Continuing Conflict and Stalemate: A note

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Abstract

This note is about the possibility of a stalemate in a continuing conflict. Following the prevailing economic literature on the topic, under some assumptions, the outcome of a conflict can be described in two ways: (i) a predetermined split of the contested output; (ii) a winner-take-all contest where the winning agent is capable to grab all the contested stake. By contrast, in reality many disputes do not have a clear or a definite outcome. A stalemate can end the conflict with the result of a draw. To allow for a stalemate some formal modifications to the classical Hirshleifer's model of conflict are needed. In particular, since the cornerstone of the economic literature on conflict is the Contest Success Function the possibility of a stalemate can be captured through a modified form of the CSF as axiomatized by Blavatskyy (2004). When a stalemate can emerge, results show that the scenario exhibits a higher degree of violence.

1. Introduction

This note is about the possibility of a stalemate in a continuing conflict. A conflict can be defined as: *a destructive interaction which involves strategic interdependent decisions in the presence of coercion and anarchy.* Jack Hirshleifer pioneered the work on modeling conflict, whose foundations are in Hirshleifer (1987, 1988, and 1989). The economic theory of conflict[1] rests upon the assumption that agents involved in conflict interactions have to choose an optimal level of resources devoted to the unproductive activity of conflict which is necessarily detrimental for welfare. The stake of the conflict is interpreted as a joint production which depends on the productive efforts of the agents and the cost function is represented by the foregone production.

Following the prevailing literature, under some assumptions the outcome of a conflict can be described in two ways: (i) a predetermined split of the contested output; (ii) a winner-take-all contest where the winning agent is capable to grab all the contested stake. In both cases the outcome of conflict is definite and have a clear outcome. By contrast, in reality many disputes do not have a clear or a definite outcome. A stalemate can end the conflict with the result of a draw. The occurrence of stalemates is a common feature of international interactions. As can be simply verified in the Militarized Interstates Disputes dataset maintained within the Correlates of War2 project at the Pennsylvania State University[2], a large part of militarized disputes (40%) over the period 1816-2001 resulted in a stalemate. A stalemate shapes a scenario where there is neither a clear victory of one party nor an effective conflict resolution mechanism.

To allow for a stalemate some formal modifications to the classical Hirshleifer's model of conflict are needed. In particular, cornerstone of the economic literature on conflict is the Contest Success Function (henceforth CSF for brevity).[3] Therefore, the existence of a stalemate can be captured through a modified form of the CSF as axiomatized by Blavatskyy (2004).

The paper is organised as follows: in a first section the Hirshleifer's basic model will be expounded. It is slightly modified with respect to the original version. This does not affect the main classical results. In a second section, the classical basic model is enriched in order to capture the emergence of a stalemate. In a third section, results of the foregoing sections are simply compared in order to highlight the impact of a possibility of a stalemate upon the destructiveness of conflict. Eventually the concept of statistical entropy will be applied as a novel measurement tools for conflicts. A final section summarizes the results and provides a tentative interpretation of the results.

2. HIRSHLEIFER'S CLASSICAL MODEL OF CONFLICT

In the classical Hirshleifer's model of continuing conflict two risk-neutral agents indexed by $i = 1,2$ make simultaneous (à la Nash-Cournot) and once-and-for-all choices about their own allocation of resources between 'butter' and 'guns'. Each agent is endowed with an initial

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[1] In more recent years several studies extended Hirshleifer's basic model. See among others: Grossman (1991/1998), Skaperdas (1992), Neary (1997), Anderton et al. (1999), Garfinkel and Skaperdas (2000), Dixit (2004), Spolaore (2004), Caruso (2006). The literature on the economics of conflict has been recently surveyed and deeply expounded in Garfinkel and Skaperdas (2007).

[2] The dataset is available at http://cow2.la.psu.edu/ (accessed March 2007). See also Bremer at al. (2004).

