Mixed Oligopoly, Partial Privatization and Subsidization

Yoshihiro Tomaru
Graduate School of Economics, Waseda University

Abstract

White (1996), Poyago-Theotoky (2001) and Myles (2002) prove that the optimal subsidy, equilibrium output level, all firms' profits and social welfare are identical before and after privatization of a public firm in a mixed oligopolistic market. We show that we can obtain these irrelevance results even though partial privatization introduced by Matsumura (1998) is considered.

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1 Introduction

Recently a number of paper have studied "mixed markets" in which public and private firms compete. These studies assume that the public firm maximizes social welfare, defined as the sum of consumer surplus and firm profits while the private firm maximizes its own profit. Under these assumptions, the existing work has investigated the consequences of privatization. DeFraja and Delbono (1989) shows that privatization of the public firm is desirable in terms of social welfare when the number of existing private firms is large. Governmental interventions such as a production subsidy considered, however, this is not always true. By using simultaneous-moves oligopoly, White (1996) showed privatization of the public firm is fruitless because if the subsidy is utilized, then under the optimal output subsidy, all firms' output, profits and social welfare are identical before and after privatization of the public firm. Also, Poyago-Theotoky (2001) and Myles (2002) show that the optimal output subsidy is identical and profits, output and social welfare are also identical irrespective of whether (i) the public firm moves simultaneously with the private firms or (ii) the public firm acts as a Stackelberg leader or (iii) all firms behave as profit-maximizers. These are called 'irrelevance results.'

They neglect the possibility of partial privatization, however. In many cases, even though public firms are partially privatized, the public sector (or the government) holds a non-negligible share in the privatized firms. In this paper, we provide a much stronger 'irrelevance' result in the sense of being taken into account partial privatization introduced by Matsumura (1998). Focusing on simultaneous-moves game, we show that the optimal subsidy, all firms' output, profits and social welfare are identical regardless of the share in the privated firm a public sector holds.

2 The model and main result

In a single market for a homogeneous good, there exist n identical private firms and one partially privatized firm which is jointly owned by both the public and private sectors. Following earlier work, we assume demand is linear, $P = a - Q$, where $Q = \sum_{i=1}^{n} q_i + q_0$

and q_i is the output of private firm i and q_0 is the output of the privatized firm. All firms have identical technologies, represented by the quadratic cost function $C(q_j) = c + (1/2)kq_j^2$ $(j = 0, 1, 2, \cdots, n)$ and $k > 0$[1]. We assume $c = 0$ with no loss of generality since we do not consider entry issues.

Each private firm i $(i = 1, 2, \cdots, n)$ chooses its output q_i to maximize its own profit, given by

$$\pi_i = q_i \left(a - \sum_{i=1}^{n} q_i - q_0 \right) - \frac{1}{2} k q_i^2 + s q_i \,, \tag{1}$$

where s is the subsidy. The privatized firm's profit is given by

$$\pi_0 = q_0 \left(a - \sum_{i=1}^{n} q_i - q_0 \right) - \frac{1}{2} k q_0^2 + s q_0 \,, \tag{2}$$

and social welfare is given by

$$W = CS + \pi_0 + \sum_{i=1}^{n} \pi_i - s \left(\sum_{i=1}^{n} q_i + q_0 \right), \tag{3}$$

where $CS = (1/2)Q^2$ represents consumer surplus. Following Matsumura (1998), the public sector owns a share $\alpha \in [0, 1]$ of the partially privatized firm which chooses its output q_0 to maximize the weighted average of social welfare and its own profit. Let the partially privatized firm's objective be U_0. This is given by

$$\begin{aligned} U_0 &= \alpha W + (1 - \alpha)\pi_0 \\ &= \alpha \left[\frac{1}{2} Q^2 + \pi_0 + \sum_{i=1}^{n} \pi_i - s \left(\sum_{i=1}^{n} q_i + q_0 \right) \right] \\ &\quad + (1 - \alpha) \left[q_0 \left(a - \sum_{i=1}^{n} q_i - q_0 \right) - \frac{1}{2} k q_0^2 + s q_0 \right]. \end{aligned} \tag{4}$$

Note that if the privatized firm is completely privatized ($\alpha = 0$), then it becomes a private firm and sets its output to maximize its own profit. Also note that if the firm is completely nationalized ($\alpha = 1$), then it becomes a public firm and sets its output to maximize social welfare. To sum up, we can interpret $(1 - \alpha)$ as the level of privatization

[1]Our claim in this paper can be proved in more general setting by the same way as Myles (2002). To help understanding our model, we dare to use linear demand and the quadratic cost function.

of the privatized firm. The level of privatization becomes higher (lower) when α becomes lower (higher)[2].

2.1 Simultaneous-moves game

There are two stages: in stage 1 the government determines the output subsidy to maximize social welfare for a given α; in stage 2 all firms choose their output conditional on the output subsidy. We adopt subgame perfect Nash equilibrium (SPNE) as our equilibrium concept. Now we solve for the second stage equilibrium expressions first. Maximizing (1) and (4) simultaneously we obtain the Nash equilibrium for given s and α:

$$Q(s,\alpha) = \frac{[(n+1)k + n(1-\alpha) + 1]a + [(n+1-\alpha)k + (n+1)(1-\alpha)]s}{k^2 + (3-\alpha+n)k + n(1-\alpha) + 2 - \alpha}, \tag{5}$$

$$q_0(s,\alpha) = \frac{(1+k)a + [(1-\alpha)k + 1 - \alpha - n\alpha]s}{k^2 + (3-\alpha+n)k + n(1-\alpha) + 2 - \alpha}, \tag{6}$$

$$q_i(s,\alpha) = \frac{(1-\alpha+k)a + (1+k)s}{k^2 + (3-\alpha+n)k + n(1-\alpha) + 2 - \alpha}. \tag{7}$$

