E C O N O M I C S B U L L E T I N

Cost reducing investments and spatial competition

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Abstract

In this paper we analyze the relationship between competition and cost reducing investments in the context of a location model. In particular, we derive the symmetric subgame−perfect equilibrium of a three−stage circular city model with closed−loop strategies, and study the effects of changes in competition fundamentals under both a given number of firms and free entry

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1. Introduction

The relationship between market competition and cost reducing investments (henceforth CRIs) has been widely analyzed in both the cases of Cournot and Bertrand competition[1]. However, in the context of location models, this issue has been investigated, to the best of our knowledge, only for the case of open-loop strategies by Raith (2003), who considers CRIs in the form of non-observable incentives to managers to exert effort. In this paper, we generalize the analysis by allowing for more general functions of marginal cost and CRI cost and, above all, for closed-loop strategies. We set up a three-stage spatial competition model *à la* Salop (1979), where firms decide whether to enter the market and then invest in cost reduction and compete in prices. After deriving the symmetric subgame-perfect equilibrium, we study the effects of changes in competition fundamentals (entry costs, transport costs or product substitutability, market size) on equilibrium for both the short run (i.e. with a given number of firms) and the long run (free-entry).

With respect to the case of open-loop strategies, here a major difficulty lies in expressing the price reaction functions in terms of the vector of competitors' CRIs. For this purpose, we establish some relevant properties of the inverse of the reaction function coefficients matrix. Considering closed-loop strategies allows us to take into account a strategic commitment effect involving a lower investment in CRIs.

2. The model

Consider a market populated by *n* firms symmetrically located on the unit circle at $y_i = i/n$, with $i = 1, \dots, n$, and by a continuum of consumers of measure *m* uniformly distributed on the circle. Each consumer buys one unit of the good at most, provided that the surplus she derives from her purchase is nonnegative. Let *a* be the utility arising from consuming one unit of the good and *t* the unit transport cost. The surplus that a consumer located at *x* derives from purchasing from a firm located at y_i is:

$$V_i(x) = a - p_i - t(y_i - x)^2, \tag{1}$$

where p_i is the price set by firm *i* and $t(y_i - x)^2$ measures the quadratic transport costs borne by consumers[2].

Consumers choose the supplier that maximizes their surplus, subject to the rationality constraint $V_i(x) \geq 0$. Given the assumption of symmetric location, the marginal consumers of firm *i* (i.e., those

[1] A recent paper by Vives (2004) provides a comprehensive survey of the large body of work now existing on the topic.

[2] With quadratic transport costs the symmetric case can be proved to supply an equilibrium configuration; see Novshek (1980) and Economides (1989). Most of the following results also hold with linear transport costs.

who are indifferent between the two neighboring firms *i-1* and *i+1*) are located at $x^{-} = \frac{2i-1}{2n} + \frac{n(p_i - p_{i-1})}{2t}$ and $x^{+} = \frac{2i+1}{2n} + \frac{n(p_{i+1} - p_i)}{2t}$. Therefore, if a is sufficiently high to allow all consumers to buy one unit of the good, firm individual demand is:

$$q_i = \frac{m}{n} + \frac{mn(p_{i+1} + p_{i-1} - 2p_i)}{2t} \tag{2}$$

and profits are:

$$\pi_i = [\,p_i - c(z_i)]\left[\frac{m}{n} + \frac{mn}{2t}(p_{i-1} + p_{i+1} - 2p_i)\right] - b(z_i) - F \tag{3}$$

where F are entry costs and $c(z_i)$ and $b(z_i)$ are respectively marginal costs and the cost of CRIs. In what follows we will assume $c'(\cdot) < 0$, $c''(\cdot) > 0$, $b'(\cdot) > 0$ and $b''(\cdot) > 0$.

Firms take part in a sequential three-stage game. In the first stage, each firm decides whether to enter the market. In the second and third stage firms choose CRIs and prices.

