E C O N O M I C S B U L L E T I N

Process RDin Monopoly under Demand Uncertainty

Luca Lambertini
Department of Economics, University of Bologna

Abstract

I investigate RDefforts for process innovation in a monopoly with uncertain demand. Two different models are proposed, where either (i) the reservation price is affected by an additive shock and the marginal production cost is increasing, or (ii) a multiplicative shock on the slope of demand combines with a flat marginal production cost. In either case, price-setting behaviour generates a larger RDinvestment than quantity-setting behaviour. An Arrovian interpretation of the first result and a Schumpeterian interpretation of the second are proposed.

I thank the Editor, Thijs ten Raa, for precious suggestions. I am also grateful to Sougata Poddar, Gianpaolo Rossini and Dan Sasaki for helpful comments and discussion. The usual disclaimer applies.
Citation: Lambertini, Luca, (2006) "Process RDin Monopoly under Demand Uncertainty." *Economics Bulletin*, Vol. 15, No. 14 pp. 1-9
Submitted: December 22, 2004. **Accepted:** December 27, 2006.
URL: http://economicsbulletin.vanderbilt.edu/2006/volume15/EB-04O30007A.pdf

Process R&D in Monopoly under Demand Uncertainty*

Luca Lambertini
Dipartimento di Scienze Economiche
Università degli Studi di Bologna
Strada Maggiore 45, 40125 Bologna, Italy
phone 39-051-2092600, fax 39-051-2092664
lamberti@spbo.unibo.it
ENCORE, Faculty of Economics
University of Amsterdam
Roetersstraat 11, 1018 WB Amsterdam

December 27, 2006

Abstract

I investigate R&D efforts for process innovation in a monopoly with uncertain demand. Two different models are proposed, where either (i) the reservation price is affected by an additive shock and the marginal production cost is increasing, or (ii) a multiplicative shock on the slope of demand combines with a flat marginal production cost. In either case, price-setting behaviour generates a larger R&D investment than quantity-setting behaviour. An Arrovian interpretation of the first result and a Schumpeterian interpretation of the second are proposed.

JEL Classification: L12, O31
Keywords: technical progress, uncertainty

*I thank the Editor, Thijs ten Raa, for precious suggestions. I am also grateful to Sougata Poddar, Gianpaolo Rossini and Dan Sasaki for helpful comments and discussion. The usual disclaimer applies.

1 Introduction

The incentives to invest in R&D (either for process or for product innovation) have been extensively investigated in the existing literature. The available contributions focus upon either (i) the role of uncertainty in the R&D activity, given the prize to be awarded to the winner of the race, or (ii) the role of the type of market competition, either Cournot or Bertrand, and market structure in shaping firms' incentives (for exhaustive accounts of both strands see, e.g., Reinganum, 1989; and Martin, 2001, ch. 14).

To the best of my knowledge, the interplay between demand uncertainty and firms' R&D efforts has not been investigated thus far, although there exist several influential contributions dealing with demand uncertainty either in monopoly (see Leland, 1972; Klemperer and Meyer, 1986) or in oligopoly (Weitzman, 1974; Klemperer and Meyer, 1986) or in perfect competition (Sandmo, 1971). All of these contributions focus upon the optimal price or quantity choice and the relative profitability of such strategies. In particular, Klemperer and Meyer (1986) show that if technology is characterised by an increasing marginal production cost, then the monopolist is better off using the output level rather than the price.

Here, I rely on Klemperer and Meyer's analysis to model the relationship between the monopolist's incentive to invest in process innovation and demand uncertainty. In particular, I propose two alternative models. In the first, an additive shock appears in the demand function and the cost function is convex. Under these conditions, expected profits (gross of R&D costs) are larger under quantity-setting behaviour. In the second, a multiplicative shock affects the slope of the demand function, while production costs are linear in the output level. Under these conditions, instead, expected profits (gross of R&D costs) are larger under price-setting behaviour. In both cases, the monopolist invests in R&D in order to reduce marginal cost. I show that, irrespective of the assumptions adopted regarding the type of uncertainty and the shape of the cost function, the optimal R&D investment is larger when the monopolist sets the price than when it sets the output level. In the first model, this is due to the fact that increasing the R&D effort amounts in fact to a decrease in the uncertainty affecting the profits generated by fixing the price; it is indeed an optimal response to the expected profit loss associated with the variance of the shock, that the monopolist foresees when setting the price. In the second model, R&D cannot contribute to reduce the effects of uncertainty on the expected profits associated with quantity-setting behaviour; therefore, the larger funds available under price-setting behaviour drive the result.

The two alternative models are laid out and investigated in section 2. Concluding comments are in section 3.