[3] Selective seminal contributions are by Tullock (1980), O'Keeffe et al. (1984), Rosen (1986) and Dixit (1987). See then Skaperdas (1996) and Clark and Riis (1998) for a basic axiomatization. See also Amegashie (2006) and Peng (2006).

positive endowment of resources, $R_i \in (0,\infty)$, which can be converted into 'guns', or 'butter' according a Resources Partition Equation defined by:

$$R_i = g_i + y_i \tag{1}$$

where $g_i \in (0,\infty)$ and $y_i \in (0,\infty)$ denote 'guns' and 'butter' respectively. A contestable output – say the 'pie' - is determined through an aggregate production function, denoted by y, which is a simple linear additive function:

$$y(y_1, y_2) = y_1 + y_2 \tag{2}$$

Then, the resources allocated to productive activities determine a total contestable output, that is to be distributed according the resources allocated to 'guns'. Let me assume that agent 1 has a larger initial endowment than agent 2: $R_1 > R_2$. In particular, for sake of simplicity I set $R_1 = 1$ whereas agent 2's endowment is assumed to be a fraction $\delta \in (0,1)$ such that $R_2 = \delta R_1 = \delta$. Eventually the contestable 'pie' becomes:

$$y = 1 - g_1 - g_2 + \delta \tag{3}$$

The outcome of the conflict is determined through a Contest Success Function. It summarizes the relevant aspects of what Hirshleifer defines the *technology of* conflict. In particular, even if the CSF can take different forms, I apply the *ratio* form of the CSF.

$$p_i(g_1, g_2) = \frac{g_i}{g_i + g_j} \qquad \text{for } i = 1,2 \text{ and } j \neq i \tag{4}$$

Equation (4) is differentiable and follows the conditions below:

$$\begin{cases} p_1 + p_2 = 1 \\ \partial p_i / \partial g_i > 0 \qquad \partial p_i / \partial g_j > 0 \\ \partial^2 p_i / \partial^2 g_i < 0 \quad \partial^2 p_i / \partial^2 g_j > 0 \\ \partial^3 p_i / \partial^3 g_i > 0 \quad \partial^3 p_i / \partial^3 g_j < 0 \end{cases} \tag{4.1}$$

The functional form adopted in equation (4) implies that there is no preponderance of an agent over the other. This is of course a limiting assumption, even if many conflicts fall in this category. Under the assumption of risk-neutrality the outcome of the CSF can also denote the proportion of a deterministic appropriation of the 'pie' going to agent i for $i = 1,2$. Eventually, the income distribution equations for both agents are given by:

$$W_i = p_i(g_1, g_2)(1 - g_2 - g_2 - \delta) \tag{5}$$

The first order conditions for a maximum are:

$$\frac{\partial W_i}{\partial g_i} = \frac{(1+\delta)g_j}{(g_1 + g_2)^2} - 1 = 0, i \neq j, i = 1,2 \tag{6}$$

The second order conditions are:

$$\frac{\partial^2 W_i}{\partial^2 g_i} = -\frac{2g_j(1+\delta)}{(g_1+g_2)^3} < 0, i \neq j, i = 1,2 \tag{7}$$

Therefore, the optimal allocations to 'guns' in the classical continuing conflict scenario are given by:

$$g_1^* = g_2^* = \frac{(1+\delta)}{4} \tag{8}$$

Note that - as Hirshleifer noted in his seminal paper –there will be a critical resource ratio R_1/R_2 at which the poorer agent devotes all its resources to fighting – namely corner solutions emerge. Then, it would be possible to say that there is a critical interval $(0,\delta^*)$ such that for $\delta \in (0,\delta^*)$ there is room for corner solutions. In this simple case the upper bound is given by $\delta^* = 1/3$. Total level of 'guns' is simply defined as:

$$TG = g_1^* + g_2^* = \frac{(1+\delta)}{2} \tag{9}$$

And the level of joint production is given by:

