E C O N O M I C S B U L L E T I N

A Class of Decomposable Poverty Measures With Public Transfers

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Abstract

This paper proposes a class of decomposable poverty measures. It incorporates ideas of flexible minimum basic requirement norms, relative deprivation and the presence of public transfer systems. Public transfers oftentimes take the form of implicit transfers and are not usually reflected in the reported income figures. Depending on the access and usage of public transfer systems, real consumption possibility can be very different for different individuals. This paper demonstrates that a poverty measure can be used in a straightforward manner to derive a metric to evaluate the efficiency of the public transfer systems to reach their intended targets. Some of the policy implications are also provided.

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1. Introduction

This paper shows that a properly modified poverty measure can also serve as a measure of the target efficiency of public transfer programs. Target efficiency of a public transfer system can be defined as the ability of the public authorities to effectively target the deserving population through that transfer policy. The poverty measure proposed in this paper is a member of the generalised class of decomposable poverty measures first proposed by Foster et al (1984) [henceforth, FGT(1984)]. Therefore, it satisfies many desirable properties like monotonicity, subgroup additivity etc. as argued below. Besides, it has the added power that it can be used explicitly to measure the performance of public welfare programs thus according considerable power to the proposed measure from a policy-relevant point of view.

An appropriate notion of income for the purpose of poverty measurement has always been an area of major debate in the poverty measurement literature (Sen (1976, 1992), Atkinson (1987)). Adequate income is needed to buy goods and services enough to maintain a living standard considered normatively minimally acceptable. Any person not being able to enjoy that normative minimum level of living is adjudged to be poor. Such minimum income is usually called the "poverty line." But once we fix the poverty line at any level, the measurement of poverty is relatively straightforward (Sen (1976, 1992), FGT (1984)). For example, in the head count measure of poverty it just involves counting the people below the poverty line and expressing it as a fraction of the total population.

Oftentimes, people receive subsidies and benefits from public sources in the form of implicit transfers that are not directly reflected in reported income. Therefore, to reach an appropriate income-figure, we need to adjust the reported income figure by the value of the subsidies and benefits. This is required since benefits offered by the public authorities oftentimes enter individuals' consumption basket directly thereby increasing their real welfare. Command over real consumption basket can differ widely even at the same level of reported earning with or without access to publicly subsidised goods or services. Since, real consumption basket is a true indicator of individual welfare levels, subsidy can considerably change the welfare profile of the population if we do not take this into account explicitly.

A convenient example can well-illustrate the point just made. Take two people who are earning $100.00 a month each. Person A has no access to any subsidy and he can only buy meat from the market at a rate of $4.00/pound. Person B has access to publicly provided subsidised food and she can buy meat from a public distribution shop at a subsidised rate of $2.00/pound. However, she has also the option of buying meat from the market at the same price as person A. These two people enjoy quite different command over goods. Assuming meat is the only consumption good available, real possible consumption basket of person A amounts to 25 pounds and that of person B is 50 pounds. Suppose the meat-poverty-line is set at 26 pounds per person per month. Clearly, even at the same level of reported income, while person A will be poor and person B will not be poor if she chooses to buy sufficient amount of meat from the public distribution shop.

Many countries offer subsidies towards improving welfare of their people. Such subsidies are often geared towards alleviating suffering of the poor people. The Public Distribution System (PDS) in India is one such case in point. The aim of PDS in India is to provide food security to middle and lower income residents of urban India at a cheap govt. announced price. Such prices are usually well below the normal market price. Sometimes policymakers and researchers are concerned if the deserving population is being targeted well or not (Dutta & Ramaswami 2001, 2004; Tarozzi 2005;

Umali-Deininger & Deininger 2001.) Suppose, poverty reduction is the stated goal of a hypothetical public welfare system (like PDS in India). We may be interested to know if the program is working well or not in terms of reducing poverty.

The basic concepts of this paper is also applicable to cases of spatial variation of prices especially induced by subsidies. It is also applicable to contexts where different segments of the population faces policy mandated price variation in essential commodities (like food, fuel, medicine, housing etc.) For example, in a developed country the USA, publicly subsidized housing and medical services are quite prevalent (Grigsby & Bourassa 2003.)

