E C O N O M I C S B U L L E T I N

On the role of borrowing constraints in public and private universities' choices

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Abstract

We investigate the reasons why universities use different combinations of fees and exams to guide admission decisions, focusing on the role of borrowing constraints on such decisions. On the one hand, we show that public universities choose exams and zero fees under borrowing constraints because exams are efficient allocation devices, and the objective of public institutions is the maximization of surplus. On the other hand, private universities prefer the use of fees to guide admission policies since tuition fees are not only an allocation device but also a source of revenues. Interestingly, we find that while borrowing constraints do not affect quality and admission standards in the public university, they reduce both quality and fees in the private.

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1 Introduction

The provision of higher education has some distinctive features that differentiate this market from others. While the majority of goods and services in an economy are allocated exclusively by means of prices, the allocation of students to schools is frequently guided as well by exams. The importance of exams varies depending on the country and institution. For instance, in many European countries public universities set very low fees and admissions are guided by exams while private universities are often of lower quality and use mainly fees to allocate students. In the US, however, many universities are private and non-profit organizations, and they use both fees and exams to guide admissions.

In this paper we try to explain the observed different combinations of fees and exams used by higher education institutions to guide admission decisions. For this purpose, we provide universities with an active role in the selection of students. Our analysis is based on public, surplus-maximizing, and private, profit-maximizing, universities. In our model, higher education institutions choose their quality and admission policies (a combination of fees and exams) to pursue their objectives, and we focus on the role of borrowing constraints in shaping universities' optimal choices under monopoly.

Our results fit reasonably well the features of the higher education market in many European countries. The public university prefers exams to fees under borrowing constraints because the use of exams allows this institution to maximize total surplus. Instead, the private university uses tuition fees, independently of borrowing constraints, because fees are not only an allocation device but also a source of revenues. Interestingly, both monopolies provide the same level of educational quality under perfect capital markets. However, the tuition fee, and hence, selectivity, is higher at the private institution. We find that the presence of borrowing constraints reduces both private fee and quality, but it does not affect either quality or admission standards in the public institution.

The empirical evidence on borrowing constraints in higher education is assessed in Carneiro and Heckman (2002), who argue that long run credit constraints are more important than short run constraints in explaining the correlation between parental income and college attendance. Although their quantitative importance may be different, our analysis is based on the assumption that both types of credit constraints exist. Under such assumption, Fernández (1998) and Fernández and Galí (1999) investigate the impact of borrowing constraints on the relative efficiency of fees and exams as allocation devices. However, the existence of borrowing constraints is important not only for efficiency but also for equity reasons, since it prevents poor and talented students from attending college. In this context, Hanushek, *et al.* (2004) show that different forms of subsidization of college education tend to improve the efficiency of the economy at the expense of higher income inequality. Also related to our paper is Epple and Romano (2002), in which competition among quality-maximizing universities is analyzed. Although their model explains observed admission and price policies of private institutions, they do not investigate how different universities' objectives may affect these policies.

The paper is organized as follows: Section 2 presents a model of the higher education market. Section 3 firstly analyzes monopolies' choices under perfect capital markets, and secondly, under borrowing constraints, and compares both allocations. Section 4 concludes. Proofs and lengthy computations appear in the Appendix.

2 The Model

2.1 Individuals

The economy consists of a continuum of individuals of measure one. Each individual i is characterized by a different and unobservable ability, a_i, and an initial income endowment, w_i, uniformly and independently distributed over the interval $[0, 1]$. An individual i derives utility from his total lifetime income:

$$u_j^i = w_i - f_j + h_i, \tag{1}$$

where f_j is university j's fee, and h_i is the accumulated human capital or total earnings of individual i.

Human capital is increasing in individual's ability, a_i, and in university's educational quality, Q_j, and both inputs are complements in the determination of earnings. For simplicity of computations, we assume that human capital has the following functional form:

$$h_i = a_i Q_j. \tag{2}$$

We consider that the human capital an individual obtains if he does not attend university is normalized to zero, and then, his utility is equal to his initial endowment, i.e., $U_0^i = w_i$.