2 The setup

Consider a single-product monopolist that invests in R&D for process innovation and supplies the good to the market by setting either the price or the output level so as to maximise profits.[1] Setting up the plant involves a fixed cost k, whose size is such to allow the monopolist to survive while preventing further entries. Define as π^m the instantaneous monopoly profits, gross of R&D and setup costs. R&D activities involve costs $\Gamma(x)$, x being the R&D effort. The R&D cost function is characterised by the following properties: $\Gamma'(x) > 0$ and $\Gamma''(x) \leq 0$.

The expected net profits are:

$$E\Pi^m = E\pi^m - k - \Gamma(x). \tag{1}$$

As to the issue of modelling production costs and the demand function, I will consider two alternative cases based upon Klemperer and Meyer (1986):

- **Model I:** The market demand function is $p = a - Q + \varepsilon$. The additive shock ε has $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma^2$. The cost function is $C(Q) = cQ^2/2$, with $c = c(x)$ and $c'(x) < 0$; $c''(x) \geq 0$.[2]

- **Model II:** The market demand function is $p = a - Q/\theta$. The shock on the slope of demand, θ, has $E(\theta) = 1$ and $E(\theta^2) = s^2 > 1$. Accordingly, I may define $z \equiv E(1/\theta)$, which is larger than one (by Jensen's inequality). The cost function is $C(Q) = \gamma Q$, with $\gamma = \gamma(x)$ and $\gamma'(x) < 0$; $\gamma''(x) \geq 0$.

Concerning the setup cost, in order to prevent entry it must be that $k \in \left(E\pi^d - \Gamma(x), E\pi^m - \Gamma(x)\right)$, where $E\pi^d$ is the gross expected profit of

[1] The approach I adopt here is static, but it can be easily shown that it encompasses the case of an infinite horizon with constant discounting.

[2] As is known from Klemperer and Meyer (1986, Lemma 1), if the marginal production cost were constant, then the additive shock on demand would exert no effects on the equilibrium behaviour of the firm.

a firm in duopoly. With additive shocks and increasing marginal costs $E\pi^d$ is the outcome of a quantity-setting game, while with multiplicative shocks and constant marginal costs it is associated with a price-setting game (cf. Klemperer and Meyer, 1986, Propositions 1-2).

In both models, the monopolist may use either price or quantity as the market variable to maximise per-period profits.

2.1 Equilibrium analysis: Model I

Under the additive shock on the vertical intercept of the demand function (i.e., the reservation price), the expected gross monopoly profits are:

$$E\pi_Q^m = \frac{a^2}{2\left[2+c\left(x\right)\right]} \; ; \; E\pi_P^m = E\pi_Q^m - \frac{\sigma^2}{2} c\left(x\right) \tag{2}$$

under quantity- and price-setting behaviour, respectively (cf. Klemperer and Meyer, 1986, pp. 636-37). Therefore, the larger the variance, the larger the difference $E\pi_Q^m - E\pi_P^m$, all else equal. Profits (2) can be plugged into (1) in order to derive the first order conditions (FOCs) pertaining to the R&D activity at $t = 0$, in the two cases:

$$\frac{\partial E\Pi_Q^m}{\partial x} = \frac{\partial E\pi_Q^m}{\partial x} - \Gamma'\left(x\right) = 0 \Leftrightarrow -\frac{a^2 c'\left(x\right)}{2\left[2+c\left(x\right)\right]^2} - \Gamma'\left(x\right) = 0; \tag{3}$$

$$\frac{\partial E\Pi_P^m}{\partial x} = \frac{\partial E\pi_P^m}{\partial x} - \Gamma'\left(x\right) = 0 \Leftrightarrow -\frac{a^2 c'\left(x\right)}{2\left[2+c\left(x\right)\right]^2} - \Gamma'\left(x\right) = \frac{\sigma^2}{2} \cdot c'\left(x\right). \tag{4}$$

Second order conditions require, respectively:

$$\frac{\partial^2 E\Pi_Q^m}{\partial x^2} \leq 0 \, \forall \, c''\left(x\right) \geq \frac{2\left[\Psi - \Gamma''\left(x\right)^2\right]\left[2+c\left(x\right)\right]^2}{a^2} \equiv c_Q''\left(x\right); \tag{5}$$

$$\frac{\partial^2 E\Pi_P^m}{\partial x^2} \leq 0 \, \forall \, c''\left(x\right) \geq \frac{2\left[\Psi - \Gamma''\left(x\right)^2\right]\left[2+c\left(x\right)\right]^2}{a^2 + \sigma^2\left[2+c\left(x\right)\right]^2} \equiv c_P''\left(x\right), \tag{6}$$

where $\Psi \equiv \left(ac'\left(x\right)\right)^2 / \left[2+c\left(x\right)\right]^3$. Clearly, $c_Q''\left(x\right) > c_P''\left(x\right)$ for all positive σ. Accordingly, if the problem is concave under quantity-setting, then it is also under price-setting. Therefore, in the remainder of this subsection I will pose $c''\left(x\right) \geq c_Q''\left(x\right)$.

