A note on the suboptimality of right-of-first-refusal clauses

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Abstract

We show that, under independent private values, no mechanism that contains a right-of-first-refusal clause can maximize the sum of the utilities of the seller and the right-holder.

1   Introduction

A right of first refusal (ROFR) is a contract clause that provides its holder with the right to purchase an object at the highest price the seller is able to get from another buyer.[1] In essence, the clause awards a specific buyer the right to act after all her rivals have participated in some form of bidding competition.[2] ROFR clauses are broadly used in share transactions, lease contracts, partnerships and professional sports, among many other cases (see Walker, 1999, for more examples). In addition, a context where an ROFR arises naturally is that where the seller and the favored buyer are two firms in the same conglomerate.

One possible justification for introducing such a clause is that it could result in a higher joint expected surplus for the seller and the right-holder in the bidding process –while generating a negative externality on all other parties to the auction, since it creates an allocative distortion. For instance, Choi (2003) shows that adding an ROFR clause to any of the four most usual auctions (English, Dutch, first- and second-price) results in a higher joint expected utility for the seller and the favored bidder if there is only one unfavored rival. Along the same lines, Burguet and Perry (2005) study the first-price auction and conclude that, if the seller auctions off an ROFR and then conducts the auction with a favored bidder she will receive, under some conditions, a higher expected price than she would by using a standard first-price auction. However, Bikhchandani et al. (2005) examine the ROFR in the context of a symmetric sealed-bid second-price auction and find that under private values, with at least three bidders, the ROFR generates an increase in the expected surplus of the favored buyer that exactly equals the loss to the seller. With interdependent values, their joint surplus may rise or fall.

In this note, we complement those results. We show that, under independent private values, no mechanism that includes an ROFR clause can maximize the joint expected surplus of the seller and the right-holder. Adding such a clause to any given auction format, then, is jointly suboptimal for the two parties involved.

2   The suboptimality result

The owner of a single, indivisible object is selling it through an auction. For simplicity, we assume the seller attaches no value to the object. There are $N \geq 2$ risk-neutral bidders. Bidder i's valuation for the object, v_i, is distributed according to a c.d.f. F_i with support on the interval $[\underline{v}, \overline{v}]$ and a density f_i that is positive and bounded on the whole support. Bidders' valuations are independent.

We want to characterize a selling mechanism that maximizes the sum of the expected utilities of the seller and a specific buyer. Without loss of generality, we assume that the favored buyer is bidder 1. Our problem is a slight modification of the standard optimal auction problem

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[1] All our results are valid as well in the case of procurement auctions, where an ROFR is usually referred to as a meet-the-competition clause. For ease of exposition, however, in this note we will stick to the case of a seller favoring a specific potential buyer.

[2] We present here the simplest and most frequently used ROFR. For other possible versions of the clause, see Walker (1999) and Grosskopf and Roth (forthcoming).

with independent private values.[3] We solve it following the usual steps in the literature. Let $H_i(v_1, ..., v_N)$ $(P_i(v_1, ..., v_N))$ be the probability that bidder i gets the object (respectively, the price bidder i has to pay to the seller) if bidder valuations are given by $(v_1, ..., v_N)$. In addition, let $h_i(v_i)$ $(p_i(v_i))$ be the expected probability that bidder i gets the object (respectively, the expected price she pays) when her valuation is v_i, and the valuations of all other bidders are unknown.