$$y(y_1, y_2) = \frac{(1+\delta)}{2} \tag{10}$$

Eventually, in the interior Nash equilibrium the incomes for agent 1 and agent 2 are:

$$W_1^* = W_2^* = \frac{(1+\delta)}{4} \tag{11}$$

Summarizing, in the classical continuing conflict scenario conflict appears to be as a redistributive activity. The poorer agent will invest more in 'guns'. This is the Hirshleifer's argument of *Paradox of Power*. The conflict imposes a wastage of resources since that half of the endowments are devoted to 'guns' and consequently the size of the 'pie' shrinks.

3.CONFLICT AND STALEMATE
Hereafter I shall slightly modify Hirshleifer's basic model by means of a particular functional form of the CSF. It has been axiomatized by Blavatskyy (2004). This functional from admits the possibility that a stalemate can emerge between agents. The CSF takes the following form:

$$\widetilde{p}_{is} = \frac{g_{1s}}{1 + g_{1s} + g_{2s}}, i = 1,2 \tag{12}$$

Where the subscripts 's' denote the scenario with the possibility of a stalemate. The (12) follows the conditions presented in (4.1) but note that the probability of a stalemate is given by:

$$1 - \tilde{p}_1(g_{1s}, g_{2s}) - \tilde{p}_2(g_{1s}, g_{2s}) = \frac{1}{1 + g_{1s} + g_{2s}} \tag{13}$$

Hence, the income redistribution equations become:

$$W_{is} = \tilde{p}_{si}(g_{1s}, g_{2s})(1 - g_{1s} - g_{2s} - \delta) \tag{14}$$

The first order conditions for a maximum are:

$$\frac{\partial W_{is}}{\partial g_{is}} = \frac{\delta(g_{js} + 1) - (g_{1s} + g_{2s})^2 - 2g_{is} + 1}{(1 + g_{1s} + g_{2s})^2} = 0, i = 1,2, i \neq j \tag{15}$$

And the second order conditions are given by:

$$\frac{\partial^2 W_{is}}{\partial^2 g_i} = -\frac{2(\delta + 2)(g_{js} + 1)}{(1 + g_{2s} + g_{2s})^3} < 0, i = 1,2, i \neq j \tag{16}$$

The symmetric interior Nash equilibrium solutions for guns are given by:

$$g_{1s}^* = g_{2s}^* = \frac{(\delta^2 + 12\delta + 20)^{\frac{1}{2}} + \delta - 2}{8} \tag{17}$$

Note that $g_{1s}^* < 1$ whereas $g_{2s}^* < \delta \Leftrightarrow \delta > .43$. Hence, also in this case there is a critical interval $(0, \delta^{**})$ such that for $\delta \in (0, \delta^{**})$ there is room for corner solutions. In this case the upper bound is given by $\delta^{**} = .43$. In this symmetric equilibrium incomes of both agents are:

$$W_{1s}^* = W_{2s}^* = \frac{[(\delta^2 + 12\delta + 20)^{\frac{1}{2}} + \delta - 2][3(\delta + 2) - (\delta^2 + 12\delta + 20)^{\frac{1}{2}}]}{8[(\delta^2 + 12\delta + 20)^{\frac{1}{2}} + \delta + 2]} \tag{18}$$

In equilibrium the total level of guns is given by:

$$TG_s = g_{1s}^* + g_{2s}^* = \frac{(\delta^2 + 12\delta + 20)^{\frac{1}{2}} + \delta - 2}{4} \tag{19}$$

and the final joint production is given by:

$$y_s = 1 + \delta - TG_s = \frac{3(\delta + 2) - (\delta^2 + 12\delta + 20)^{\frac{1}{2}}}{4} \tag{20}$$

