E C O N O M I C S   B U L L E T I N

Does a Non-verifiable Imperfect Informative Binary Signal Always Create a Strictly Positive Value?

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Abstract

Owing to the difference between the status quo utility levels of a good agent and a bad agent, we find that a firm adopting a non-verifiable imperfect informative binary signal does not necessarily change its action (on trading off output efficiency against rent extraction). Hence, the signal does not always create a strictly positive value for the firm.

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1. Introduction:

Riordan and Sappington (1988) indicate that, under mild assumptions, the adoption of an *ex post* verifiable signal does not cause the agent to receive excess rent whatever his type is, and that the complete information optimal output levels can be implemented. Hence, the *ex post* verifiable signal does create a strictly positive value for the firm whenever the second-best contract - for the case whereby the signal is not adopted - entails some output distortion and/or some excess information rent(s) given up for some type(s).[1]

On this basis we wonder, without considering the costs of signal acquisition, whether an (*ex ante*) non-verifiable imperfect informative binary signal always creates a strictly positive value for a firm under asymmetric information. This paper proposes a contractual game to analyze such an issue. Owing to the difference between the status quo utility levels of a good agent and a bad agent, we find that a firm adopting a non-verifiable imperfect informative binary signal does not necessarily change its action (on trading off output efficiency against rent extraction). Hence, the signal does not always create a strictly positive value for the firm. Furthermore, if the binary signal σ (defined in Section 2) contains a higher probability for the occurrence of a good agent, then the chance is less likely that the signal σ creates no strictly positive value.

The remainder of this paper is organized as follows. Section 2 introduces the information structure of the binary signal and the model of a firm's contracting problem. Section 3 analyzes the main finding that is alluded to above. The conclusion is in Section 4.

2. Model:[2]

Consider a firm (the principal) that wants to delegate an agent to produce q units of a certain product. The value of the product to the principal is characterized by $S(q)$, where $S^{'} > 0$, $S^{''} < 0$, and $S(0) = 0$. The agent's production cost is $C(q,\theta) = \theta q$, where θ is a constant marginal cost that is observed by the agent, but which is unobservable to the firm. Nonetheless, it is common knowledge that the cost function is either:

$$C(q,\underline{\theta}) = \underline{\theta} q \qquad \text{with probability } \nu \tag{1}$$

or

$$C(q,\overline{\theta}) = \overline{\theta} q \qquad \text{with probability } 1-\nu . \tag{2}$$

Here, $\underline{\theta} < \overline{\theta}$.

We define here the low cost (high cost) agent as the good (bad) agent, and we denote by $\Delta\theta = \overline{\theta} - \underline{\theta} > 0$ the spread of uncertainty on the agent's marginal cost. If the firm pays t for producing q units of the good, then the good (bad) agent's benefit is $U = t - \underline{\theta} q$ ($U = t - \overline{\theta} q$), and the firm's profit is $\pi \equiv S(q) - t$. For simplicity, we normalize the status quo utility of the high cost agent $(\overline{\theta})$ to zero and

[1] Cremer and McLean (1998) generalize this idea by considering correlated information in their multi-agent model.
[2] We draw heavily on the model used in Laffont and Martimort (2004).

assume that the status quo utility level of the low cost agent ($\underline{\theta}$) is $U_0 \geq 0$.

Before proposing a contract to the agent, the firm first decides whether to adopt an imperfect non-verifiable binary signal σ about θ. The signal takes two possible values: σ_g and σ_b. Suppose that the conditional probabilities of the realizations of σ_g and σ_b are $\mu_g = \Pr(\sigma = \sigma_g / \theta = \underline{\theta}) \geq 1/2$ and $\mu_b = \Pr(\sigma = \sigma_b / \theta = \overline{\theta}) = 1$, respectively. Note that, for any binary signal σ' with $\mu_g' = \mu_g$ and μ_b'<1, the firm's profit of adopting the signal is weakly less than that under the situation whereby the available signal is σ. Therefore, one will only focus on the case of adopting signals of this kind, such as σ.

