E C O N O M I C S   B U L L E T I N

On Firms' Preferences for Product Differentiation

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Abstract

We examine firms' preferences for product differentiation when a firm has a demand−side and/or a cost−side advantage over its competitor. We show that if the magnitude of these advantages is small, then both firms prefer more differentiated products. However, if the magnitude of demand−side (cost−side) advantage is larger, then only the advantaged (disadvantaged) firm prefers more differentiated products.

Citation: Tyagi, Rajeev, (2005) "On Firms' Preferences for Product Differentiation." *Economics Bulletin,* Vol. 12, No. 17 pp. 1−8
Submitted: August 2, 2005. **Accepted:** October 6, 2005.
URL: http://www.economicsbulletin.com/2005/volume12/EB−05L10023A.pdf

1. Introduction

It is generally accepted in oligopoly theory that firms prefer to differentiate their products since that helps them avoid harsh price competition. In an influential paper, D'Aspremont et al (1979) build on Hotelling (1929) to formally show that if two symmetric firms choose values of a product characteristic before competing on prices, then they prefer to maximally differentiate on that product characteristic. This famous principle of "maximal differentiation" has since been shown to not hold in many circumstances, helping explain the presence of undifferentiated products in many markets. For example, De Palma et al (1985) show that when firms have sufficiently large uncertainty about consumer choice rules, then firms choose to minimally differentiate on the product characteristic that is under their control; Irmen and Thisse (1998) show that if firms compete on multiple product characteristics, then they need to maximally differentiate on only one of those dimensions; and Bester (1998) shows that if firms also compete on product quality that is not perfectly observable to consumers, then firms maintain high prices to signal quality and hence the need to differentiate on the observable horizontal product characteristic disappears.

One feature common to these models is that firms *share* their preferences for product differentiation – either both prefer more product differentiation or both prefer less product differentiation. The purpose of this paper is explore whether and when firms in a market may not have the same preference for product differentiation. We build a model in which one firm is stronger than the other one owing to a demand-side and/or a cost-side advantage. We show that if the extent of this advantage is small, then, consistent with the principle of maximal differentiation, both firms prefer more differentiated products. However, if the extent of this advantage is large, then firms do not share their preferences for product differentiation. Specifically, if a firm in a market has a large cost-side advantage over the other firm, then only the disadvantaged firm prefers more differentiated products; on the other hand, if a firm has a large demand-side advantage over the other one, then only the advantaged firm prefers more differentiated products.

Thus, we show that the *magnitude* and the *source* of competitive advantage of the stronger firm in a market determines *whether* firms share their preference for product differentiation and *which* firm prefers less product differentiation.

2. The Model

Consider a market with two firms, S (*strong* firm) and W (*weak* firm). The strong firm gets larger market share and profit owing to its competitive advantage on (i) the demand side, (ii) the cost side, or (iii) both the demand- and cost-sides. We explore firms' preferences for product differentiation under each of these three cases next.

2.1. Cost-Side Advantage

Let the marginal cost of production of the strong firm be c_s and that of the weak firm be $c_w > c_s$. For simplicity, we set $c_s = 0$ and $c_w = d > 0$ so that d represents the extent of cost asymmetry.

On the demand side, we use the following symmetric demand function proposed by Shubik and Levitan (1980):[1]

$$q_i = \frac{1}{2}\left[1 - p_i + \beta\left(\overline{p} - p_i\right)\right], \qquad i = \{s, w\}, \tag{1}$$

where $\overline{p} = \left(p_s + p_w\right)/2$ is the average market price and parameter $\beta \in [0, \infty]$ represents the degree of product substitutability between the two products. In particular, $\beta = 0$ represents the case where products are completely independent and $\beta \to \infty$ represents the case where products are completely substitutable. As shown in Shubik and Levitan (1980), this demand function results from the following concave utility function of the representative consumer:

$$U\left(q_s, q_w\right) = q_s + q_w - \frac{1}{2\left(1+\beta\right)}\left[q_s^2 + q_w^2 + \beta\left(q_s + q_w\right)^2\right] + I$$

where I is the numeraire good.