3. Equilibrium

Consider the price subgame first. At this stage, firms have already stated their CRIs so that marginal costs are known. Maximizing (3) with respect to p_i yields the reaction function of firm *i*:

$$p_i = \frac{t}{2n^2} + \frac{p_{i-1}}{4} + \frac{p_{i+1}}{4} + \frac{c(z_i)}{2} \tag{4}$$

To find the vector of equilibrium price, we therefore need to solve the system:

$$\mathbf{\Lambda p} = \mathbf{h} \tag{5}$$

where:

$$\underset{(n\times n)}{\mathbf{\Lambda}} = \begin{bmatrix} 1 & -1/4 & 0 & 0 & 0 & 0 & .... & -1/4 \\ -1/4 & 1 & -1/4 & 0 & 0 & 0 & .... & 0 \\ 0 & -1/4 & 1 & -1/4 & 0 & 0 & .... & 0 \\ 0 & 0 & -1/4 & 1 & -1/4 & 0 & .... & 0 \\ .... & ..... & ..... & ..... & ..... & ..... & .... & ..... \\ .... & .... & .... & .... & .... & .... & .... & .... \\ -1/4 & 0 & 0 & 0 & .... & 0 & -1/4 & 1 \end{bmatrix}; \quad \underset{(n\times 1)}{\mathbf{h}} = \begin{bmatrix} 1/2\left(\frac{t}{n^2} + c(z_1)\right) \\ .... \\ .... \\ 1/2\left(\frac{t}{n^2} + c(z_i)\right) \\ .... \\ .... \\ 1/2\left(\frac{t}{n^2} + c(z_n)\right) \end{bmatrix}$$

The reaction function coefficients matrix $\mathbf{\Lambda}$ is symmetric and circulant. It is positive definite and invertible, so that system (5) admits solution. As is well known, its inverse $\mathbf{\Lambda^{-1}}$ is in turn symmetric and circulant. We can therefore state the following lemma.

Lemma 1: Let λ^{ij} be a generic element of matrix $\mathbf{\Lambda^{-1}}$. For any value of *n* and *i*, the following properties hold: (1) $\lambda^{ii} > 1$; (2) $\sum_j \lambda^{ij} = 2$; (3) $0 \le \lambda^{ij} < 1$ for any $j \ne i$; (4) $\lambda^{i-1,i} = \lambda^{i+1,i}$; (5) $1 - \lambda^{i-1,i}/2 = \lambda^{i,i} - \lambda^{i-1,i} = 2\left(1 - \lambda^{i,i}/2\right)$.

Proof: Property 1 derives from the fact that positive definite matrices have $\lambda^{i,i} \ge 1/\lambda_{ii}$, with equality if and only if $\lambda_{ii} = 0 \quad \forall j \ne i$ (see Rao, 1973, page 74, property 20.2). In our case clearly $\lambda_{ii} = 1$, $\lambda_{ij} \ne 0$ for $j = i \pm 1$ and $\lambda_{ij} = 0$ otherwise, so that $\lambda^{i,i} > 1$. To prove property (2), consider a generic row of matrix $\mathbf{\Lambda}$, say, without loss of generality, the first row. Multiplying by $\mathbf{\Lambda^{-1}}$, one gets the system

$$\begin{array}{l} \lambda_1\lambda^1 + \lambda_2\lambda^n + \ldots + \lambda_{n-1}\lambda^3 + \lambda_n\lambda^2 = 1 \\ \lambda_1\lambda^2 + \lambda_2\lambda^1 + \ldots + \lambda_{n-1}\lambda^4 + \lambda_n\lambda^3 = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \lambda_1\lambda^n + \lambda_2\lambda^{n-1} + \ldots + \lambda_{n-1}\lambda^2 + \lambda_n\lambda^1 = 0 \end{array}$$

where λ_j and λ^j are respectively the *j-th* element of the first row of $\mathbf{\Lambda}$ and $\mathbf{\Lambda^{-1}}$. By summing by column, one obtains:

$$\lambda_1\sum\lambda^j + \lambda_2\sum\lambda^j + \ldots + \lambda_{n-1}\sum\lambda^j + \lambda_n\sum\lambda^j = 1$$