2.2 Universities

There exist two types of universities that produce educational services of quality Q_j, where $j = \{b,\ v\}$ stand respectively for *public* and *private*. Educational quality may be interpreted as the *prestige* of the higher education institution.[1]

Public and private universities differ in their objectives; while the public university maximizes public surplus, the private institution maximizes profits.[2] Universities have the same cost technology, given by per-student costs, $C(Q_j)$, which are specified as follows:[3]

$$C(Q_j) = Q_j^k,\ \ k > 1. \tag{3}$$

One explanation for the convexity of costs is the following: if school quality depends positively on teacher quality, and we abstract from the existence of externalities in research among faculty members, increasingly higher wages are required to attract better teachers, provided that the supply of teacher quality is not perfectly elastic.

2.3 Allocation Mechanisms

2.3.1 Exams

Universities may use an entry exam to select the best students among those who are willing to attend the university. The exam consists of a minimum score and students who obtain a

[1] Empirical measures of school quality include pupil/teacher ratios, relative wages of teachers, education expenditures and students' performance in standardized tests.

[2] Public surplus is the difference between the sum of the earnings of students attending the public university and the costs incurred to provide education.

[3] This specification simplifies calculations and ensures the concavity of the universities' optimization problem.

score equal or higher are accepted into the university. We assume that the exam technology is able to perfectly reveal student's ability, which means that students who obtain a score higher or equal than the minimum required by the university are those of ability $a_i \geq a_j^E$, $j = \{b,\ v\}$.

2.3.2 Fees

Students may be allocated to schools also by means of tuition fees. This mechanism selects students according to their willingness and ability to pay university's fee. Individuals decide whether to attend university or remain uneducated by means of comparing their utility with and without education. Let $\hat{a}_j$ be the ability of the student who is indifferent between attending school j and remaining uneducated, i.e., $\widehat{u}_0 = \widehat{u}_j$:

$$\hat{a}_j = \frac{f_j}{Q_j}. \tag{4}$$

Students of ability $a_i \geq \hat{a}_j$ are willing to attend university j while students of ability $a_i < \hat{a}_j$ prefer to remain uneducated.

3 Universities' Optimal Choices under Monopoly

In this section we analyze universities' optimal choices of quality, fees and exams, in the case in which there is only one institution, either public or private, in the higher education market. Our benchmark is an economy with perfect capital markets, in which students can borrow any amount to finance their education investments. For simplicity, the interest rate is constant and equal to zero. Next, we introduce borrowing constraints in the economy and compare the results with those obtained for the benchmark economy. The timing of decisions is the following: in the first stage, each monopoly chooses educational quality, Q_j. In the second stage, the tuition fee, f_j, is decided. In the third stage, each institution decides whether to run an exam or accept all applications. We solve the universities' problem by backward induction.[4]

3.1 Perfect Capital Markets

3.1.1 Public University

The public monopoly aims at maximizing public surplus:

$$U_b^p = \int_0^1 \int_{a_b}^1 \left(a_i Q_b - C\left(Q_b\right)\right)\ da\ dw, \tag{5}$$

where the superscript p stands for *perfect capital markets.*

Exams. In the third stage, the public university decides whether to run an exam or accept

[4]Convavity and the solution to the universities' optimization problem are shown in the second part of the Appendix.

all applications, given f_b and Q_b. The optimal public exam determines a critical level of ability, a_b^p, that satisfies

$$\begin{array}{l} a_b^p = \arg\max\ U_b^p, \\ \text{s.t. } a_b^p \geq \widehat{a}_b. \end{array} \tag{6}$$

The optimal limiting admission grade is the following:

$$a_b^p = \begin{cases} \frac{C(Q_b)}{Q_b} & \text{if } f_b \leq C(Q_b), \\ \frac{f_b}{Q_b} & \text{if } f_b > C(Q_b). \end{cases} \tag{7}$$

Hence, the public university runs an exam if $f_b \leq C(Q_b)$, and accepts all applicants otherwise.

Fees. In the second stage, the public monopoly chooses f_b^p to maximize (5), where $a_b = a_b^p$, given by (7). We obtain the following result:

Proposition 1 *Under perfect capital markets, the public university chooses exams and charges anything below or equal to the cost, including a zero tuition fee.*

In the presence of perfect capital markets, tuition fees do not limit admissions and then, exams and fees are both efficient allocation devices because they select students according to their ability (see Fernández (1998) and Fernández and Galí (1999)). Since the optimal allocation of students is independent of the instrument chosen, the public university would choose a fee equal to the cost in case of accepting all applications. In such case, and according to (7), the public university runs an exam and charges any fee below or equal the cost.