The profit-maximization problems for the two firms are $Max_{p_s}\left[p_s q_s\right]$, and $Max_{p_w}\left[\left(p_w - d\right)q_w\right]$, where the expressions for q_s and q_w are given in (1). The first-order conditions for the strong and weak firms are

$$p_s = \frac{2 + \beta p_w}{4 + 2\beta}, \text{ and } p_w = \frac{2 + \beta p_s + d\left(2+\beta\right)}{4 + 2\beta}.$$

Solving the first-order conditions together, we get the following equilibrium expressions:

$$p_s^* = \frac{8 + 6\beta + \beta\left(2+\beta\right)d}{\left(4+\beta\right)\left(4+3\beta\right)}; \quad p_w^* = \frac{8 + 6\beta + 2\left(2+\beta\right)^2 d}{\left(4+\beta\right)\left(4+3\beta\right)}; \tag{2}$$

$$q_s^* = \frac{\left(2+\beta\right)\left[8 + 6\beta + d\left(2+\beta\right)\beta\right]}{4\left(4+\beta\right)\left(4+3\beta\right)}; \quad q_w^* = \frac{\left(2+\beta\right)\left[8 + 6\beta - d\left(8 + 8\beta + \beta^2\right)\right]}{4\left(4+\beta\right)\left(4+3\beta\right)}; \tag{3}$$

$$\pi_s^* = \frac{\left(2+\beta\right)\left[8 + 6\beta + d\left(2+\beta\right)\beta\right]^2}{4\left(4+\beta\right)^2\left(4+3\beta\right)^2}; \quad \pi_w^* = \frac{\left(2+\beta\right)\left[8 + 6\beta - d\left(8 + 8\beta + \beta^2\right)\right]^2}{4\left(4+\beta\right)^2\left(4+3\beta\right)^2}. \tag{4}$$

From the first-order conditions and the equilibrium prices in (2), we note that in the presence of cost asymmetry, the inefficient firm's price is higher than the efficient firm's; further, this price difference is larger for larger cost asymmetry. Thus, as expected, the equilibrium quantities in (3) show that the introduction of cost asymmetry leads to inefficient firm losing

[1] This demand function is similar to the popular demand system proposed by Singh and Vives (1984).

sales and the efficient firm gaining sales; further this sales-shifting effect is larger when cost asymmetry is larger and/or products are less differentiated. In fact, when

$$d \geq \overline{d}(\beta) = \frac{8 + 6\beta}{8 + \beta (8 + \beta)}, \tag{5}$$

then the inefficient firm cannot sell anything profitably and the efficient firm becomes a monopoly. Further, note that $\overline{d}(\beta)$ decreases in β, going from 1 to 0 as β goes from 0 to ∞, and hence the sales-shifting effect is stronger for less-differentiated products.[2]

Using (4), we get

$$\text{(i)}\ \frac{\partial \pi_w^*}{\partial \beta} < 0 \text{ for } d \leq \overline{d}(\beta), \quad \text{but} \quad \text{(ii)}\ \frac{\partial \pi_s^*}{\partial \beta} \lesseqqgtr 0 \text{ for } d \lesseqqgtr \widehat{d}(\beta) \leq \overline{d}(\beta)$$

$$\text{where } \widehat{d}(\beta) = \frac{2\beta (4 + 3\beta)^2}{(2 + \beta) [64 + \beta \{80 + 3\beta (12 + \beta)\}]}.$$

Thus, we have the following result.

Proposition 1. *If firms are asymmetric on the cost-side, then (i) the weak firm always prefers more differentiated products, but (ii) the strong firm prefers less differentiated products if the extent of its cost advantage is large.*

Intuitively, decreased product differentiation hurts both firms by increasing the intensity of competition, whether firms have the same or different marginal costs. However, when firms differ in their marginal costs, then decreased product differentiation has the above-described additional effect of shifting sales from the high-cost firm to the low-cost firm, with this effect increasing in magnitude for larger cost asymmetries. As a result, while the higher-cost firm always benefits from increased product differentiation, the lower-cost firm prefers less differentiated products when the extent of its cost advantage is large.