Bidder i's expected utility when her valuation is v_i and she announces that it is v_i' is

$$\widetilde{U}_i(v_i, v_i') = h_i(v_i')v_i - p_i(v_i').$$

Besides, let

$$U_i(v_i) = \widetilde{U}_i(v_i, v_i) = h_i(v_i)v_i - p_i(v_i)$$

Then, our problem is[4]

$$\max_{\{H_i(.), P_i(.)\}_{i=1}^N} \sum_{i=1}^N \int_{\underline{v}}^{\overline{v}} p_i(v_i) f_i(v_i) dv_i + \int_{\underline{v}}^{\overline{v}} U_1(v_1) f_1(v_1) dv_1$$

subject to the standard incentive compatibility and participation constraints

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\begin{aligned}
U_i(v_i) &\geq \widetilde{U}_i(v_i, v_i') && \text{for all } i, \text{ for all } v_i, v_i' \\
U_i(v_i) &\geq 0 && \text{for all } i, \text{ for all } v_i
\end{aligned}
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Let $\widetilde{v}_i(v_i)$ be the valuation that bidder i announces optimally when her true valuation is v_i. Clearly, by incentive compatibility, it has to be true that $\widetilde{v}_i(v_i) = v_i$ and $U_i(v_i) = \widetilde{U}_i(v_i, \widetilde{v}_i(v_i))$. The envelope theorem then implies that

$$U_i'(v_i) = \frac{\partial}{\partial v_i} \widetilde{U}_i(v_i, \widetilde{v}_i(v_i)) = h_i(v_i).$$

Therefore, it follows that $U_i(v_i) = \int_{\underline{v}}^{v_i} h_i(s)ds + U_i(\underline{v})$. Stated in a way that is more convenient to us in what follows, and noting that, in the solution to our problem, $U_i(\underline{v}) = 0$ for all $i > 1$,[5] we have

$$p_i(v_i) = h_i(v_i)v_i - \int_{\underline{v}}^{v_i} h_i(s)ds \tag{1}$$

for all $i > 1$. Replacing in the objective function yields

$$\int_{\underline{v}}^{\overline{v}} h_1(v_1)v_1 f_1(v_1) dv_1 + \sum_{i \neq 1} \int_{\underline{v}}^{\overline{v}} \left[h_i(v_i)v_i - \int_{\underline{v}}^{v_i} h_i(s)ds \right] f_i(v_i) dv_i.$$

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[3]See Myerson (1981) and Riley and Samuelson (1982).

[4]This can be thought as an extension to the N-bidder context of a particular case of the analysis in Naegelen and Mougeot (1998), when there is no consumer surplus, the shadow cost of public funds is zero and the domestic firm profit weight is one.

[5]Note that $U_1(\underline{v})$ may be zero or positive in a solution to our problem. Given that we are adding the expected utilities of the seller and bidder 1, how much the latter pays (as long as incentive compatibility holds) does not affect the objective function. There is a solution, however, where $U_1(\underline{v}) = 0$.

Integrating by parts, we have

$$\int_{\underline{v}}^{\overline{v}} h_1(v_1)v_1 f_1(v_1)dv_1 + \sum_{i \neq 1} \int_{\underline{v}}^{\overline{v}} h_i(v_i)J_i(v_i)f_i(v_i)dv_i$$

where $J_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$ is bidder i's "virtual" valuation, which we assume increasing. Alternatively, we can express the objective function as

$$E_{v_1,...,v_N}\left[H_1(v_1,...,v_N)v_1 + \sum_{i \neq 1} H_i(v_1,...,v_N)J_i(v_i)\right]$$

The allocation rule that maximizes the joint expected surplus is then

$$H_1(v_1,...,v_N) = \begin{cases} 1 & \text{if } v_1 > \max_{i \neq 1} J_i(v_i) \\ 0 & \text{otherwise} \end{cases}$$

$$H_i(v_1,...,v_N) = \begin{cases} 1 & \text{if } J_i(v_i) > \max\{v_1, \max_{j \neq i} J_j(v_j)\} \\ 0 & \text{otherwise} \end{cases}$$

for $i > 1$. That is, the unfavored bidder with the highest virtual valuation gets the object unless her virtual valuation is lower than the favored bidder's *actual* valuation. In the latter case, the favored bidder gets the object. A standard revenue-maximizing auction would compare all bidders' virtual valuations and select the highest, while this mechanism replaces the favored bidder's virtual with her actual valuation in that comparison.[6] Since we are maximizing the sum of the expected utilities of the seller and the favored bidder, we can interpret v_1 as the seller's valuation. Thus, the allocation rule that follows is the same as in a revenue-maximizing auction when the seller has a positive (but not known in advance) valuation for the object.