Sen (1976, 1992) proposed some properties that an "ethical" measure of poverty should satisfy. I'll discuss them in the context of the current paper in Section 3. It will be shown that the measure proposed in this paper satisfies the "ethical" properties proposed by Sen (1976, 1992). We will closely follow the formulation in FGT(1984) and extend their ideas towards a more direct measure that can measure poverty and target efficiency of public transfer systems simultaneously. FGT (1984) remains a seminal study in the literature of poverty measurement in that it allows us to explicitly address the question of sub-group poverty and study the contribution of group specific poverty to the total population poverty.

The poverty measure proposed in this paper may be considered as an extension of the seminal paper, FGT (1984.) As will be elaborated below, this paper extends the notion of income in FGT (1984) to the domain of consumer expenditure based poverty measurement especially in the situation where some implicit transfer through subsidy is not explicitly accounted. Thus the message of this paper is complements FGT (1984.)

Rest of this paper is organised in the following way: Section 2 provides the basic definitions used in this paper and proposes construction of a particular poverty measure; Section 3 discusses the properties of the proposed measure; Section 4 discusses how the proposed measure could be used to evaluate the efficiency of different public transfer regimes and Section 5 concludes the paper.

2. The Measure

Consider an economy of N people where the income of the person i is denoted by x_i, $i = 1,2,....,N$. Let $x = (x_1, x_2, ..., x_N)$ be an ordered vector of personal incomes in increasing order. There is a government/public-authority that indulges in public support activities with direct benefits accruing to the individuals. To support its activities the govt. raises taxes. Therefore, the income of the individual may be viewed as a *net disposable income*.

Public support projects may include Public Distribution System (PDS), education subsidy, public health provision, unemployment benefit, social security transfer etc. In this paper I am concerned more with the kind of transfers that directly affect consumption of an individual. Clearly, some of the transfers mentioned above involve direct nominal transfer & others take the form of implicit subsidy. To bring them to the same platform, we need to be concerned only with the amount of benefit measured in nominal (monetary) terms, going to the individual *i*. *x* therefore, shall include incomes from labour or services or from accumulated assets & not any public transfer.

Suppose, individual *i* needs a socially acceptable normative minimum amount of $\overline{W_i}$. Among other things $\overline{W_i}$ depends on age, gender, health condition, physical abilities, nature of occupation etc. of the concerned individual. Clearly, this is the most generalised framework in that it allows for a

completely different poverty line for each individual. In real world, poverty lines are usually kept constant over population groups. The framework proposed here is easily amenable to multiple poverty lines defined over mutually exclusive subgroups.

It is assumed that the socially acceptable minimum norm to be exogenously given. Contingent upon the same demographic and other criteria mentioned above, i^{th} individual is allowed a maximum public benefit of $\bar{q}_i$. This is measured in the same nominal units as income. It is also assumed that $\bar{q}_i$ is exogenously given. Let, individual *i* avail a public benefit of q_i where $q_i \leq \bar{q}_i$. Therefore, the non-usage of public benefit (q_i^e) can be defined to be: $q_i^e = \bar{q}_i - q_i \geq 0$.

Let, $W_i = x_i + q_i \equiv$ *effective earning of the individual i*. Therefore, for individual *i*, shortfall of income from the socially acceptable normative minimum income requirement may be defined as:

$$\begin{aligned} &\overline{W_i} - W_i \\ &\equiv \overline{W_i} - x_i - q_i \equiv (\overline{W_i} - \bar{q}_i) - (x_i - q_i^e) \\ &\equiv A_i - B_i \equiv Z_i \end{aligned}$$

Where $A_i \equiv \overline{W_i} - \bar{q}_i$ & $B_i \equiv x_i - q_i^e$

A_i, therefore, is the income needed net of maximum possible public benefit available to live at the socially acceptable normative minimum level. The explanation for B_i is not as straightforward as A_i. B_i may be interpreted as the nominal loss that individual *i* suffers due to the non-usage of public benefit. If in some situation, $q_i = \bar{q}_i$, the individual suffers no loss and enjoys all the possible benefits that she is entitled to coming from her personal income and public transfer. Now, we can characterise the poor and non-poor in the population in the following manner:

Individual i is poor if $A_i \geq B_i$ or, $(\overline{W_i} - \bar{q}_i) \geq (x_i - q_i^e)$ .....(1)

Individual i is non-poor $A_i < B_i$ or, $(\overline{W_i} - \bar{q}_i) < (x_i - q_i^e)$ .....(2)

(See Appendix for a formal discussion).