Setting $s = 0$ and $\alpha = 1$ in the above expressions yields the result which many existing studies such as DeFraja and Delbono (1989) obtain, that is, $q_0(0,1) > q_i(0,1)$. When setting only $s = 0$, we can also find that $q_0(0,\alpha) \geq q_i(0,\alpha)$ with equality if the privatized firm is completely privatized ($\alpha = 0$). As the level of α becomes higher, the privatized firm becomes interested in consumer surplus. The firm has an incentive to produce more than a private firm so as to increase total output and raise consumer surplus. Thus the firm's output exceeds private firm's and the privatized firm's marginal cost exceeds private firm's without the output subsidy. The effects of the output subsidy on private firm's output and total output are positive while that of the output subsidy on the privatized firm's is ambiguous.

[2]Usually, if an entity owns more than 50 % of a firm then it obtains total control. We can modify our model in such a way, but our results do not depend on how we model the objective function of the partially privatized firm.

Now consider the effect of the output subsidy on social welfare. This can be written in a general form as

$$\frac{\partial W}{\partial s} = (P - C'(q_0))\frac{\partial Q}{\partial s} + \sum_{k=1}^{n}(C'(q_0) - C'(q_k))\frac{\partial q_k}{\partial s}. \tag{8}$$

The first term in (8) represents *resource allocation effect*. The effect is the one which the subsidy generates through increase in total output. The second term in (8) represents *reallocation effect*. This effect is generated by shifting production from low-productivity firms to high-productivity firms. If price exceeds the privatized firm's marginal cost, then administering the subsidy improves social welfare through resource allocation effect, since we know that the subsidy raises total output. Similarly administering the subsidy improves social welfare through reallocation effect as long as the privatized firm's marginal cost exceeds private firm's. Subsidization raises social welfare through these two effects.

In stage 1, taking into account how all firms will react to the subsidy, the government sets the output subsidy to maximize (3). We obtain the optimal subsidy as follows:

$$s^* = \frac{a}{1+k+n}\,. \tag{9}$$

Notice that the optimal subsidy does not depend on α. Substituting (9) into (5) and (6) yields SPNE outcomes:

$$q_0(s^*, \alpha) = \frac{a}{1+k+n} = q_i(s^*, \alpha)\,, \tag{10}$$

$$Q(s^*, \alpha) = \frac{a(1+n)}{1+k+n}\,, \tag{11}$$

$$P(s^*, \alpha) = \frac{ak}{1+k+n}\,, \tag{12}$$

$$\pi_0(s^*, \alpha) = \frac{a^2(2+k)}{2(1+k+n)^2} = \pi_i(s^*, \alpha)\,, \tag{13}$$

$$W(s^*, \alpha) = \frac{a^2(1+n)}{2(1+k+n)}\,. \tag{14}$$

We find two facts from these outcomes. First, the optimal subsidy achieves the first-best as price equals marginal cost, $P = kq_j$ $(j = 0, 1, 2, \cdots, n)$. Second, these equilibrium outcomes do not depend on α. These facts may seem surprising, but it can be explained by using (8). In order to maximize social welfare, government sets the output subsidy

to equalize $\partial W/\partial s$ in (8) with zero. Thus the government decides the subsidy so as to satisfy $P = C'(q_0)$ and $C'(q_0) = C'(q_k)\,(k = 1, 2, \cdots n)$. This indicates that subsidization makes all firms' costs equal and as a result, total cost in a whole industry are minimized. It is also indicated that subsidization compels all firms to equalize their marginal cost to price. These two facts do not depend on α. Thus the SPNE does not rely on α. The preceding results are summarized in Proposition 1.

Proposition 1 *The optimal output subsidy is independent of the share of the partially privated firm that is owned by the public. Under the optimal subsidy, the output and profits of all firms are the same.*

This proposition includes the result of White (1996), Poyago-Theotoky (2001) and Myles (2002) as a special case. In fact, they compare the case where the privatized firm is completely privatized ($\alpha = 0$) to the case where it is completely nationalized ($\alpha = 1$).

3 Conclusion

This paper has demonstrated that the optimal output subsidy, all firms' profits, and social welfare are identical regardless of how many shares in a firm public sector has, in other words, the level of privatization of a public firm. This analysis has extended the irrelevance result of Poyago-Theotoky (2001) to allow for partial privatization. Our findings has been obtained for linear demand and a quadratic cost function, but these can be obtained for more general functions. In this sense, our irrelevance results are robust.

References

DeFraja, Giovanni and Flavio Delbono, "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers*, 1989, *41*, 302–11.

Matsumura, Toshihiro, "Partial Privatization in Mixed Duopoly," *Journal of Public Economics*, 1998, *70*, 473–83.

Myles, Gareth, "Mixed Oligopoly, Subsidization and the Order of Firm's Moves: An Irrelevance Result for the General Case," *Economics Bulletin*, 2002, *12*, 1–6.

Poyago-Theotoky, Joanna, "Mixed Oligopoly, Subsidization and the Order of Firm's Moves: An Irrelevance Result," *Economics Bulletin*, 2001, *12*, 1–5.

White, Mark D., "Mixed Oligopoly, Privatization and Subsidization," *Economics Letters*, 1996, *53*, 189–95.