Quality. In the first stage, the university chooses educational quality to maximize (5), where $a_b^p = \frac{C(Q_b)}{Q_b}$. The optimal level of public quality, Q_b^p, is determined as follows:

$$C'(Q_b^p) = \frac{1 + \frac{C(Q_b^p)}{Q_b^p}}{2}. \tag{8}$$

Using the specification of the cost function, given by (3), and solving (8) for Q_b^p we obtain that $0 < Q_b^p < 1$. The optimal level of public educational quality depends positively on the mean ability of the students attending the public university, $\frac{1+a_b^p}{2}$. This result illustrates the fact that students are not only consumers of higher education, but also inputs in its production, as described by Rothschild and White (1995).

3.1.2 Private University

The private monopoly aims at maximizing profits:

$$U_v^p = \int_0^1 \int_{a_v}^1 (f_v - C(Q_v))\ da\ dw. \tag{9}$$

Exams. In the third stage, the private institution chooses the critical level of ability, a_v^p, that maximizes (9) subject to $a_v^p \geq \frac{f_v}{Q_v}$. The university decides to accept all applications and

$a_v^p = \frac{f_v}{Q_v}$ if $f_v \geq C(Q_v)$, and shut down if $f_v < C(Q_v)$.

Fees. In the second stage, whenever $f_v \geq C(Q_v)$, the private monopoly chooses the fee, f_v^p, that maximizes (9), where $a_v = \frac{f_v}{Q_v}$. The optimal fee is

$$f_v^p = \frac{Q_v + C(Q_v)}{2}, \tag{10}$$

if $0 \leq Q_v \leq 1$. Otherwise, the private university decides to shut down.[5]

Quality. In the first stage, the private monopoly decides the level of educational quality that maximizes (9), where f_v is given by (10) and $a_v = \widehat{a}_v^p = \frac{f_v^p}{Q_v}$, as follows:

$$C'(Q_v^p) = \widehat{a}_v^p = \frac{1 + \frac{C(Q_v^p)}{Q_v^p}}{2}. \tag{11}$$

The following proposition compares admission policies and quality in both monopolies:

Proposition 2 *In the presence of perfect capital markets, public and private universities provide the same quality under monopoly, although the private institution is more selective than the public.*

Selectivity is measured by the ability of the least able student accepted into the university. While the public university charges any fee below or equal the cost under perfect capital markets, the private university chooses a fee above the cost in order to make positive profits. Since educational quality is equal in both universities, the higher fee chosen by the private monopoly attracts students of higher ability than those attending the public.

3.2 Borrowing Constraints

We next turn to analyze universities' choices in the presence of borrowing constraints. We assume that students cannot borrow at all to finance their investments in education, and then, only individuals with income $w^i \geq f_j$ are able to attend university j.

3.2.1 Public University

In the presence of borrowing constraints, the public institution maximizes

$$U_b^c = \int_{f_b}^1 \int_{a_b}^1 (a_i Q_b - C(Q_b)) \; da \; dw, \tag{12}$$

where the superscript c stands for *borrowing constraints.*

Exams. In the third stage, the university decides whether to run an exam or accept all applicants, taking as given f_b and Q_b. The level of ability, a_b^c, that maximizes (12) subject to $a_b \geq \frac{f_b}{Q_b}$, is the same as in the case of perfect capital markets, and is given by (7).

Fees. In the second stage, the university chooses the tuition fee anticipating the optimal choice of exams in the following stage, and taking educational quality as given. The solution to this problem is the following:

[5] Notice that $Q_v \leq 1$ is required for $f_v^p \geq C(Q_v)$ to hold since $C(Q_v) \leq Q_v$ if and only if $Q_v \leq 1$.

Proposition 3 *In the presence of borrowing constraints, admission decisions in the public university are guided by exams and the optimal tuition fee is equal to zero.*

The existence of complementarities between ability and quality implies that total income is maximized when students are allocated to schools according to their ability. The proof shows that whenever $0 \leq Q_b \leq 1$ the public institution chooses an admission policy based on exams and a zero tuition fee, and shuts down the school otherwise. The policy chosen allows the public university to maximize total surplus provided that any positive fee would prevent some poor and talented individuals from attending the university in the presence of borrowing constraints.