Let's define an Efficient Public Transfer Regime (EPTR, henceforth) as a situation where $\bar{q}_i \leq \overline{W_i} - x_i$ & $\bar{q}_i \geq 0$. In other words, under EPTR, transfers are directed only to them who are living below the socially acceptable normative minimal level. (We shall see the use of this assertion in section 3). This also means that the public authorities need not provide any transfer to those for whom $\overline{W_i} \leq x_i$.

The EPTR assumption is quite logical given the fact that any public transfer system involves drain of costly resources on the part of the public authorities and may subsequently involve higher economic burden for the population as a whole. It is also assumed that the sole idea behind any public transfer system is to ensure some basic minimum level of living for the population. But, more often than not, public transfers are also directed to people who are living above the socially acceptable normative minimum level. But, in this paper I shall only be concerned about the transfers going to the needy ones. A direct example of such transfer would include poverty alleviating subsidies. Therefore, efficiency of a public transfer system shall only try to capture the ability of public authorities to reach the needy (poor) people and eliminating poverty from the economy.

Clearly, Z_i is a function of $\overline{W_i}, \overline{q_i}, x_i \& q_i$. I allow the functional forms to be different for different individuals to accommodate for the differences in individual tastes and needs. Although such functional forms will not play any role in the context of the current paper, I will assume that they exist in the background. Note that $\overline{W_i}, \overline{q_i} \& x_i$ are given for an individual and he is only allowed to vary q_i. Clearly, this is an ex-post way of looking at things. I am not taking into account the problems of endogenous labor supply, occupational choice and hence income. Similarly, it is implicitly assumed that it is costless to monitor individual needs and hence we can figure out the basic minimum needs of any given individual. Again, such an assumption need not be that extreme. All we need are different "poverty lines" for different segment of the population.

People may differ in their notion about public transfer system. They may therefore choose not to utilise the benefits. Distance, waiting time, time of distribution, insufficient quality etc. may prompt people to stay away from public distribution of grains, for example. It may also be the case that no distribution shop is available in a wide area. People may not take the benefit of the transfer system because they do not feel any need for it. There may be social stigma attached towards participation in such transfer systems. In a society where interpersonal income may not be commonly known, the observable fact that somebody participates in the public transfer system may give out the credible information that a particular person is poor and the person under consideration may not like it. However, due to the presence of binding wealth/consumption constraints, poor people are always assumed to be better off if they use the public transfers.

Let K be the number of poor in the economy. Now I propose a measure $P_\alpha(.)$ defined as $P_\alpha(.) = \frac{1}{N}\sum_{i=1}^{K}(Z_i)^\alpha$. This measure is akin to FGT(1984) measure. But, given the special kind of construction we followed above, it is significantly wider in its scope. In the above expression α can be interpreted as measure closely resembling Atkinson-ian measure of poverty aversion. A large α gives more weight to the poor. As α approaches infinity, the poverty measure approaches an ideal "Rawlsian" measure where only the poorest person (a situation where absolute deprivation is maximum) is given maximum weight. Also note that the concept of relative deprivation of poverty is embedded in the poverty measure. I will come back to the point in section 3.

The poverty measure proposed is not bounded in [0, 1]. In fact, given the current construction it takes any non-negative value. This might look a little unattractive at the outset. But this poses no particular problem as such. Please note that an alternate specification of $P_\alpha(.) = \left[\frac{1}{N}\sum_{i=1}^{K} Z_i^\alpha\right] / \left[\frac{1}{N}\sum_{i=1}^{K}\overline{W}_i^\alpha\right] \in [0,1]$. As long as the socially acceptable minimum consumption amounts are exogenously given, $\left[\frac{1}{N}\sum_{i=1}^{K}\overline{W}_i^\alpha\right]$ is just a constant. Also note that this constant just the upper bound of $\left[\frac{1}{N}\sum_{i=1}^{K} Z_i^\alpha\right]$. This pertains to the extreme situation where all the poor people have nothing.

