E C O N O M I C S B U L L E T I N

Ederington's ratio with production flexibility

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Abstract

The impact of flexibility upon hedging decision is examined for a competitive firm under demand uncertainty. We show that if the firm can adapt its production subsequently to its hedging decision, the standard minimum variance hedge ratio from Ederington (Journal of Finance 34, 1979) is systematically biased. This resulting bias depends on the statistical relation between demand and futures prices.

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1 Introduction

Since the pioneering contributions by Rotschild and Stiglitz (1970), Baron (1970) and Sandmo (1971), the theory of the competitive firm under price uncertainty has been a focus of much attention from financial literature. In the so-called 'unbiased' case, Danthine (1978), Holthausen (1979) and Feder *et al.* (1980) studied consequences of the introduction of forward markets by establishing the now well-known *separation property*[1]. Relaxing the non-realistic assumption of *unbiasedness* when markets are organized and standardized – futures markets – Ederington (1979) elicited the optimal minimum variance hedge ratio[2]. Taking into account the inescapable basis risk[3], this ratio is still today the most widely used, because of limited improvements provided by numerous suggested alternatives[4].

However, these alternatives, as the original Ederington's paper, consider a fixed amount of output[5]. In other words, the quantity to hedge is perfectly known before the hedging decision is made. Naturally, economic situations that do not have this property are frequent. It is not difficult to find examples where the production decision is posterior to the hedging decision: (*i*) farmers never know precisely the volume of their future crop, because they depend on meteorology and other factors; (*ii*) because of non-expected variations in demand, power producers and petroleum companies face uncertainty in quantity ; (*iii*) multinational firms do not know exactly the amount they will receive in foreign currencies in advance. On this subject, few contributions can be mentioned.

First, McKinnon (1967) in an agricultural framework shows the importance of the covariance between quantity and price for variance minimization . The problem is that McKinnon does not benefit from Ederington's work and then does not put forward covariance between spot and futures prices. Losq (1982) generalizes McKinnon's model in an expected utility framework. Preferences are then not necessarily quadratic and joint probability distribution not necessarily normal. But Losq's analysis does not assume any production cost and the model is built exactly as if the decision-maker only consider its income. To some extent, the analysis of Kerkvliet and Moffett (1991) is near enough

[1]Production decisions are not affected by changes in risk aversion or in price expectations. However, hedging decisions depend on both risk preferences and price expectations. See also Ethier (1973), equation 3, p 496 for a first formulation

[2]Ederington showed the relationship existing between optimal hedging and the futures prices/spot prices covariance.

[3]Basis risk occurs because of location, timing or quality differences between production and futures contracts specifications.

[4]Among others: expected utility ratio, mean-variance ratio, semivariance ratio, Sharpe's ratio, mean-Gini coefficient. For a survey, see Chen *et al.* (2003).

[5]We can precise here that output problem and input problem are symmetric under assumption of input inflexibility (see Anderson and Danthine (1983), p 379).

to Losq's one because of the non production cost assumption. These authors take into consideration a particular case of a multinational firm, which will receive an amount of foreign currency in the future, which is uncertain. The firm is assumed to be risk-averse and plans a risk-minimizing hedge. The optimal hedge ratio is derived and shown to be dependant on the covariance between prices and quantities. Lapan and Moschini (1994) provide a general model in an expected-utility framework. They assume a production cost proportional to the crop area but not to the harvest, which corresponds to the agricultural reality. Effectively, when considering the farming of a land, no real adjustment can be realized once the surface area is decided.

This paper studies the case of a firm facing both price and output uncertainties, but whose production perfectly matches demand. An example is the case of a power producer, where supply corresponds exactly to consumption, and no kWh is produced without demand. Thus variability on demand – quantity – leads to variability on production cost, and cost cannot therefore been looked at as a fixed amount. Accordingly, previous analyses are not relevant. The aim of this paper is to show the differences between previous optimal hedge ratios (OHR) and optimal ratio with (perfect) flexibility. We then indicate that without taking into account flexibility, the OHR is systematically biased, following the intuition that effectiveness of the hedge is depending on the statistical relationship between the hedge instrument and the product to be hedged.

The paper is set out as follows. Section 2 presents the model with the introduction of a variable cost function. Section 3 gives analytical results. Section 4 summarizes the main conclusions of the paper.

2 Notations

The model is a standard two-periods model $t = 0, 1$. Consider a competitive firm with a given – deterministic – production technology, which produces a sole commodity. Its production capacity is chosen prior to the model and cannot be modified. Output is produced at a cost $C(q)$, increasing, but indifferently concave or convex. The cost function is also assumed to be deterministic. The firm is assumed to face a stochastic spot price $\tilde{p_1}$ for its single output in the second period $(t = 1)$[6].