Quality. In the first stage, the public university decides the quality level, Q_b^c, that maximizes (12), subject to $a_b = \frac{C(Q_b)}{Q_b}$ and $f_b = 0$, whenever $0 \leq Q_b \leq 1$. The maximization problem at this stage is exactly the same as in the presence of perfect capital markets, and then, the optimal level of public quality also satisfies (8). Hence, quality and admission standards are not affected by borrowing constraints.

3.2.2 Private University

In the presence of borrowing constraints, the private university maximizes

$$U_v^c = \int_{f_v}^{1} \int_{a_v}^{1} (f_v - C(Q_v)) \, da \, dw. \tag{13}$$

Exams. The private institution chooses the ability threshold, a_v^c, that maximizes (13). As in the presence of perfect capital markets, the institution decides to accept all applicants if $f_v \geq C(Q_v)$, and shuts down otherwise.

Fees. The private institution decides the fee that maximizes (13) whenever $f_v \geq C(Q_v)$, taking private quality as given. The optimal private fee under borrowing constraints is

$$f_v^c = \frac{1 + Q_v + C(Q_v) - \sqrt{(Q_v - C(Q_v))^2 + (1 - C(Q_v))(1 - Q_v)}}{3}, \tag{14}$$

whenever $0 \leq Q_v \leq 1$. Otherwise, the private university shuts down.

Quality. The university chooses Q_v to maximize (13), where $f_v = f_v^c$ and $a_v = \widehat{a}_v^c = \frac{f_v^c}{Q_v}$. The optimal level of educational quality is implicitly defined as follows:

$$C'(Q_v^c) = \widehat{a}_v^c \left(1 - \left(\frac{f_v^c - C(Q_v^c)}{1 - f_v^c}\right)\right). \tag{15}$$

The following proposition shows how private university's optimal fee and quality are affected by the presence of borrowing constraints.

Proposition 4 *In the presence of borrowing constraints, the private university chooses a lower tuition fee and a lower level of educational quality than under perfect capital markets.*

The presence of borrowing constraints prevents some poor and talented individuals from attending the private university since this institution uses fees as allocation device. Hence, the demand of the university is lower than under perfect capital markets. The optimal reaction of the private institution to the presence of borrowing constraints is to decrease both its fee and quality to compensate for such decrease in student demand.

Summarizing, we have shown that universities' different objectives help explain why, in the presence of borrowing constraints, public universities use exams as allocation device and set a zero fee for its educational services, while private institutions prefer tuition fees. Interestingly, the presence of borrowing constraints has a different impact on public and private institutions' choices. Public quality and admission standards are not affected, but both quality and standards are lower in the private university due to the presence of borrowing constraints.

4 Concluding Remarks

In this paper we have provided an explanation, based on universities' objectives, for the different combinations of fees and exams observed across countries and higher education institutions. We have also analyzed the role of borrowing constraints in universities' decisions. In order to simplify the analysis, we have focused on two types of institutions; public and surplus-maximizing, and private and profit-maximizing universities. While our model is a first step to explain universities' admission policies, some of the assumptions made deserve some discussion.

Firstly, we have assumed that technology is the same across universities while there may be differences between public and private institutions. The reason for such assumption is to isolate the effects that universities' different objectives may have on their choices of quality and admission policies. Notice that even if private universities were more productive than public, our qualitative results would not change. In particular, differences in the choice of fees and exams, as well as in the reaction to the presence of borrowing constraints, would persist across public and private universities. Secondly, the assumption that private universities are profit maximizers may be questionable, provided that many private institutions are non-profit organizations. However, this does not mean that non-profit organizations cannot make profits, but that profits cannot be distributed to outsiders (see Winston (1999)). An alternative objective function for private universities would be the maximization of educational quality, as in Epple and Romano (2002).

The model presented here can be enriched with the inclusion of peer group effects in the human capital production function. The presence of peer effects does undoubtedly affect the equilibrium allocation of students to schools. However, the fundamental differences among public and private institutions are likely to persist. Our model also provides a useful framework to study the interaction among public and private universities when they compete for students (see Romero and Del Rey (2004)). The introduction of competition between institutions in the model may also be important to explain universities' choices and improve the understanding of higher education markets.