hence:

$$\sum\lambda^j = 1/\sum\lambda_j \ .$$

In our specific case, $\sum\lambda_j = 1/2$ whence $\sum\lambda^j = 2$. Since $\mathbf{\Lambda}$ can be seen as a Leontief matrix, its inverse has only nonnegative elements and therefore, from (1) and (2), property (3) follows. Property (4) follows from the fact that $\mathbf{\Lambda^{-1}}$ is symmetric and circulant. Finally, property (5) can be proved by multiplying $\mathbf{\Lambda}$ times $\mathbf{\Lambda^{-1}}$. Considering, without loss of generality, row *i* and column *i*, one obtains the equality $\lambda^{i,i} - \dfrac{\lambda^{i-1,i}}{4} - \dfrac{\lambda^{i+1,i}}{4} = 1$, whence by property (4) and after some trivial algebra, property (5) follows. □

On the basis of Lemma 1, we can write the price reaction function of firm *i* in terms of the CRIs of all the other firms

$$p_i = \frac{t}{n^2} + \frac{1}{2}\boldsymbol{\lambda}^i\mathbf{c(z)} \tag{6}$$

where $\boldsymbol{\lambda}^i$ denotes the i-th row of matrix $\boldsymbol{\Lambda}^{-1}$, and $\mathbf{c(z)}$ is the column vector of marginal costs $\left(c(z_1), c(z_2), ..., c(z_n)\right)$.

Moving to the CRIs stage, each firm sets its CRIs by maximizing profits under the optimal price strategy (6). Substituting (6) into (3) we obtain:

$$\pi_i = \left[\frac{t}{n^2} + \frac{1}{2}\boldsymbol{\lambda}^i\mathbf{c(z)} - c(z_i)\right]\left[\frac{m}{n} + \frac{mn}{2t}\left(\frac{1}{2}\boldsymbol{\lambda}^{i+1}\mathbf{c(z)} + \frac{1}{2}\boldsymbol{\lambda}^{i-1}\mathbf{c(z)} - \boldsymbol{\lambda}^i\mathbf{c(z)}\right)\right] - b(z_i) - F \,. \tag{7}$$

By using properties (2), (4) and (5) of Lemma 1, the conditions for z_i^* to be a maximum can be written as

$$\frac{\partial \pi_i}{\partial z_i} = -c'(z_i)\,\frac{m}{n}\left(1 - \frac{\lambda^{i-1,i}}{2}\right)\left[1 - \frac{n^2}{4t}\left(2c(z_i) - \frac{1}{2}\boldsymbol{\lambda}^{i+1}\mathbf{c(z)} - \frac{1}{2}\boldsymbol{\lambda}^{i-1}\mathbf{c(z)}\right)\right] - b'(z_i) = 0 \tag{FOC}$$

$$\frac{\partial^2 \pi_i}{\partial z_i^2} = \frac{mn}{2t}\left(1 - \frac{\lambda^{i-1,i}}{2}\right)^2\left[c'(z_i)\right]^2 - \frac{m}{n}\left(1 - \frac{\lambda^{i-1,i}}{2}\right)c''(z_i)\left[1 - \frac{n^2}{4t}\left(2c(z_i) - \frac{1}{2}\boldsymbol{\lambda}^{i+1}\mathbf{c(z)} - \frac{1}{2}\boldsymbol{\lambda}^{i-1}\mathbf{c(z)}\right)\right] - b''(z_i) < 0 \tag{SOC}$$

Limiting our attention to the symmetric equilibrium, it is easy to show that, since $\left(\partial^2\pi_i / \partial z_i^{\,2}\right)$ is monotonically increasing in *n*, a sufficient condition for the SOC to hold is that the unit transport cost is sufficiently high, i.e.