Given that the signal is adopted, the firm applies Bayes law to compute the posterior belief of dealing with a good agent, namely:

$$\nu_g = \Pr(\theta = \underline{\theta} / \sigma = \sigma_g) = \frac{\nu\mu_g}{\nu\mu_g + (1-\nu)(1-\mu_b)} = 1, \tag{3}$$

$$\nu_b = \Pr(\theta = \underline{\theta} / \sigma = \sigma_b) = \frac{\nu(1-\mu_g)}{\nu(1-\mu_g) + (1-\nu)\mu_b} < \nu. \tag{4}$$

3. The optimal contracts:

In the case of complete information, the efficient production levels $\underline{q}^*$ and $\overline{q}^*$ are given by the following first-order conditions:

$$S'(\underline{q}^*) = \underline{\theta} \tag{5}$$

and

$$S'(\overline{q}^*) = \overline{\theta}. \tag{6}$$

Under the situation of asymmetric information, the firm - without observing the type of agent it confronts - offers a menu of contracts $\{(\underline{t}, \underline{q}); (\overline{t}, \overline{q})\}$ (on a take-it-or-leave-it basis) in order to maximize its expected profit.

3.1 The case where the signal is not adopted:

Define the information rents by $\underline{U} = \underline{t} - \underline{\theta}\underline{q}$ and $\overline{U} = \overline{t} - \overline{\theta}\overline{q}$. We now replace the payments in the firm's objective function as functions of information rents and outputs so that the new optimal variables are $\{(\underline{U}, \underline{q}), (\overline{U}, \overline{q})\}$. The firm solves the following optimization problem (P_n) and obtains the optimal profit π_n:

(P_n): $$\max_{\{(\underline{U},\underline{q});(\overline{U},\overline{q})\}} \nu(S(\underline{q}) - \underline{\theta}\underline{q}) + (1-\nu)(S(\overline{q}) - \overline{\theta}\overline{q}) - (\nu\underline{U} + (1-\nu)\overline{U})$$

subject to

$$\underline{U} \geq \overline{U} + \Delta\theta\overline{q} \quad (\underline{IC}) \tag{7}$$

$$\overline{U} \geq \underline{U} - \Delta\theta\underline{q} \quad (\overline{IC}) \tag{8}$$

$$\underline{U} \geq U_0 \quad (\underline{IR}) \tag{9}$$

$$\overline{U} \geq 0. \qquad (\overline{IR}) \qquad (10)$$

Depending on the value of U_0, the solution to problem (P_n) falls into one of the following five different regimes (R1-R5). These regimes (characterized by the binding constraints in (7) through (10)) are as follows.

R1	$(\underline{IC})(\overline{IR})$	binding
R2	$(\underline{IC})(\overline{IR})(\underline{IR})$	binding
R3	$(\overline{IR})(\underline{IR})$	binding
R4	$(\overline{IR})(\underline{IR})(\overline{IC})$	binding
R5	$(\underline{IR})(\overline{IC})$	binding.

Proposition 1 states all the complete results. In what follows we index the solution to the problem with a superscript "SB" and define $\overline{q}^{SB*}$ and $\underline{q}^{CI}$ by the following equations, respectively:

$$S'(\overline{q}^{SB*}) = \overline{\theta} + \frac{\nu}{1-\nu}\Delta\theta \qquad (11)$$

and

$$S'(\underline{q}^{CI}) = \underline{\theta} - \frac{1-\nu}{\nu}\Delta\theta. \qquad (12)$$

Proposition 1: (For the proof, see Laffont and Martimort (2002), pp 101-104.)

R1. If $U_0 < \Delta\theta\overline{q}^{SB*}$, $\underline{q}^{SB} = \underline{q}^*, \overline{q}^{SB} = \overline{q}^{SB*}, \underline{U}^{SB} = \Delta\theta\overline{q}^{SB*}$ and $\overline{U}^{SB} = 0$.

R2. If $\Delta\theta\overline{q}^* \geq U_0 \geq \Delta\theta\overline{q}^{SB*}$, $\underline{q}^{SB} = \underline{q}^*, \overline{q}^{SB} = \dfrac{U_0}{\Delta\theta}, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = 0$.

R3. If $\Delta\theta\underline{q}^* > U_0 > \Delta\theta\overline{q}^*$, $\underline{q}^{SB} = \underline{q}^*, \overline{q}^{SB} = \overline{q}^*, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = 0$.

R4. If $\Delta\theta\underline{q}^{CI} \geq U_0 \geq \Delta\theta\underline{q}^*$, $\underline{q}^{SB} = \dfrac{U_0}{\Delta\theta}, \overline{q}^{SB} = \overline{q}^*, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = 0$.