Appendix

A. Proofs of Proposition

Proof of Proposition 1. Under perfect capital markets, the public university decides f_b anticipating the optimal choice of exams in the next stage. We may distinguish two cases:
CASE 1. The public university runs an exam, $a_b^p = \frac{C(Q_b)}{Q_b}$ and $f_b \leq C(Q_b)$. Plugging $a_b^p = \frac{C(Q_b)}{Q_b}$ into (5) we obtain that $U_b^p = \frac{Q_b}{2} - C(Q_b) + \frac{C^2(Q_b)}{2Q_b}$. Since utility is independent of the fee, the university chooses any fee satisfying $f_b \leq C(Q_b)$.
CASE 2. The public university accepts all applicants, $a_b^p = \frac{f_b}{Q_b}$ and $f_b > C(Q_b)$. In this case, the public university chooses f_b to maximize $U_b^p = \frac{Q_b}{2} - C(Q_b) - \frac{f_b^2}{2Q_b} + \frac{C(Q_b)f_b}{Q_b}$. The fee that maximizes utility in this case satisfies $f_b = C(Q_b)$, which contradicts the acceptance of all applicants by the public university.
The optimal decision of the public university is to run an exam and set $f_b \leq C(Q_b)$. ■

Proof of Proposition 2. The first result follows directly from the comparison of (8) with (11). Since educational quality is provided at the same level by both monopolies, i.e., $Q_b^p = Q_v^p = Q^p < 1$, we observe that the private university is more selective than the public since $a_v^p = \frac{1+\frac{C(Q^p)}{Q^p}}{2} > \frac{C(Q^p)}{Q^p} = a_b^p$ provided that $\frac{C(Q^p)}{Q^p} < 1$ whenever $0 < Q^p < 1$. ■

Proof of Proposition 3. In the second stage, we have two cases:
CASE 1. The public university runs an exam, $a_b^c = \frac{C(Q_b)}{Q_b}$ and $f_b \leq C(Q_b)$.
In this case, the university chooses f_b to maximize $U_b^c = \left(\frac{Q_b}{2} - C(Q_b) + \frac{C^2(Q_b)}{2Q_b}\right)(1 - f_b)$, and the optimal fee, f_b^c, is the following:

$$f_b^c = \begin{cases} 0 & \text{if } \frac{Q_b}{2} - C(Q_b) + \frac{C^2(Q_b)}{2Q_b} \geq 0, \\ 1 & \text{otherwise.} \end{cases} \tag{16}$$

Using the fact that $C(Q_b) = Q_b^k$, we obtain that $\frac{Q_b}{2} - C(Q_b) + \frac{C^2(Q_b)}{2Q_b} \geq 0$ if $0 \leq Q_b \leq 1$. Notice that if $Q_b > 1$, the public university sets $f_b^c = 1$ and shuts down the school.
CASE 2. The public university accepts all applicants, $a_b^c = \frac{f_b}{Q_b}$ and $f_b > C(Q_b)$. The tuition fee that maximizes $U_b^c = \left(\frac{Q_b}{2} - C(Q_b) - \frac{f_b^2}{2Q_b} + \frac{C(Q_b)f_b}{Q_b}\right)(1 - f_b)$ is $f_b \leq C(Q_b)$ if $\frac{Q_b}{2} - C(Q_b) - \frac{f_b^2}{2Q_b} + \frac{C(Q_b)f_b}{Q_b} \geq 0$, and $f_b = 1$, otherwise. Notice that $f_b \leq C(Q_b)$ contradicts the fact that the public monopoly accepts all applicants.
Hence, the public university runs an exam and sets a zero fee in the presence of borrowing constraints. ■