Let us now turn to the ROFR clause. As mentioned above, the favored bidder has the right to match the highest price the seller is able to obtain from any of her rivals. Naturally, the right-holder will match whenever the highest standing price is lower than or equal to her valuation, and she will not match otherwise. Hence, if a mechanism including an ROFR maximized joint expected surplus, the price that the favored bidder would have to match to win would always be the highest among her rivals' virtual valuations. Therefore, we would necessarily have, for all $i > 1$, that $P_i(v_1,...,v_N) = J_i(v_i)$ whenever bidder i gets the object. Let $l_i(v_i) = E_{v_{-i}}[P_i(v_i, v_{-i}) \mid J_i(v_i) < \max\{v_1, \max_{j \neq i} J_j(v_j)\}]$ be the expected price that bidder i pays given that she does not get the object and her valuation is v_i. By incentive compatibility,

$$p_i(v_i) = h_i(v_i)J_i(v_i) + [1 - h_i(v_i)]l_i(v_i) \tag{2}$$

If there is a way to make an auction with an ROFR clause maximize joint expected surplus of the seller and the right-holder, both equations (1) and (2) must hold for all $i > 1$. But we know that, for those bidders, we must have

$$h_i(v_i) = F_1(J_i(v_i)) \prod_{j \neq i, j > 1} F_j(J_j^{-1}(J_i(v_i)))$$

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[6]Note as well that in our simplified setting the object is always awarded to some bidder, since the favored bidder's valuation cannot be negative.

If, as is most usual, only the bidder that gets the object pays a positive price, $l_i(v_i) = 0$ for all i, v_i. Then, it is clear that, since

$$\int_0^{v_i} h_i(s)ds \neq h_i(v_i)\frac{1 - F_i(v_i)}{f_i(v_i)}$$

equations (1) and (2) cannot be satisfied at the same time. So it follows that no standard auction with an ROFR clause can achieve joint surplus maximization.

If $l_i(v_i) \neq 0$ for some i, v_i, from (1) and (2) we have

$$U_i(v_i) = \int_0^{v_i} h_i(s)ds = h_i(v_i)\frac{1 - F_i(v_i)}{f_i(v_i)} - [1 - h_i(v_i)]l_i(v_i)$$

for all $i > 1$. Evaluating this expression at $v_i = \overline{v}$, we conclude that $U_i(\overline{v}) = 0$, which is absurd.

Hence, no auction with an ROFR clause maximizes the sum of the utility of the seller and the favored bidder. The intuition is clear. Obtaining the payment scheme that maximizes joint surplus determines the allocation rule and the expected payment of each bidder conditional on her valuation. Having an ROFR clause that satisfies joint surplus maximization, if it were achievable, would force each nonfavored bidder to pay her own virtual valuation when winning, which does not coincide with the payment rule determined by the allocation rule and incentive compatibility.

Many mechanisms implement the allocation that maximizes joint surplus, although they are necessarily more complex than adding an ROFR clause to a standard auction. For instance, the seller could ask the favored bidder to announce her valuation (either directly or by making a bid), and then conduct an English auction among unfavored bidders, with individual reserve prices set in such a way that only bidders whose virtual valuation exceeds the favored bidder's actual valuation decide to participate. The favored bidder's expected payment, of course, should follow from incentive compatibility. Alternatively, the seller could run a first -or second- price auction with an adequately chosen advantage for the favored bidder: she would lose only if a rival's bid were higher than hers by a margin that reveals that the rival's virtual valuation exceeds her actual one.

To conclude, let us note that our result on the suboptimality of ROFR clauses reinforces the conclusions in Bikhchandani et al. (2005). ROFR clauses should be explained by reasons beyond the simple one-time interaction between the seller and a favored buyer, and should not be awarded lightly by sellers.

References

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