R5. If $U_0 > \Delta\theta\underline{q}^{CI}$, $\underline{q}^{SB} = \underline{q}^{CI}, \overline{q}^{SB} = \overline{q}^*, \underline{U}^{SB} = U_0$ and $\overline{U}^{SB} = U_0 - \Delta\theta\underline{q}^{CI} > 0$.

We note that, even though the firm is ready to accept some distortion away from efficiency in order to reduce the agent's information rent, this incurs no output distortion and no excess information rent received by any agent in regime R3. Therefore, there is no reason for the firm to adopt the signal when the condition $\Delta\theta\underline{q}^* > U_0 > \Delta\theta\overline{q}^*$ holds. In other regimes, each optimal second-best menu of contracts entails output distortions and/or excess information rent given up. In the following, we will answer whether the binary signal always creates a strictly positive value in these regimes.

3.2 The case where the signal is adopted:

Suppose that the binary signal is adopted. If $\sigma = \sigma_g$, then the firm selects the complete information contract $\underline{q} = \underline{q}^*$ and $\underline{U} = U_0$ (since the firm is completely informed). The firm's profit, denoted by π_g, is $S(\underline{q}^*) - \underline{\theta}\underline{q}^* - U_0$. If $\sigma = \sigma_b$, then the conditional probability of dealing with a good agent is ν_b (given by (4)). The firm now solves the following optimization problem (P_b).

$$(P_b): \qquad \max_{\{(\underline{U},\underline{q});(\bar{U},\bar{q})\}} \nu_b(S(\underline{q}) - \underline{\theta}\underline{q}) + (1-\nu_b)(S(\bar{q}) - \bar{\theta}\bar{q}) - (\nu_b \underline{U} + (1-\nu_b)\bar{U})$$

subject to (7)-(10).

We next denote the firm's optimal profit by π_b. Note that the structures of the solutions to problem (P_b) and problem (P_n) are the same. Proposition 2 below states all the complete results. Define $\bar{q}^{SB*}(\nu_b)$ and $\underline{q}^{CI}(\nu_b)$ by the following equations, respectively:

$$S'(\bar{q}^{SB*}(\nu_b)) = \bar{\theta} + \frac{\nu_b}{1-\nu_b}\Delta\theta \tag{13}$$

and

$$S'(\underline{q}^{CI}(\nu_b)) = \underline{\theta} - \frac{1-\nu_b}{\nu_b}\Delta\theta. \tag{14}$$

Note that $\bar{q}^{SB*}(\nu_b) \geq \bar{q}^{SB*}$, $\underline{q}^{CI}(\nu_b) \geq \underline{q}^{CI}$, and the expected profit of adopting the binary signal is:

$$\pi_a = \nu\nu_g\pi_g + (\nu(1-\nu_g) + (1-\nu))\pi_b. \tag{15}$$

Proposition 2: (For the proof, see Laffont and Martimort (2002), pp 101-104.)

r1. If $U_0 < \Delta\theta\bar{q}^{SB*}(\nu_b)$, $\underline{q}^{SB} = \underline{q}^*, \bar{q}^{SB} = \bar{q}^{SB*}(\nu_b), \underline{U}^{SB} = \Delta\theta\bar{q}^{SB*}(\nu_b)$, and $\bar{U}^{SB} = 0$.

r2. If $\Delta\theta\bar{q}^* \geq U_0 \geq \Delta\theta\bar{q}^{SB*}(\nu_b)$, $\underline{q}^{SB} = \underline{q}^*, \bar{q}^{SB} = \frac{U_0}{\Delta\theta}, \underline{U}^{SB} = U_0$, and $\bar{U}^{SB} = 0$.

r3. If $\Delta\theta\underline{q}^* > U_0 > \Delta\theta\bar{q}^*$, $\underline{q}^{SB} = \underline{q}^*, \bar{q}^{SB} = \bar{q}^*, \underline{U}^{SB} = U_0$, and $\bar{U}^{SB} = 0$.

r4. If $\Delta\theta\underline{q}^{CI}(\nu_b) \geq U_0 \geq \Delta\theta\underline{q}^*$, $\underline{q}^{SB} = \frac{U_0}{\Delta\theta}, \bar{q}^{SB} = \bar{q}^*, \underline{U}^{SB} = U_0$, and $\bar{U}^{SB} = 0$.

r5. If $U_0 > \Delta\theta\underline{q}^{CI}(\nu_b)$, $\underline{q}^{SB} = \underline{q}^{CI}(\nu_b), \bar{q}^{SB} = \bar{q}^*, \underline{U}^{SB} = U_0$, and $\bar{U}^{SB} = U_0 - \Delta\theta\underline{q}^{CI}(\nu_b) > 0$.