Proof of Proposition 4. The optimal private fee under borrowing constraints is given by (14), and using $C(Q_v) = Q_v^k$, we see that $Q_v > f_v^c > C(Q_v)$ if $0 < Q_v < 1$ since in this interval $Q_v > C(Q_v)$.
From the comparison of f_v^c, given by (14), with f_v^p, given by (10), we obtain that $f_v^c \leq f_v^p$

if and only if

$$\left(1-\left(\frac{Q_v+C(Q_v)}{2}\right)\right)^2 \leq (Q_v-C(Q_v))^2+(1-C(Q_v))(1-Q_v), \tag{17}$$

which is satisfied if $0 \leq Q_v \leq 1$, and holds with strict inequality if $0 < Q_v < 1$.
Local quality maxima are the global maxima since private utility is equal to zero in the extremes of the quality interval $Q_v = \{0,\ 1\}$, and strictly positive if $0 < Q_v < 1$. Convexity of costs in school quality implies that $Q_v^c < Q_v^p < 1$ if and only if $C'(Q_v^c) < C'(Q_v^p)$. Comparing Q_v^c, given by (15), with Q_v^p, given by (11), we find that $Q_v^c < Q_v^p$ holds if and only if $\widehat{a}_v^c \left(1-\left(\frac{f_v^c-C(Q_v)}{1-f_v^c}\right)\right) < \widehat{a}_v^p$. This last inequality is satisfied since $\widehat{a}_v^c = \frac{f_v^c}{Q_v} < \frac{f_v^p}{Q_v} = \widehat{a}_v^p$ and $1-\left(\frac{f_v^c-C(Q_v)}{1-f_v^c}\right) < 1$ whenever $Q_v < 1$. Therefore, both private educational quality and tuition fees are lower in the presence of borrowing constraints. ■

B. Solution to the Universities' Optimization Problem

Public University: Perfect Capital Markets
Exams. The critical level of ability that maximizes $U_b^p = \left(\frac{1-a_b^2}{2}\right) Q_b - C(Q_b)(1-a_b)$ subject to $a_b \geq \frac{f_b}{Q_b}$ satisfies:

$$\frac{dU_b^p}{dQ_b} = -a_b^p Q_b + C(Q_b) = 0, \tag{18}$$

$$\frac{d^2U_b^p}{dQ_b^2} = -Q_b < 0. \tag{19}$$

Fees. The fee that maximizes $U_b^p = \left(\frac{1-a_b^2}{2}\right) Q_b - C(Q_b)(1-a_b)$ depends on whether $f_b \leq C(Q_b)$ or $f_b > C(Q_b)$:
CASE 1. $a_b = \frac{C(Q_b)}{Q_b}$ and $f_b \leq C(Q_b)$. Utility is independent of fees and any fee satisfying $f_b \leq C(Q_b)$ is optimal.
CASE 2. $a_b = \frac{f_b}{Q_b}$ and $f_b > C(Q_b)$. The optimal fee in this case satisfies

$$\frac{dU_b^p}{df_b} = -\frac{f_b}{Q_b} + \frac{C(Q_b)}{Q_b} = 0, \tag{20}$$

$$\frac{d^2U_b^p}{df_b^2} = \frac{-1}{Q_b} < 0. \tag{21}$$

Quality. The level of quality that maximizes $U_b^p = \left(\frac{1-a_b^{p2}}{2}\right) Q_b - C(Q_b)(1-a_b^p)$ is given by

$$\frac{dU_b^p}{dQ_b} = (1-a_b^p)\left(\frac{1+a_b^p}{2} - C'(Q_b^p)\right) = 0, \tag{22}$$

$$\frac{d^2U_b^p}{dQ_b^2} = -\frac{da_b^p}{dQ_b}\left(\frac{1+a_b^p}{2} - C'(Q_b^p)\right) + (1-a_b^p)\left(\frac{1}{2}\frac{da_b^p}{dQ_b} - C''(Q_b^p)\right) < 0. \tag{23}$$

Notice that $\frac{d^2U_b^p}{dQ_b^2}\left(Q_b = Q_b^p\right) < 0$ since $\frac{1+a_b^p}{2} - C'(Q_b^p) = 0$ and substituting $C(Q_b^p) = (Q_b^p)^k$, we obtain that $\frac{1}{2}\frac{da_b^p}{dQ_b} - C''(Q_b^p) = \frac{1}{2}\frac{d\left(\frac{C(Q_b^p)}{Q_b^p}\right)}{dQ_b} - C''(Q_b^p) = (k-1)\left(Q_b^p\right)^{k-2}\left(\frac{1}{2} - k\right) < 0$ since $k > 1$ and $Q_b^p > 0$. Hence, Q_b^p is the maximum of U_b^p.

Private University: Perfect Capital Markets

Exams. In the first stage, the private monopoly decides to accept all applicants if $f_v \geq C(Q_v)$, and shuts down otherwise.

