky (2001) does no longer apply to this context.[4] Hence, it is obvious that public policy can lead to a second-best optimum with two targets to control for and only one policy instrument available.

2.2 The privatized Stackelberg oligopoly with asymmetric subsidies

How are the previous results affected when the government can provide asymmetric rather than symmetric subsidies? Providing an answer to this question forms the subject of the present section.

The public firm, realizing the reaction function of each follower as in (4), chooses its output to maximize profits for given subsidies. The solution to this problem gives rise to the following output

$$q_0(s_0,s)=\frac{a(1+k)-ns+(1+k+n)s_0}{2+k^2+k(3+n)}. \tag{8}$$

Substituting (8) into (4), we obtain the output of each private firm

$$q_i(s_0,s)=\frac{a\left[1+k^2+k(2+n)\right]+(1+k)(2+k+n)s-(1+k+n)s_0}{(1+k+n)\left[2+k^2+k(3+n)\right]}. \tag{9}$$

In stage one, the government maximizes welfare with respect to s_0 and s, leading to the following subsidies

[3] The assumption of increasing marginal and total cost is crucial for this result.

[4] When the public firm is privatized and acts simultaneously with the private firms to maximize profits, the optimal subsidy can restore the first-best allocation by tackling both market failures.

$$s_0^* = \frac{a(1+k)}{(1+k+n)^2}, \quad s^* = \frac{a}{1+k+n}. \tag{10}$$

Using (10), straightforward calculations yield the SPNE solutions of the entire game

$$\begin{aligned} q_0^* &= \frac{a}{1+k+n} = q_i^* = q^* \\ p^* &= \frac{ak}{1+k+n} \\ \pi_0^* &= \frac{a^2\left[2+k^2+k(3+n)\right]}{2(1+k+n)^3} \\ \pi_i^* &= \frac{a^2(2+k)}{2(1+k+n)^2} \\ W^* &= \frac{a^2(1+n)}{2(1+k+n)}. \end{aligned} \tag{11}$$

The use of asymmetric subsidies restores the first-best optimum, since price equals marginal cost ($p^* = kq^*$). Moreover, the irrelevance result is generated with the exception of the public firm's profits. In this case, each follower makes a larger profit compared to the leader's profit (see Appendix A).

2.3 Comparison and discussion of the results

Comparing the case of symmetric subsidies with the case of asymmetric ones, we obtain that in the former the public leader receives a larger subsidy, but the followers receive a smaller subsidy. In addition to that, the leader produces more output and the market price is higher. It also comes at no surprise that welfare is strictly higher under the implementation of asymmetric subsidies. These results are shown below

$$\begin{aligned} s - s^* &= -\frac{anG}{(1+k+n)B} \\ s - s_0^* &= \frac{an^2K}{(1+k+n)^2B} \\ q_0^* - q_0 &= -\frac{an^2\left[F+k(3+3n+n^2)\right]}{(1+k+n)B} \\ q_i^* - q_i &= \frac{an\left[F+k(3+4n+n^2)\right]}{(1+k+n)B} \\ p^* - p &= -\frac{akn^3}{(1+k+n)B} \\ W^* - W &= \frac{a^2kn^3}{2(1+k+n)B} \end{aligned} \tag{12}$$

$G = (1+2n) + 2k(2+3n+n^2) + k^2(6+6n+n^2) + 2k^3(2+n) + k^4,$
$K = (1+2n)+k(5+7n+2n^2)+k^2(10+12n+5n^2+n^3)+k^3(10+10n+3n^2)+k^4(5+3n)+k^5$
$F = (1+2n) + k^2(3+2n) + k^3.$

The logic behind these results can be seen by recalling the objectives of public policy. When it comes to the optimal intervention, it is clear that the government faces two distortions (as mentioned previously). This implies that two policy instruments are required in order to attain the first-best allocation. The implementation of asymmetric subsidies serves this purpose. Indeed, the public leader is subsidized at a lower rate than the followers and therefore, the share of the private firms in total production increases (see Appendix A). Conversely, the optimal subsidies imply a cost redistribution effect, in addition to raising the total output level. With a two-fold role of subsidization, this positively influences welfare and restores the first-best optimum.

3 Conclusion

In this paper, we showed that in contrast to previous treatments of the same topic, the order of the firms' moves after privatization may not be a crucial determinant of the results. What matters though is whether it is possible to provide firm-specific subsidies for the public and the private firms. In particular, when the public leader becomes a private leader subsequent to privatization, this mandates lower subsidies for the leader than for its private counterparts, which implies that the first-best allocation can be restored.

Our results do not overturn the findings of Fjell and Heywood (2004), but rather do complement them. In this respect, the use of asymmetric subsidies increases the effectiveness of intervention in that all market failures can be addressed. Indeed, our study suggests that privatization can still be socially desirable, even if the public leader retains its position post-privatization, provided that the private followers can receive larger subsidies than the leader.

Appendix A The public leader makes a smaller profit than each follower

$$\pi_0^* - \pi_i^* = -\frac{a^2 n}{(1+k+n)^3}$$

The leader is subsidized to a lesser extent than each follower

$$s^* - s_0^* = \frac{an}{(1+k+n)^2}.$$

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