2.2. Demand-Side Advantage

We next consider the case where both firms have the same marginal cost of production, which is set to zero, but the stronger firm has a demand-side advantage over the weak firm. Specifically, we use the following asymmetric demand function proposed by Shubik and Levitan (1980):[3]

$$q_i = w_i [1 - p_i + \beta (\overline{p} - p_i)], \qquad i = \{s, w\} \tag{6}$$

[2] The condition in (5) can alternatively be rewritten as $\beta \geq \left[3 - 4d + \sqrt{9 - 8(2 - d)d}\right]/d$, which decreases in d, going from ∞ to 0 as d goes from 0 to 1.

[3] All our qualitative results hold for other similar popular linear demand functions, such as $q_i = a_i - p_i + \beta (p_j - p_i)$, where $a_s > a_w$ in the asymmetric demand case and $a_s = a_w$ in the symmetric demand case.

where w_i is the strength of firm i, and $\overline{p} = \sum w_i p_i$ is the weighted average price in the market. Weights w_is introduce asymmetry in the demand structure, as influenced by price, and sum to unity ($\sum w_i = 1$) so that if both firms were to charge the same price, then w_i would be the firm i's market share.[4] Note that the symmetric demand function in (1) is the special case of (6) with $w_s = w_w = 0.5$. Here we let $w_s = \lambda$ and $w_w = 1 - \lambda$, with parameter $\lambda \in (0.5, 1]$ representing the extent of demand asymmetry. As before, parameter $\beta \in [0, \infty]$ represents the degree of product substitutability between the two products. Finally, as shown in Shubik and Levitan (1980), this demand function results from the following concave utility function of the representative consumer:

$$U(q_s, q_w) = q_s + q_w - \frac{1}{2(1+\beta)}\left[\frac{q_s^2}{w_s} + \frac{q_w^2}{w_w} + \beta(q_s + q_w)^2\right] + I$$

where I is the numeraire good.

The profit-maximizing problems for the two firms are now $Max_{p_i}[\pi_i = p_i q_i]$, $i = \{s, w\}$, where q_i is as in (6). The first-order conditions for the strong and weak firms are

$$p_s = \frac{1 + (1-\lambda)\beta p_w}{2[1 + (1-\lambda)\beta]}, \text{ and } p_w = \frac{1 + \lambda\beta p_s}{2[1 + \lambda\beta]}.$$

Solving these together, we get the following equilibrium expressions:

$$p_s^* = \frac{2 + (1+\lambda)\beta}{4 + \beta[4 + 3\beta(1-\lambda)\lambda]}; \; p_w^* = \frac{2 + (2-\lambda)\beta}{4 + \beta[4 + 3\beta(1-\lambda)\lambda]}; \quad (7)$$

$$q_s^* = \frac{\lambda[1 + \beta(1-\lambda)][2 + \beta(1+\lambda)]}{4 + \beta[4 + 3\beta(1-\lambda)\lambda]}; \quad q_w^* = \frac{(1-\lambda)[1 + \beta\lambda][2 + \beta(2-\lambda)]}{4 + \beta[4 + 3\beta(1-\lambda)\lambda]}; \quad (8)$$

$$\pi_s^* = \frac{\lambda[1 + \beta(1-\lambda)][2 + \beta(1+\lambda)]^2}{[4 + \beta[4 + 3\beta(1-\lambda)\lambda]]^2}; \quad \pi_w^* = \frac{(1-\lambda)[1 + \beta\lambda][2 + \beta(2-\lambda)]^2}{[4 + \beta[4 + 3\beta(1-\lambda)\lambda]]^2}. \quad (9)$$

From the first-order conditions and the equilibrium prices in (7), we note that in the presence of demand asymmetry, the stronger firm's price is higher than the weaker firm's; further, this price difference is larger for larger demand asymmetry. Then, since the price difference between the firms affects their sales more when products are less differentiated, the weaker firm can benefit from reduced product differentiation when the demand asymmetry is large.[5]

[4] The reasons for the innate demand-side asymmetry, captured by weights w_is, are exogenous to our model and can be seen, for example, as long-run factors such as differences in firms' ages, the effectiveness of their past advertising history, etc.