Assumption 1: $t > \dfrac{\gamma^2 m\bar{n}^2 (c'(z(\bar{n})))^2}{2[\bar{n}b''(z(\bar{n})) + \gamma m c''(z(\bar{n}))]}$

where $\gamma = 1 - \lambda^{i-1,i}/2$ and $\bar{n} = am/F$ is the largest conceivable equilibrium number of firms[3].

The symmetric optimal CRIs z^* are therefore given by

$$-c'(z^*)\frac{m}{n} + c'(z^*)\frac{\lambda^{i-1,i}}{2}\frac{m}{n} - b'(z^*) = 0 \tag{8}$$

The second term in (8) accounts for the strategic commitment effect, which is absent in the case of open-loop strategies. Due to the convexity of $c(z_i)$ and $b(z_i)$, the strategic effect involves lower CRIs, consistently with the taxonomy of Bulow *et al.* (1985). As is known from other contexts, when firms compete *à la* Bertrand, they can gain by taking actions at a prior stage that commit themselves to higher costs (see Fudenberg and Tirole, 1984).

Recalling property (2) of Lemma 1, symmetric equilibrium prices and profits are therefore

$$p^* = \frac{t}{n^2} + c(z^*) \tag{9}$$

[3] Given the assumption that the gross utility *a* is sufficiently high to allow all consumers to buy the good, the largest overall revenues are *am* and therefore the largest possible number of firms is $\bar{n} = am/F$.

$$\pi^* = \frac{mt}{n^3} - b(z^*) - F \tag{10}$$

CRIs and prices as determined by (8) and (9) constitute a unique symmetric subgame perfect Nash equilibrium. With respect to the case of open-loop, here the presence of a strategic effect on CRIs involves higher marginal costs, prices and profits.

Simple exercises of comparative statics allow us to derive the effects of changes in the fundamentals of competition on equilibrium:

Result 1 (given number of firms): *When market size m increases, CRIs increase, prices decrease and profits may increase or decrease. When the transport cost t increases, CRIs are unaffected while both prices and profits increase. When entry costs F increase, profits decrease.*

The proof is straightforward and therefore omitted. The economic intuition is simple as well: increases in *m* involve larger z^*, because with larger individual market size, the marginal benefit of CRIs increases. The consequent reduction in production costs lowers prices. The ambiguous effect on profits is due to the fact that a larger market also induces an increase in CRIs costs $b(z^*)$ which may outweigh the larger revenues. Changes in *t* and *F* do not alter marginal costs and benefits of CRIs, and so they have the same impact on prices and profits as in the standard Salop model.

4. Endogenous market structure

Moving to an endogenous market structure, we must take into account that the elements of the matrix $\mathbf{\Lambda}^{-1}$ (in particular $\lambda^{i-1,i}$) change over *n*. Numerical computations indicate that $\lambda^{i-1,i}$ is equal to 0.4 for $n = 3$, slightly decreases until $n = 7$ and then becomes approximately constant at the value 0.309.

This behavior is intuitively sensible. Since decisions are strategic complements, when the number of firms is larger than 3, the reactions of firm *i*'s closest competitors to the CRIs of firm *i* tend to be weaker because they have to take into account the reaction of their own rivals. Therefore, the under-investment in CRIs of firm *i* due to strategic effect is lower. However, the marginal impact of this "trickle-around" effect on the size of CRIs is lower and lower as the number of competitors increases, and vanishes when $n \geq 7$.

Assuming $\lambda^{i-1,i}$ constant[4] and treating *n* as a continuous variable, the impact of changes in the number of firms on the equilibrium values of CRIs can be calculated by implicitly deriving (8):

[4] This assumption is introduced only for the sake of simplicity. Numerical computations indicate that all the following results also hold for $n < 7$.

$$\frac{\partial z^*}{\partial n} = \frac{mc'(z^*)\gamma}{n^2 b''(z^*) + n\gamma mc''(z^*)} \tag{11}$$

Given the convexity of $c(z)$ and $b(z)$, increases in the number of firms lead to decreasing CRIs because the individual market size decreases.