In addition, the firm faces a quantity uncertainty in that the demand $\tilde{q}$ is not known in the first period $(t = 0)$. Because of its flexibility property, the firm can match perfectly the demand level. In this way the issue differs fundamentally from the standard *newsboy problem* examined throughout operational research literature and initiated by Arrow *et*

[6]Throughout the paper, random variables have a tilde.

al. (1951)[7].

The only decision variable for the firm is the amount of output hedged h in the futures market. Futures contract is the only type of hedging instrument or insurance available for the firm. The current futures price f_0 is perfectly known, whereas the second period's one $\tilde{f}_1$ is not. The realized total profit is then:

$$\widetilde{\Pi} = \tilde{p_1}\tilde{q} - C(\tilde{q}) + h(\tilde{f}_1 - f_0) \tag{1}$$

Consider the firm as infinitely risk-averse; its aim is to minimize the profit's variance without taking into account consequences of the hedge on profit's expectation:

$$\min_h [var(\widetilde{\Pi})] \tag{2}$$

Taking into account variability of production cost, profit variance is:

$$var[\widetilde{\Pi}] = var[\tilde{p_1}\tilde{q}] + var[C(\tilde{q})] + h^2 var[\tilde{f}_1] - 2cov[\tilde{p_1}\tilde{q}, C(\tilde{q})] + 2hcov[\tilde{p_1}\tilde{q}, \tilde{f}_1] - 2hcov[C(\tilde{q}), \tilde{f}_1] \tag{3}$$

An expression of the variance of a product of random variables can be found in Bohrnstedt and Goldberger (1969, p 1439, equation (5)). However, this result is not essential because the firm has no power to reduce this variance by hedging. Clearly, there is no relation between the variance of the revenue and h, the number of futures contracts purchased or sold. From a certain viewpoint, this term can be seen as an irreducible risk, a risk on which the firm has no control.

Considering that the only way the firm can reduce its profit's variance is hedging. Considering further that only one futures contract is available. The first order condition (henceforth FOC) for program (2) is[8]:

$$h^* var[\tilde{f}_1] + cov[\tilde{p_1}\tilde{q}, \tilde{f}_1] - cov[C(\tilde{q}), \tilde{f}_1] = 0 \tag{4}$$

A simplification of equation (4) is essential to make the hedge ratio apparent. Let us consider the production as a random variable. For any pair of random variables x and y, $cov(x, y) = E(xy) - E(x)E(y)$. We can then rewrite $cov[C(\tilde{q}), \tilde{f}_1]$ as the difference between $E[C(\tilde{q})\tilde{f}_1]$ and $E[C(\tilde{q})]E[\tilde{f}_1]$. To further reduce this result, a preliminary lemma is useful.

Lemma 1 (Price's Theorem, 1958) [9] *Let x and y be bivariate normally distributed*

[7]See also Leland (1972).

[8]The second-order condition is satisfied by the sign of a variance.

[9]For a similar result in a more general framework, see also Middleton (1948).

with covariance σ_{xy}. Then if $h(x)$ and $g(x)$ are two functions square integrable with respect to the normal density and with derivatives of all orders,

$$E[h(x)g(y)] = E[h(x)]E[g(y)] + \sigma_{xy}E[h'(x)]E[g'(y)]+$$

$$\sigma_{xy}^2 E[h''(x)]E[g''(y)] + ... + \sigma_{xy}^i E[h^{(i)}(x)]E[g^{(i)}(y)] + ...$$

In particular, $E[xg(y)] = E[x]E[g(y)] + \sigma_{xy}E[g'(y)]$

Applying this result to the cost function with quantity and futures prices as random variables, we obtain:

$$E[C(\tilde{q})\tilde{f}_1] = E[C(\tilde{q})]E[\tilde{f}_1] + cov(\tilde{f}_1, \tilde{q})E[C'(\tilde{q})] \tag{5}$$

Using this last result, the product of expectations vanishes in the last expression and:

$$cov[C(\tilde{q}), \tilde{f}_1] = E[C'(\tilde{q})]cov(\tilde{f}_1, \tilde{q}) \tag{6}$$

To further simplify equation (4), another preliminary result is useful[10].