Fees. The fee that maximizes $U_v^p = (f_v - C(Q_v))\left(1 - \frac{f_v}{Q_v}\right)$ satisfies

$$\frac{dU_v^p}{df_v} = 1 - 2\frac{f_v^p}{Q_v} + \frac{C(Q_v)}{Q_v} = 0, \tag{24}$$

$$\frac{d^2U_v^p}{df_v^2} = \frac{-2}{Q_v} < 0. \tag{25}$$

Hence, $f_v^p = \frac{Q_v + C(Q_v)}{2}$ is the maximum of U_v^p whenever $0 \leq Q_v \leq 1$, which is required for $f_v^p \geq C(Q_v)$ to hold.

Quality. The level of quality that maximizes $U_v^p = \left(f_v^p - C(Q_v)\right)\left(1 - \widehat{a}_v^p\right)$, where $\widehat{a}_v^p = \frac{f_v^p}{Q_v} = \frac{1+\frac{C(Q_v)}{Q_v}}{2}$, is determined as follows:

$$\frac{dU_v^p}{dQ_v} = -C'(Q_v^p)\left(1 - \widehat{a}_v^p\right) + \widehat{a}_v^p\left(\widehat{a}_v^p - \frac{C(Q_v^p)}{Q_v^p}\right) = 0, \tag{26}$$

$$\begin{aligned}\frac{d^2U_v^p}{dQ_v^2} = & -C''(Q_v^p)\left(1 - \widehat{a}_v^p\right) - \frac{d\left(\frac{C(Q_v^p)}{Q_v^p}\right)}{dQ_b}\widehat{a}_v^p \\ & + \frac{d\widehat{a}_v^p}{dQ_v}\left(C'(Q_v^p) + 2\widehat{a}_v^p - \frac{C(Q_v^p)}{Q_v^p}\right).\end{aligned} \tag{27}$$

Substituting $\widehat{a}_v^p = \frac{f_v^p}{Q_v} = \frac{1+\frac{C(Q_v^p)}{Q_v^p}}{2}$ into (26) we obtain that $C'(Q_v^p) = \widehat{a}_v^p = \frac{1+\frac{C(Q_v^p)}{Q_v^p}}{2}$ since $1 - \widehat{a}_v^p = \widehat{a}_v^p - \frac{C(Q_v^p)}{Q_v^p}$. Using this result we find that $\frac{d\widehat{a}_v^p}{dQ_v}\left(2\widehat{a}_v^p\right) = \frac{d\left(\frac{C(Q_v^p)}{Q_v^p}\right)}{dQ_b}\widehat{a}_v^p$ and $C'(Q_v^p) - \frac{C(Q_v^p)}{Q_v^p} = 1 - \widehat{a}_v^p = \frac{1-\frac{C(Q_v^p)}{Q_v^p}}{2}$ and then, (27) simplifies to

$$\frac{d^2U_v^p}{dQ_v^2} = \left(\frac{1 - \frac{C(Q_v^p)}{Q_v^p}}{2}\right)\left(-C''(Q_v^p) + \frac{1}{2}\frac{d\left(\frac{C(Q_v^p)}{Q_v^p}\right)}{dQ_b}\right) < 0, \tag{28}$$

since $C''(Q_v^p) > \frac{1}{2}\frac{d\left(\frac{C(Q_v^p)}{Q_v^p}\right)}{dQ_b}$ using $C(Q_v^p) = \left(Q_v^p\right)^k$. Therefore, Q_v^p is the maximum of U_v^p.

Public University: Borrowing Constraints

Exams. The optimal public exam is the same as under perfect capital markets.

Fees. To determine the fee that maximizes $U_b^c = \left(\left(\frac{1-\left(a_b^c\right)^2}{2}\right) Q_b - C(Q_b)\,(1-a_b^c)\right)(1-f_b)$, we distinguish two cases:
CASE 1. $a_b^c = \frac{C(Q_b)}{Q_b}$ and $f_b \leq C(Q_b)$. The fee that maximizes U_b^c in this case is $f_b^c = 0$, whenever $0 \leq Q_b \leq 1$, because public utility is strictly decreasing in f_b. Otherwise, the optimal fee is $f_b = 1$ and then, the public university shuts down the school.
CASE 2. $a_b^c = \frac{f_b}{Q_b}$ and $f_b > C(Q_b)$. Utility is $U_b^c = \left(\frac{Q_b}{2} - C(Q_b) - \frac{f_b^2}{2Q_b} + \frac{C(Q_b)f_b}{Q_b}\right)(1-f_b)$ and the optimal fee satisfies