3. Properties of $P_\alpha(.)$

Sen (1976, 1992) proposed two axioms for an "ethical measure" of poverty to satisfy:

Monotonicity Axiom: *Ceteris paribus*, a reduced income for a poor person should result in an increase in the poverty measure.

Transfer Axiom: *Ceteris paribus*, a pure income transfer from a poorer to a richer person should increase the poverty measure.

Proposition 1: *For* $\alpha > 0, P_\alpha$ *satisfies monotonicity and for* $\alpha > 1, P_\alpha$ *satisfies transfer axioms.*

Proof: Since, $P_\alpha(.)$ is a monotonic function of Z_i it is sufficient to show that Z_i follows monotonicity. A reduced income for a household can be interpreted in two ways either as a reduction in x_i or as a reduction in $\bar{q}_i$. It is easy to verify that $\frac{\delta Z_i}{\delta x_i} \& \frac{\delta Z_i}{\delta \bar{q}_i} \leq 0$ meaning thereby that a reduction in income for the poorer household will lead to an increase in the poverty measure.

To check the transfer axiom part, let's consider two different individuals *j* and *k* whose shortfalls are given by Z_j and Z_k respectively where $Z_k > Z_j$. Since, the shortfall of person *k* is higher hence, *k* is poorer than *j*. Thus, $P_\alpha = \frac{1}{N}\sum_{i=1}^{K-2} Z_i{}^{\alpha} + \frac{1}{N}(Z_j{}^{\alpha} + Z_k{}^{\alpha})$ where *i, j* and *k* are different. Now, consider a transfer *t>0* from *k* to *j* (poorer person to richer person). As a result of this transfer, the shortfall of person *k* will increase and that of person *j* will get reduced. The new value of the poverty measure is now readily calculated to be

$$P_\alpha = \frac{1}{N}\sum_{i=1}^{K-2} Z_i{}^{\alpha} + \frac{1}{N}\{(Z_j - t)^{\alpha} + (Z_k + t)^{\alpha}\}$$

$$= \frac{1}{N}\sum_{i=1}^{K-2} Z_i{}^{\alpha} + (1/N)\{Z_j{}^{\alpha} + Z_k{}^{\alpha} + R\}$$

Where "*R*" contains other residual terms from the binomial expansion. It is easy to check that R is strictly positive for $\alpha > 1$. **Q.E.D**

Proposition 2: P_α *satisfies the "sub-group monotonicity axiom" proposed in FGT(1984).*

Proof: Consider *n* mutually exclusive subgroups where N_i denotes the total population in subgroup *I* and K_i denotes the number of poor people in the same. Hence,

$$P_\alpha = (1/N)[N_1(1/N_1)\sum_{i=1}^{K_1} Z_i{}^{\alpha} + N_2(1/N_2)\sum_{j=1}^{K_2} Z_j{}^{\alpha} + \ldots\ldots\ldots + N_n(1/N_n)\sum_{l=1}^{K_n} Z_l{}^{\alpha}]$$

where $i \neq j \neq \ldots\ldots \neq l$ and $\sum_{m=1}^{n} K_m = K$ and $\sum_{p=1}^{n} N_p = N$

It is easy to see that the aggregate poverty measure is a weighted average of the subgroup poverty measures. Thus, other things remaining constant, and increased poverty in any subgroup will lead to a rise in the aggregate measure and vice-versa. **Q.E.D**

FGT(1984) has laid out in detail about various attractive aspects of the "sub-group monotonicity" property. They have also pointed out that Sen measure and its variants do not satisfy this property because their overwhelming emphasis on ranking individuals.

The construction followed here allows one to incorporate maximum possible variability in the *minimum requirement* norms. This means that it is possible to conceptualise completely different norm for every individual. At one extreme it means that every person can be thought of as a different subgroup. However, the flexibility in the minimum norm requirement allows us to consider subgroups defined along ethnic, geographical or any other criteria. Also note that in the proposed poverty measure the minimum requirement norm does not appear as a normalising factor. Only deviations from the "norm" matter. So long as alternative norms are scale-preserving, the measure, appropriately scaled, will convey an invariant picture. This opens up scope for cross-sectional studies involving any degree of heterogeneity.