From (9) and (10), and taking into account (8) and (11), the effects of changes in *n* on profits and prices are

$$\frac{\partial \pi^*}{\partial n} = -\frac{m}{n^2}\left[\frac{3t}{n^2} - \frac{m\gamma^2 (c'(z^*))^2}{nb''(z^*) + \gamma mc''(z^*)}\right] \tag{12}$$

$$\frac{\partial p^*}{\partial n} = -\frac{1}{n}\left[\frac{2t}{n^2} - \frac{m\gamma (c'(z^*))^2}{nb''(z^*) + \gamma mc''(z^*)}\right] \tag{13}$$

Concerning (12), Assumption 1 is sufficient to ensure that the expression in the square brackets is positive and so that profits are decreasing with *n*. In fact, even if the reduction in CRIs cost $b(z_i)$ tends to raise profits, the greater competition pushes them downward, always dominating the former effect. Conversely, changes in the number of competitors *n* has an overall ambiguous impact on prices. Indeed, due to larger marginal costs following the reduction in CRIs, prices may go up; instead, if the effect of greater competition prevails, prices will decrease. This result is different from that obtained in the case of open-loop strategy where, as the strategic commitment effect is absent, the competition effect always dominates the cost effect.

We are now able to study the impact of changes in competition fundamentals F, t and m on the number of firms under free-entry and therefore on the equilibrium values of CRIs and prices for an endogenous market structure.

Result 2 (free entry): *For higher entry costs, the free-entry number of firms decreases and therefore CRIs increase. For higher transport costs, the free-entry number of firms increases and therefore CRIs decrease. In both cases, the effect on prices is ambiguous. For larger market size, the free-entry number of firms may increase or decrease and so do CRIs and prices.*

Proof: From (12), $\partial \pi^* / \partial n < 0$. Therefore, applying the implicit function theorem, $sign\frac{\partial n^*}{\partial v} = sign\frac{\partial \pi^*}{\partial v}$, where *v* = *F*, *t*, *m*. From (10), it follows that $\frac{\partial n^*}{\partial F} < 0$, $\frac{\partial n^*}{\partial t} > 0$, while, after some algebra, $sign\frac{\partial n^*}{\partial m} = sign\left(\frac{t}{n^2} - \frac{m\gamma^2 (c'(z^*))^2}{nb''(z^*) + \gamma mc''(z^*)}\right)$. The effects of competition fundamentals on CRIs and prices follow immediately from (11) and (13). □

Therefore, when n is endogenously determined by the free-entry condition, the effects of greater competition on CRIs depend on the specific fundamental of competition that one is considering. CRIs are higher in markets with lower transport costs, but lower with lower entry costs. The effect of larger markets on CRIs is ambiguous just because a larger market size has ambiguous effects on profits and n^*. Moreover, since the impact of changes in the number of firms on prices is ambiguous, the effects of changes in F, t and m on p^* are ambiguous as well.

References

Bulow, J., J. Geanakoplos, and P. Klemperer (1985) “Multimarket Oligopoly: Strategic Substitutes and Complements” *Journal of Political Economy* **93** (3), 488-511.

Economides, N. (1989) “Symmetric Equilibrium Existence and Optimality in Differentiated Product Markets” *Journal of Economic Theory* **47** (1), 178-94.

Fudenberg, D., and J. Tirole (1984) “The Fat Cat Effect, the Puppy Dog Ploy and the Lean and Hungry Look” *American Economic Review*, *Papers and Proceedings* **74**, 361-8.

Novshek, W. (1980) “Equilibrium in Simple Spatial (or Differentiated Product) Models” *Journal of Economic Theory* **22**, 313-26.

Raith, M. (2003) “Competition, Risk and Managerial Incentives” *American Economic Review* **93** (4), 1425-36.

Rao, C.R. (1973) *Linear Statistical Inference and Its Applications*, Wiley & Sons: New York.

Salop, S.C. (1979) “Monopolistic Competition with Outside Goods” *Bell Journal of Economics* **10** (1), 141-56.

Vives, X. (2004) “Innovation and Competitive Pressure” CEPR discussion paper 4369.