4. COMPARISON AND MEASUREMENT

Results of the foregoing section allows for a simple comparison between the two scenarios. First, consider the level of guns. In both scenarios, both agents devote the same amount of resources to 'guns', but in a conflict under the possibility of stalemate they arm more. Namely $g_{is}^* > g_i^*, i = 1,2$. Trivial to say that $TG_s > TG$. Moreover, in spite of a higher level of 'guns' the probability of winning the conflict is lower under the possibility of a stalemate. Namely $\tilde{p}_{is}^* < p_i^*, i = 1,2$, where $\tilde{p}_{is}^*$ and p_i^* are (12) and (4) evaluated in equilibrium respectively. Note also that differently form classical basic model under the possibility of a stalemate the total level of guns is no longer equal to the final level of production, $TG_s \neq y_s$. In particular, $TG_s > y_s$, that is the total amount of resources devoted to fighting is higher than the level of the final joint production.

Eventually let me use a simple index of *intensity of violence* (DV). By intensity of violence I mean the ratio of total level of guns on the sum of endowments. In formal terms it is possible to write:

$$DV = \frac{TG}{(1+\delta)}$$

(21)

It is clear that in the basic model $DV = 1/2$ whereas in the presence of a stalemate it is given by:

$$DV_s = \frac{(\delta^2 + 12\delta + 20)^{\frac{1}{2}} + \delta - 2}{4(1+\delta)}$$

(22)

Then, the intensity of violence is unambiguously higher in the second scenario: $DV_s > DV$. Last but not least, another point of interest is the critical interval for δ allowing for corner solutions. In particular note that $\delta^{**} > \delta^*$. This means that under the possibility of a stalemate even a less unequal resources endowments can lead to the corner solution where the poorer agent devote all its resources to fighting. To summarise it would be possible to write:

PROPOSITION 1: *Consider a conflict when agents are equal in their fighting abilities and the conflict is not decisive. Therefore: (i) the possibility of a stalemate makes the conflict more destructive. Formally $TG_s > TG$; (ii) each agent attains a lower probability of winning the conflict. Formally $\tilde{p}_{is}^* < p_i^*, i = 1,2$; (iii) the range of the Nash interior equilibrium shrinks and there is a larger room for corner solutions at which the poorer party invest all its resources in 'guns'.*

However, a conflict can be susceptible of further measurement and evaluation. In Caruso (2007a) and Caruso (2007b) I proposed a novel measurement to analyze the realm of conflicts. An appealing idea can be related to those of disorder and randomness. In fact, since conflict is a destructive interaction between two or more parties, it seems reasonable to consider also the degree of uncertainty it spreads. In actual violent appropriative conflicts, uncertainty about the final outcome does clearly constitute a characteristic element that should be considered in developing devices to solve the conflict itself. In order to capture the degree of uncertainty and disorder I apply the idea of *statistical entropy* which is commonly

adopted in communication theory and physical sciences.[4] The famous reference is the work of Shannon and Weaver (1949), which posed the quantitative foundations of information theory. Hence, entropy is defined as:

$$E(p_1,...,p_n) = -k\sum_{i=1}^{n} p_i \ln p_i,$$

(23)

where k is an arbitrary constant which can be set to unity without loss of generality.[5] The greatest disorder would occur when all outcomes have the same probability, i.e. $p_i = 1/n$ for $i = 1,...n$. The degree of disorder is given by: $E(1/n,...,1/n) = k\ln n$. For instance, in the limiting case of $n = 2$ and $k = 1$ the degree of disorder will be given by $E = \ln(2)$. However, it would also be more useful to introduce the concept of *relative entropy*. Relative entropy is defined as the ratio of the actual to the maximum entropy in a system. That is, it would be useful to recognize the extent to which the degree of disorder approaches the maximum level attainable. In formal terms it is possible to write the relative entropy as: $RE = E/Ln(n)$. Then, relative entropy in classical model of continuing conflict will exactly reaches its maximum level, namely $RE(p_1^*, p_2^*) = 1$, whereas under the possibility of a stalemate it will be

$$RE_s(\tilde{p}_{1s}^*, \tilde{p}_{2s}^*) = \frac{(D+\delta-2)Ln(D+\delta-2)-(D+\delta+2)Ln(D+\delta+2)-(D+\delta-10)Ln(2)}{(D+\delta+2)Ln(3)}$$