4. Target Efficiency Evaluation

Let us assume an Efficient Public Transfer Regime (EPTR). Target efficiency means the success of the public authority to reach the fruits of transfer to the actual target. Since, under EPTR, transfers are directed only to them who need it, this also means that given the same amount of resources available for transfer, a more successful public transfer system will reduce poverty more than a less successful one. Intuitively, success of the public transfer system should get reflected in the changes of the magnitude of poverty. The assumption that poor people are always better off availing public transfer because of the presence of the binding constraints plays a very crucial role in this context.

Under a given distributions of $\overline{W}_i s$ and $x_i s$, $Z_i \equiv \overline{W}_i - x_i - q_i \equiv C_i - q_i$ where, $C_i \equiv \overline{W}_i - x_i$. Clearly, $\frac{\delta Z_i}{\delta q_i} \leq 0$ for $q_i \in (0, \overline{q}_i)$ and for $q_i = \overline{q}_i, Z_i = 0$. It is easy to note that under EPTR, P_α is bounded below by 0. Let $P_\alpha^{\max}$ denote the value of the poverty measure when $\forall i, q_i = 0$ or, $Z_i = C_i$ for all *i*. Hence, $0 \leq P_\alpha \leq P_\alpha^{\max}$. Let us define $E = \frac{P_\alpha}{P_\alpha^{\max}}, P_\alpha^{\max} > 0$. Clearly, $E \in [0,1]$. Thus, lower the value of *E*, higher is the target efficiency of public transfer system. A completely target inefficient public transfer system is characterised by *E=1*. A value of *E=0* is attained if the public authorities have enough money available to eliminate poverty altogether and perhaps more importantly, they can target the available resources precisely without any lose.

The intuition goes in the following manner: For given distribution of income, maximum poverty is attained in a state where no public poverty alleviation system is available or even if it exists, it fails completely to reach the target population. If the poverty alleviation system reaches its targets perfectly well then, nobody will remain poor in this economy. Again, the statement just made assumes away any resource constraint at the public transfer system level. Also, it is easy to see that even with limited resources and well-targeting, *E* should be falling. That implies more and more people are effectively taken out of the poverty net through the poverty alleviation scheme.

5. Conclusion

I developed in this paper a measure of poverty taking into consideration the presence of public transfer system. Public transfers (like subsidies) allow people to consume above their current level of income. Thus a purely observed income based poverty measure in the presence of public transfer will overestimate poverty. The construction followed above also allows us to find a measure to evaluate target efficiency of the public transfer system. This measure may have a good use in lot of developing countries (like India) where a large number of public transfer systems are maintained just in the name of poor but in reality a large part of them are misdirected. Given sub-group monotonicity property of the poverty measure developed, it is possible to construct E-values for any number of sub-groups and thus there is an large scope for fine-tuning the policy measures especially if we are interested in finding out if certain groups are properly targeted or not.

Appendix

Suppose, $\overline{W_i} - q_i = x_i \& q_i^e > 0$ and hence, $\overline{W_i} - \overline{q}_i > x_i - q_i^e$. It looks as if the person concerned is earning the minimum tolerable level of earning and yet he is considered poor. But, $\overline{W_i} - W_i \equiv x_i + \overline{q}_i - x_i - q_i \equiv q_i^e > 0 \Leftrightarrow Z_i > 0 \Leftrightarrow$ the person is poor.

It can also be the case that a person is earning more than the net (of public benefit) minimum income but not being able to reach the socially acceptable normative minimum one. Technically, $\overline{W_i} - \overline{q}_i < x_i \& q_i^e > 0$ such that $\overline{W_i} - \overline{q}_i > x_i - q_i^e \Leftrightarrow \overline{W_i} > x_i + q_i$. Thus, inequality 1 is sufficient condition for a person to be poor and similarly inequality 2 is sufficient for a person to be non-poor.

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