3.3 The binary signal does not always create a strictly positive value:

By Propositions 1 and 2, if $U_0 \in [0, \Delta\theta\bar{q}^{SB*}(\nu_b)) \cup (\Delta\theta\underline{q}^*, \infty)$, then the firm's profit is strictly larger when the signal is adopted rather than when it is not adopted (i.e. $\pi_a > \pi_n$). If $U_0 \in [\Delta\theta\bar{q}^*, \Delta\theta\underline{q}^*]$, then the second-best contract without adopting the signal entails no output distortion and no excess information rent given up. Therefore, we should focus on the case of $\Delta\theta\bar{q}^{SB*}(\nu_b) \le U_0 \le \Delta\theta\bar{q}^*$.

Proposition 3: When $\Delta\theta\bar{q}^{SB*}(\nu_b) \le U_0 \le \Delta\theta\bar{q}^*$, the binary signal σ creates no strictly positive value for the firm, although the second-best contract for the case whereby the signal is not adopted entails some output distortion for the bad type agent in this region.

It is easy to see that, while U_0 falls into the interval, the expected profits of adopting and not adopting the binary signal are the same, i.e. $\pi_a = \pi_n$. The reasons are as follows. First, if the signal is not adopted, then the optimal second-best contract entails no output distortion and no excess information rent received by the good type. Hence, if the signal is adopted and $\sigma = \sigma_g$, then the signal does not entail any additional value. Second, for the case where the signal is adopted and $\sigma = \sigma_b$, the firm knows *ex post* that the probability of confronting a good agent is lower, and hence the fear of giving up information rent for the good agent is accordingly lower. However, the information rent obtained in the optimal second-best contract corresponding to regime r1 is not high enough to induce participation of the good agent. This prompts the firm (that adopts the signal) to take the same action (as when the signal is not adopted) in order to trade off output efficiency against rent extraction.

Note that for any signal σ' with $\mu_g' = \mu_g$ and μ_b'<1, the firm's profit of adopting the signal is always weakly less than the profit for the case where the signal is σ. It is quite clear that the signal σ' does not always create a strictly positive value for the firm either. Moreover, it is easy to see that any binary signal does create a strictly (more or less) positive value if there is no difference between both types' status quo utility levels.

We interpret μ_g here as an index for the informativeness of a signal of this kind, like σ. When μ_g is increasing, the posterior belief ν_b becomes closer to zero, $\bar{q}^{SB*}(\nu_b)$ turns closer to the first-best output level $\bar{q}^*$ by (13), and hence the length of the interval $[\Delta\theta\bar{q}^{SB*}, \Delta\theta\bar{q}^*]$ gets shorter. This observation is recorded as Proposition 4.

Proposition 4: The higher μ_g is, the less often the binary signal σ creates no strictly positive value.

4. Conclusion:

We propose a contractual game to analyze whether an (*ex ante*) non-verifiable imperfect informative binary signal always create a strictly positive value for a firm under asymmetric information. Owing to the difference between the status quo utility levels of a good agent and a bad agent, we find that a firm adopting a non-verifiable imperfect informative binary signal does not necessarily change its action (on trading off output efficiency against rent extraction), and hence the signal does not always create a strictly positive value for the firm. Furthermore, if the binary signal σ contains a higher probability for the occurrence of a good agent, then the chance is less likely that the signal σ creates no strictly positive value.

Our work differs from Riordan and Sappington (1988)'s well-known result about the *ex post* verifiable signal which asserts that, under mild assumptions, the agent receives no excess rent whatever his type is, and that the complete information optimal output levels can be implemented. Hence, the *ex post* verifiable signal does create a strictly positive value for the firm whenever the second-best contract - for the case when the signal is not adopted - entails some output distortion and/or some excess information rent(s) given up for some type(s). An ongoing investigation is aimed at discerning the impacts of various improvements in a firm's more general information system on the optimal contract under more general type-dependent status quo settings.

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