$$\frac{dU_b^p}{df_b} = -\left(\frac{Q_b}{2} - C(Q_b) - \frac{f_b^2}{2Q_b} + \frac{C(Q_b)f_b}{Q_b}\right) + \left(\frac{-f_b + C(Q_b)}{Q_b}\right)(1-f_b) = 0, \quad (29)$$

$$\frac{d^2U_b^p}{df_b^2} = -2\left(\frac{-f_b + C(Q_b)}{Q_b}\right) - \frac{1-f_b}{Q_b} < 0. \quad (30)$$

Whenever $\frac{Q_b}{2} - C(Q_b) - \frac{f_b^2}{2Q_b} + \frac{C(Q_b)f_b}{Q_b} \geq 0$, the interior maximum, $0 < f_b < 1$, satisfies $-f_b + C(Q_b) \geq 0$. Otherwise, it is optimal to set $f_b = 1$ and shut down the school. Notice that $-f_b + C(Q_b) \geq 0$ contradicts CASE 2.

Quality. The level of quality that maximizes U_b^c, subject to $a_b^c = \frac{C(Q_b)}{Q_b}$ and $f_b^c = 0$, is the same as under perfect capital markets. Using the specification of the cost function, we obtain that $Q_b^c = Q_b^p = \left(\frac{1}{2k-1}\right)^{k-1} < 1$ since $k > 1$.

Private University: Borrowing Constraints
Exams. The private university decides to accept all applicants if $f_v \geq C(Q_v)$, and shuts down, otherwise.

Fees. If $f_v \geq C(Q_v)$, the private fee is optimally determined as follows:

$$\frac{dU_v^c}{df_v} = \left(1 - 2\frac{f_v^c}{Q_v} + \frac{C(Q_v)}{Q_v}\right)(1-f_v^c) - (f_v^c - C(Q_v))\left(1 - \frac{f_v^c}{Q_v}\right) = 0. \quad (31)$$

Solving (31) for f_v^c we obtain two candidates to maximum, and we find that $f_v^c = \frac{1+Q_v+C(Q_v)-\sqrt{(Q_v-C(Q_v))^2+(1-C(Q_v))(1-Q_v)}}{3}$ is the interior maximum whenever $0 < Q_v < 1$, since in such case $Q_v > C(Q_v)$ and

$$\frac{d^2U_v^c}{df_v^2} = -2\left(\frac{1 + Q_v + C(Q_v) - 3f_v^c}{Q_v}\right) < 0, \quad (32)$$

while $f_v = \frac{1+Q_v+C(Q_v)+\sqrt{(Q_v-C(Q_v))^2+(1-C(Q_v))(1-Q_v)}}{3}$ is the minimum.
Non-interior maxima appear whenever $Q_v = 0$ and then, $f_v^c = 0$, and $Q_v = 1$, in which case $f_v^c = 1$.

Quality. The quality that maximizes $U_v^c = (f_v^c - C(Q_v))\left(1 - \frac{f_v^c}{Q_v}\right)(1-f_v^c)$ satisfies

$$\frac{dU_v^c}{dQ_v} = (1-f_v^c)\left(-C'(Q_v^c)\,(1-\widehat{a}_v^c) + \widehat{a}_v^c\left(\widehat{a}_v^c - \frac{C(Q_v^c)}{Q_v^c}\right)\right) = 0. \quad (33)$$

The existence of a global maximum is guaranteed provided that U_v^c is a continuous function of Q_v in the interval $0 \leq Q_v \leq 1$. The maximum cannot be attained in the extremes because $U_v^c(Q_v = 0) = U_v^c(Q_v = 1) = 0$. Since U_v^c is derivable and strictly positive in the interval $0 < Q_v < 1$, provided that $Q_v > f_v^c > C(Q_v)$ in this interval, the global maximum is attained in a local maximum, $0 < Q_v^c < 1$, satisfying (33), which is strictly lower than the local (and global) maximum under perfect capital markets, Q_v^p, as shown in Proposition 4.

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