[5] Alternatively, using the Envelope theorem, we get $d\pi_i^*/d\beta = p_i\left[\partial q_i/\partial\beta + (\partial q_i/\partial p_j)\left(\partial p_j^*/\partial\beta\right)\right]$. Since $\partial q_i/\partial p_j > 0$ and $\partial p_j^*/\partial\beta < 0$, $d\pi_i^*/d\beta$ can be positive only when $\partial q_i/\partial\beta > 0$. Using the demand function in (6) and the fact that $p_s^* > p_w^*$ in the presence of demand asymmetry, we have $\partial q_s/\partial\beta < 0$ and $\partial q_w/\partial\beta > 0$. Therefore, $d\pi_s^*/d\beta < 0$, and $d\pi_w^*/d\beta$ can be positive.

Formally, using (9), we get

$$\text{(i)}\ \ \frac{\partial \pi_s^*}{\partial \beta} < 0, \text{ but (ii) } \frac{\partial \pi_w^*}{\partial \beta} \lesseqgtr 0 \text{ for } \lambda \lesseqgtr \widehat{\lambda}(\beta) \text{ where } \widehat{\lambda}(\beta) \in [0.667, 1).$$

We state this result below, relegating the expression of $\widehat{\lambda}(\beta)$ to the Appendix.

Proposition 2. *If firms are asymmetric on the demand-side, then (i) the strong firm always prefers more differentiated products, but (ii) the weak firm prefers less differentiated products if the other firm has a large demand advantage.*

Intuitively, decreased product differentiation hurts both firms by increasing the intensity of competition, whether firms have the same or different strengths on the demand side. However, when firms differ in their demand strengths and hence charge different prices, then decreased product differentiation has the additional effect of increasing the emphasis on firms' relative prices. Since the stronger firm charges a higher price, both these effects of decreased product differentiation hurt it and hence it always prefers more differentiated products. The weaker firm, on the other, does benefit from more emphasis on relative prices since it charges the lower price in the market, and hence it prefers less differentiated products when the other firm has a strong demand advantage.

2.3. Both Demand-Side and Cost-Side Advantages

We next confirm that both the results identified in the last Sections carry over when the stronger firm possesses both the demand-side and the cost-side advantages.

The objectives functions for the two firms are now $Max_{p_s}[p_s q_s]$, and $Max_{p_w}[(p_w - d)\, q_w]$, where the expressions for q_s and q_w are as given in (6). The first-order conditions for the strong and weak firms are

$$p_s = \frac{1 + (1-\lambda)\,\beta p_w}{2\,[1 + (1-\lambda)\,\beta]}, \text{ and } p_w = \frac{1 + \lambda\beta p_s + d\,(1+\lambda\beta)}{2\,[1+\lambda\beta]}.$$

Solving these, we get the following equilibrium outcomes:

$$p_s^* = \frac{2 + \beta\,[1 + \lambda + d\,(1-\lambda)\,(1+\beta\lambda)]}{4 + \beta\,[4 + 3\beta\,(1-\lambda)\,\lambda]}; \quad p_w^* = \frac{2\,(1+d) + \beta\,[2 - \lambda + 2d\,\{1 + \beta\,(1-\lambda)\,\lambda\}]}{4 + \beta\,[4 + 3\beta\,(1-\lambda)\,\lambda]}; \tag{10}$$

$$q_s^* = \frac{\lambda\,[1 + \beta\,(1-\lambda)]\,[2 + \beta\,\{1 + \lambda + d\,(1-\lambda)\,(1+\beta\lambda)\}]}{4 + \beta\,[4 + 3\beta\,(1-\lambda)\,\lambda]}; \tag{11}$$

$$q_w^* = \frac{(1-\lambda)\,(1+\beta\lambda)\,[2(1-d) + \beta\,\{2 - \lambda - d\,(2 + \beta\lambda\,(1-\lambda))\}]}{4 + \beta\,[4 + 3\beta\,(1-\lambda)\,\lambda]}; \tag{12}$$