(24)

where $(\delta^2 + 12\delta + 20)^{\frac{1}{2}} = D$ for compactness. Note that relative entropy is inversely related to the degree of asymmetry in initial resources endowment. Being narrative, the more the agents are equal in their initial endowments the more turbulent appears to be the scenario under the possibility of a stalemate. To summarize:

PROPOSITION 2: *Consider a conflict when agents are equal in their fighting abilities and the conflict is not decisive. Therefore (i) the conflict under the possibility of a stalemate appears to be less turbulent than the classical model of conflict where relative entropy reaches its maximum level; (ii) the degree of turbulence is inversely related with the parameter capturing the asymmetry in the initial endowment. The more the agents are similar the more turbulent appears to be the scenario.*

5. PRELIMINARY CONCLUSIONS AND INTERPRETATIONS

This brief note was intended to shed light on particular aspect of conflict interactions. The emergence of stalemate in conflict interactions. In fact, differently from political science the economic theory of conflict disregarded the occurrence of a stalemate. This analysis is grounded upon a particular functional form of CSF as expounded in Blavatskyy (2004). The point of interest is that this kind of scenario exhibits a higher intensity of violence. The rationale should be that agents try to avoid the emergence of a stalemate and then increase their own level of 'guns' in order to increase their own probability of winning. A higher level of 'guns' clearly makes the interaction more destructive than the classical Hirshleifer's basic

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[4] Consider, among others, some applications of entropy to social sciences: the Nobel graduate in physic Dennis Gabor applied entropy to the measurement of social and economic freedom in Gabor and Gabor (1958). Entropy has also been proposed as a measure of competitiveness and diversification in market structure: see Attaran and Zwick (1989) and Horowitz and Horowitz (1968).

[5] The form adopted here is the one presented in Campiglio (1999), ch.4.

model. Albeit interesting, this note is nothing but a very preliminary result which has to be considered as a 'spare part' of a further analysis of conflicts.

REFERENCES

AMEGASHIE A.J., (2006), A contest Success Function with a Tractable Noise Parameter, *Public Choice*, vol. 126, pp.135-144.

ANDERTON C. (2000), An Insecure Economy under Ratio and Logistic Conflict Technologies, *The Journal of Conflict Resolution,* vol. 44, no. 6, pp. 823-838.

ANDERTON, C. H., ANDERTON R. A., CARTER J., (1999), Economic Activity in the Shadow of Conflict, *Economic Inquiry*, vol. 17, n. 1, 166-17.

ATTARAN M., ZWICK M., (1989), An Information Theory Approach to Measuring Industrial Diversification, *Journal of Economic Studies,* vol. 16, no. 1, pp. 19-30.

BLAVATSKYY P., (2004), Contest Success Function with the possibility of a draw: axiomatization, Institute for Empirical Research in Economics, University of Zurich, working paper n. 208.

BREMER S., FATEN, G., PALMER, G., (2004), The MID3 Data Set, 1993–2001: Procedures, Coding Rules, and Description, *Conflict Management and Peace Science*, vol. 21, no.2 pp. 133-154.

CAMPIGLIO L., (1999), *Mercato, Prezzi e Politica Economica*, Il Mulino, Bologna.

CARUSO R., (2007b), Reciprocity in the Shadow of Threat, paper presented at the Conference, Reciprocity, Theories and Facts, February 22-24, Verbania.

CARUSO R., (2007a), Conflict and Conflict Management with Asymmetric Stakes, (The Good-Cop and the Bad- Cop part II), paper presented at the AEA/ASSA conference 5-7, January 2007, Chicago.