Lemma 2 (Bohrnstedt and Goldberger, 1969) *Let x, y and z be jointly distributed random variables, then (with $cov(.,.)$ and $E(.)$ respectively covariance and expectation operator):*

$$cov(xy, z) = E(x)cov(y, z) + E(y)cov(x, z) + E[(x - E(x))(y - E(y))(z - E(z))]$$

Further, under multivariate normality, all third moments vanish. We have $E[(x - E(x))(y - E(y))(z - E(z))] = 0$ and last equation is reduced to: $cov(xy, z) = E(x)cov(y, z) + E(y)cov(x, z)$

Then the second quantity on the left-hand side of equation (4) becomes:

$$cov(\tilde{p_1}\tilde{q}, \tilde{f}_1) = E(\tilde{q})cov(\tilde{p_1}, \tilde{f}_1) + E(\tilde{p_1})cov(\tilde{q}, \tilde{f}_1) + E[(\tilde{q} - E(\tilde{q}))(\tilde{p_1} - E(\tilde{p_1}))(\tilde{f}_1 - E(\tilde{f}_1))] \tag{7}$$

Using Bohrnstedt and Goldberger's hypothesis concerning multivariate normality, equation (7) becomes:

$$cov(\tilde{p_1}\tilde{q}, \tilde{f}_1) = E(\tilde{q})cov(\tilde{p_1}, \tilde{f}_1) + E(\tilde{p_1})cov(\tilde{q}, \tilde{f}_1) \tag{8}$$

[10]This result is commonly used in this kind of problems with multiple sources of uncertainty, as soon as one of the risks applies in a multiplicative manner. See Lapan and Moschini (1994) or Kerkvliet and Moffett (1991).

By introducing (6) and (8) into condition (4), we obtain:

$$h^* var[\tilde{f}_1] + [E(\tilde{q})cov(\tilde{p_1}, \tilde{f}_1) + E(\tilde{p_1})cov(\tilde{q}, \tilde{f}_1)] - [E[C'(\tilde{q})]cov(\tilde{f}_1, \tilde{q})] = 0 \tag{9}$$

Condition (9) permits to determine the optimal hedge ratio which minimizes the profit's variance.

3 Optimal hedge with perfect flexibility

As mentioned in the introduction, flexibility can often be observed in economics, especially in network activities. For many of these activities, production matches exactly the demand. Hence, the variable cost roughly corresponds to the quantity supplied, as soon as cost function is deterministic. The following proposition derives from equation (9).

Proposition 1 *Exact variance minimizing optimal hedge ratio with perfect flexibility is given by:*

$$h^* = \frac{E[C'(\tilde{q})]cov(\tilde{f}_1, \tilde{q}) - E(\tilde{p_1})cov(\tilde{f}_1, \tilde{q}) - E(\tilde{q})cov(\tilde{p_1}, \tilde{f}_1)}{var(\tilde{f}_1)} \tag{10}$$

Corollary 1 *If prices and quantities are positively correlated, the optimal hedge ratio is lower if cost variability is taken into account.*

The impact of the new element on the OHR depends on the sign of $cov(\tilde{f}_1, \tilde{q})$ because $E[C'(\tilde{q})]$ is always positive. There is a difference with the agricultural approach here. Following McKinnon (1967), "*any particular farmer expects his own output to be positively correlated with the aggregate output of all farmers and hence negatively correlated with prices*". This assumption appears particularly relevant when there is a relationship between considered uncertainty and meteorology. In our case, the opposite may occur. Power markets are today often managed by auctions. A high-level demand logically leads to higher profits for electricity producers, because of the uniform price system. A positive relation between individual output and prices can therefore be expected. As a consequence, the hedge ratio is decreased if we assume that the hedger position is short in futures contracts. An immediate conclusion is that when production cost variability is not taken into account, the hedge ratio is systematically overvalued. The difference between the initial ratio and the ratio proposed here varies according to the marginal cost value in the area of uncertainty. There is an intuition here. In an area of high marginal cost, the hedge is statistically less adapted – it means in probabilities – and the hedge ratio is then lower compared to a low marginal cost area, where a variation in quantity has a lower impact on the variation of production cost.

Previous results provided in the literature can be identified. Firstly, if production are completely ignored, the result is similar to Kerkvliet and Moffett (1991) (equation 16). Secondly, if the firm is infinitely risk-averse the solution is then identical to the Lapan and Moschini's (1994) ratio (equation 34 and equation 41 for the mean-variance extension). Finally, the original result from Ederington is derived by assuming a non random quantity.

4 Conclusion

Absence of flexibility means that effective cost of production is determined *ex ante*, without any dependence *vis-à-vis* the realized demand. In this case, the production cost is not assumed to be random and optimal hedge ratio can be derived following Kerkvliet and Moffett (1991) for instance. However, the non flexibility case is a bit particular. Previous articles gave a random characteristic to quantity by using a stochastic production function[11] or an uncertain amount of money. Here, uncertainty comes from demand. In this case, residual cost variability must unambiguously be taken into account particularly if marginal costs are high.

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[11]See Just and Pope (1979) for a first formalization.

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