$$\pi_s^* = \frac{\lambda \left[1 + \beta \left(1 - \lambda\right)\right] \left[2 + \beta \left\{1 + \lambda + d \left(1 - \lambda\right) \left(1 + \beta\lambda\right)\right\}\right]^2}{\left[4 + \beta \left[4 + 3\beta \left(1 - \lambda\right) \lambda\right]\right]^2}; \text{ and} \tag{13}$$

$$\pi_w^* = \frac{\left(1 - \lambda\right) \left(1 + \beta\lambda\right) \left[2(1 - d) + \beta \left\{2 - \lambda - d \left(2 + \beta\lambda \left(1 - \lambda\right)\right)\right\}\right]^2}{\left[4 + \beta \left[4 + 3\beta \left(1 - \lambda\right) \lambda\right]\right]^2}. \tag{14}$$

From (10)-(12), we need the following condition on the extent of cost asymmetry to ensure that both firms can sell positive quantities at positive prices:

$$d < \overline{d}_1(\beta) = \frac{2 + \beta \left(2 - \lambda\right)}{2 + \beta \left[2 + \beta\lambda \left(1 - \lambda\right)\right]}. \tag{15}$$

Using (13) and (14), we find that

$$\text{(i)} \ \frac{\partial \pi_s^*}{\partial \beta} \lesseqqgtr 0 \text{ for } d \lesseqqgtr \widehat{d}_1(\beta), \text{ and (ii)} \ \frac{\partial \pi_w^*}{\partial \beta} \lesseqqgtr 0 \text{ for } d \gtreqqless \widehat{d}_2(\beta), \text{ where } \widehat{d}_1(\beta) > \widehat{d}_2(\beta).$$

We state this result below, relegating the expressions for $\widehat{d}_1(\beta)$ and $\widehat{d}_2(\beta)$ to the Appendix.

Proposition 3. *If the strong firm has both demand-side and cost-side advantage over the weak firm, then (i) for small cost- and demand-asymmetries, both firms prefer more differentiated products; (ii) for large cost asymmetries, the stronger firm prefers less differentiated products; and (iii) for large demand asymmetries, the weaker firm prefers less differentiated products.*

The intuition remains the same as that given for Propositions 1 and 2, and is not repeated here. Figure 1 shows both firms' preferences for product differentiation.

3. Conclusion

We have shown that the magnitude and source of competitive advantage possessed by the stronger firm in a market determines whether firms share their preferences for product differentiation, and which firm prefers less differentiated products. Thus, depending on these two features of competitive advantage in a market, one can expect to see various product differentiation strategies in use by firms.