CARUSO R., (2006), A Trade Institution as a Peaceful Institution? A Contribution to Integrative Theory, *Conflict Management and Peace Science,* vol. 23, no.1, pp. 53-72.

CLARK D.J., RIIS C. (1998),Contest Success Functions: an extension, *Economic Theory*, vol. 11, pp. 201-204.

DIXIT A., (2004), *Lawlessness and Economics, Alternative Modes of Governance*, Princeton, Princeton University Press.

DIXIT A., (1987), Strategic Behavior in Contests, *The American Economic Review*, vol. 77, no.5, pp. 891-898.

GABOR A., GABOR D., (1958), L'entropie comme Mesure de la Liberté Sociale et Économique, *Cahiers de L'Institut de Science Économique Appliquée,* no.72, pp. 13-25.

GARFINKEL M., (2004), On the stable formation of groups: Managing the conflict within, *Conflict Management and Peace Science*, vol. 21, no.1, pp. 43-68.

GARFINKEL M. R., SKAPERDAS S., (2007), Economics of Conflict: An Overiew, in Sandler T., Hartley K. (eds.), *Handbook of Defense Economics, (forthcoming).*

GROSSMAN H.I., (1991), A General Equilibrium Model of Insurrections, *The American Economic Review*, vol. 81, no.4, pp. 912-921.

GROSSMAN H. I., KIM M., (1995), Swords or Plowshares? A Theory of the Security of Claims to Property, *The Journal of Political Economy,* vol. 103, no. 6, pp. 1275-1288.

HIRSHLEIFER J., (1987), *Economic Behaviour in Adversity*, Brighton, Wheatsheaf Books Ltd.

HIRSHLEIFER J. (1988), The Analytics of Continuing Conflict, *Synthese*, vol. 76, no. 2, pp. 201-233. reprinted by Center for International and Strategic Affairs, CISA, University of California.

HIRSHLEIFER J., (1989), Conflict and Rent-Seeking Success Functions, Ratio vs. Difference Models of Relative Success, *Public Choice*, no. 63, pp.101-112.

HIRSHLEIFER J., (1991), The Paradox of Power, *Economics and Politics*, vol. 3, pp. 177-20. re-printed in Hirshleifer (2001), pp. 43-67.

HIRSHLEIFER J., (2001), *The Dark Side of the Force, Economic Foundations of Conflict Theory*, Cambridge University Press.

HOROWITZ A., HOROWITZ I., (1968), Entropy, Markov Processes and Competition in the Brewing Industry, *The Journal of Industrial Economics*, vol. 16, pp. 196-211.

NEARY H. M., (1997), Equilibrium Structure in an Economic Model of Conflict, *Economic Inquiry*, vol. 35, no. 3, pp.480-494.

PENG B., (2006), On the Number of Contestants and Equilibrium Individual Effort, *Topics in Theoretical Economics*, vol.6, no.1, art.14. Available at: http://www.bepress.com/bejte/topics/vol6/iss1/art14

SHANNON C.E., WEAVER W., (1949), *The Mathematical Theory of Communication*, The University of Illinois Press, Urbana.

SKAPERDAS S., (1992), Cooperation, Conflict, and Power in the Absence of Property Rights, *The American Economic Review*, vol. 82, no. 4, pp. 720-739.

SKAPERDAS S., (1996), Contest Success Functions, *Economic Theory*, vol. 7, pp. 283-290.

SPOLAORE E., (2004)*,* Economic Integration, International Conflict and Political Unions, *Rivista di Politica Economica*, vol. IX-X, pp. 3-50.

TULLOCK G., (1980), Efficient Rent Seeking, in Buchanan, J. M., Tollison R., D., Tullock G., (eds.), *Toward a Theory of the Rent-seeking Society*, Texas A&M University, College Station, pp. 97-112.