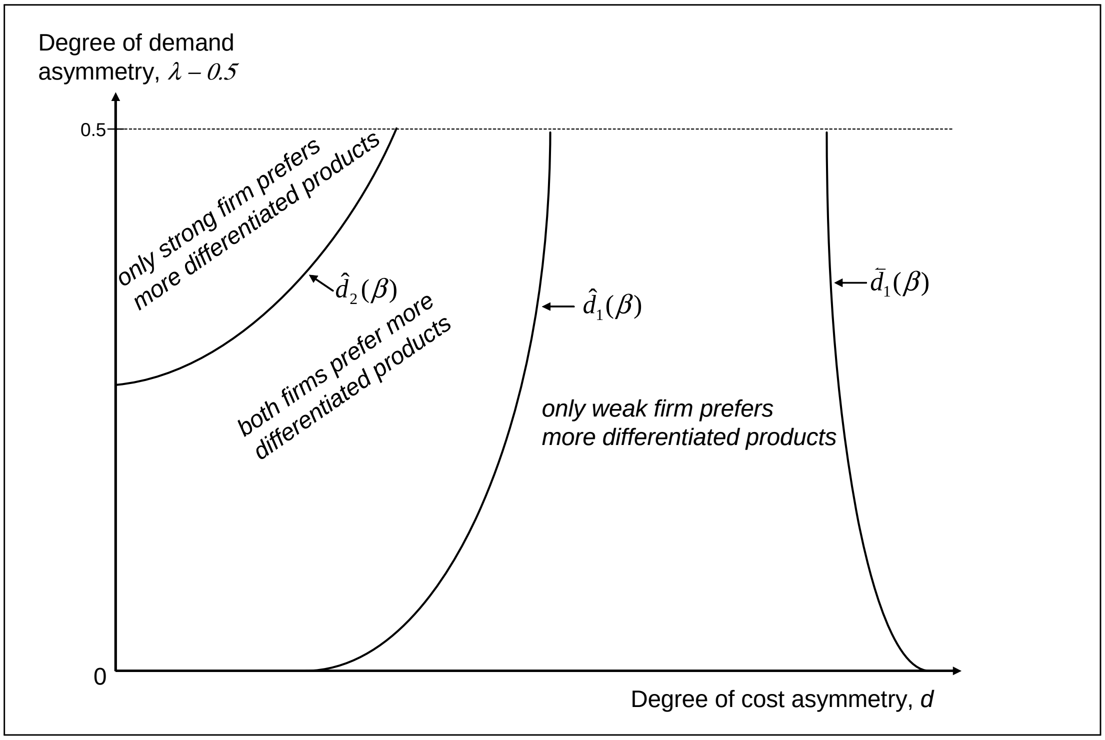

Figure 1: *Firms' Preferences for Product Differentiation.*

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Appendix

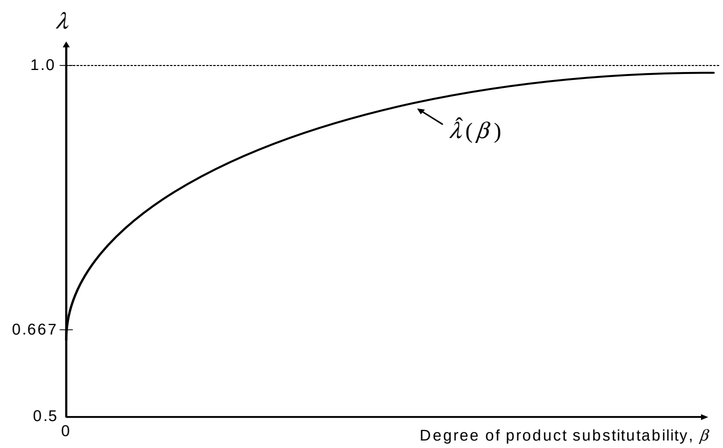
We give below some lengthy expressions that were omitted from the main text.

1. In Section 2.2

$$\widehat{\lambda}(\beta) = \frac{1}{54\beta^2}\left[18\beta\,(4+3\beta) + \frac{9\left(-1+\sqrt{-3}\right)\beta^2\,(2+\beta)\,(14+3\beta)}{X^{1/3}} - 9\left(1+\sqrt{-3}\right)X^{1/3}\right] \text{ where}$$

$$X = \sqrt{-\beta^6\left[3456+\beta\left\{9504+\beta\left(11820+\beta\left(8144+9\beta\left(340+3\beta\left(20+\beta\right)\right)\right)\right)\right\}\right]} + 2\beta^3\left[68+3\beta\left(23+6\beta\right)\right].$$

Although there are some imaginary components above, the whole expression for $\widehat{\lambda}(\beta)$ is a real number. We show its plot below.



2. In Section 2.3

$$\widehat{d_1}(\beta) = \frac{\beta\left[-4+12\lambda+\beta\left\{-4+\lambda\left(4\left(5-3\lambda\right)+3\beta\left(1-\lambda^2\right)\right)\right\}\right]}{8+\beta\left[4\left(3+\lambda\right)+\beta\left\{4+\lambda\left\{18-14\lambda+3\beta\left(1-\lambda\right)\left(3+\lambda\left(1+\beta\left(1-\lambda\right)\right)\right)\right\}\right\}\right]}, \text{ and}$$

$$\widehat{d_2}(\beta) = \frac{\beta\left[4\left(-2+3\lambda\right)-\beta\left\{4+\lambda\left\{4+3\beta\left(2-\lambda\right)\left(1-\lambda\right)-12\lambda\right\}\right\}\right]}{8+\beta\left[8\left(1+\lambda\right)+\beta\left\{4+\lambda\left\{6-2\lambda+3\beta\left(1-\lambda\right)\left(2+\beta\lambda\left(1-\lambda\right)\right)\right\}\right\}\right]}.$$