Now define as x_Q the R&D effort that satisfies (3), i.e., consider the profit-maximising investment under quantity-setting. Then, plugging x_Q into (4), one obtains:

$$\left.\frac{\partial E\Pi_P^m}{\partial x}\right|_{x=x_Q} = -\frac{\sigma^2}{2}\cdot c'(x) > 0 \tag{7}$$

given that $c'(x) < 0$. That is, in correspondence of the R&D effort that satisfies $\partial E\Pi_Q^m/\partial x = 0$, the corresponding FOC under price-setting behaviour, $\partial E\Pi_P^m/\partial x$, is still positive, i.e., the expected marginal profit from R&D under price-setting behaviour is itself still positive. Accordingly, when the firm sets the price it has a higher incentive to invest in R&D as compared to the quantity-setting case.

2.2 Equilibrium analysis: Model II

Now examine the setup where the shock affects the slope of the demand function. In this case, expected gross profits are (cf. Klemperer and Meyer, 1986, pp. 636-37):

$$E\pi_Q^m = \frac{[a-\gamma(x)]^2}{4z}\ ;\ E\pi_P^m = z\cdot E\pi_Q^m = \frac{[a-\gamma(x)]^2}{4}\,. \tag{8}$$

Here, $E\pi_Q^m < E\pi_P^m$ since $z > 1$. Proceeding as in the previous subsection, one has to calculate the FOCs pertaining to the R&D phase at $t = 0:$

$$\frac{\partial E\Pi_Q^m}{\partial x} = -\frac{[a-\gamma(x)]\,\gamma'(x)}{2z} - \Gamma'(x) = 0; \tag{9}$$

$$\frac{\partial E\Pi_P^m}{\partial x} = -\frac{[a-\gamma(x)]\,\gamma'(x)}{2} - \Gamma'(x) = 0. \tag{10}$$

Second order conditions require, respectively:

$$\frac{\partial^2 E\Pi_Q^m}{\partial x^2} \le 0\,\forall\,\gamma''(x) \ge \frac{\gamma'(x)^2 - 2z\Gamma''(x)}{a-\gamma(x)} \equiv \gamma_Q''(x)\,; \tag{11}$$

$$\frac{\partial^2 E\Pi_P^m}{\partial x^2} \le 0\,\forall\,\gamma''(x) \ge \frac{\gamma'(x)^2 - 2\Gamma''(x)}{a-\gamma(x)} \equiv \gamma_P''(x)\,, \tag{12}$$

with $\gamma_Q''(x) \ge \gamma_P''(x)$ for all $\Gamma''(x) \le 0$ and all $z > 1$. Once again, if the problem is concave under quantity-setting, then it is also under price-setting.

Consequently, to ensure concavity, in the remainder of this subsection I will suppose $\gamma''(x) \geq \gamma''_Q(x)$.

Then, one can proceed by solving (9) and plugging the expression $\Gamma'(x) = -[a-\gamma(x)]\gamma'(x)/(2z)$ into (10) to verify that

$$\frac{\partial E\Pi_P^m}{\partial x} = -\frac{(z-1)[a-\gamma(x)]\gamma'(x)}{2z} > 0 \tag{13}$$

given that $\gamma'(x) < 0$. That is, when $\partial E\Pi_Q^m/\partial x = 0$, $\partial E\Pi_P^m/\partial x > 0$ which entails that the level of x that maximises the net expected profits is higher under price-setting than under quantity-setting behaviour.

The discussion of the two models can be summarised in the following Proposition:

Proposition 1 *Provided concavity conditions are satisfied, then irrespective of whether (i) the marginal cost is increasing and demand is affected by an additive shock, or (ii) the marginal cost is flat and uncertainty affects the slope of demand, the R&D investment is larger when the monopolist sets the price than when it sets the output level.*

However, the source of the result is different in the two cases. With increasing marginal cost and an additive shock on the vertical intercept of demand, gross profits are larger under quantity-setting behaviour, so that the firm invests more under price-setting in order to reduce the negative bearings of the reservation price variance on profits. In the limit, as the marginal cost tends to zero, the effect of the shock disappears altogether. That is, by increasing the intensity of the R&D effort, the monopolist gets two eggs in one basket: a more efficient technology as well as a reduction in the negative effects of uncertainty on profits. This is an insurance policy against the uncertainty affecting the demand function. On the contrary, in the presence of a constant marginal cost coupled with a multiplicative shock on the slope of the demand function, setting the price allows for higher expected gross profits than setting the output level. Given that in this case a larger investment does not bring about a reduction in the degree of uncertainty while it entails an increase in gross profits for any given z, the incentive to invest in R&D is driven by a 'deep purse' argument.

These results can be interpreted in the following way. In the existing literature on R&D, a wide attention has been devoted to the relationship between market power (or the structure of the industry) and the associated incentives to invest in innovative activities. This assessment is to be traced back to the debate between Schumpeter (1942) and Arrow (1962) and the balance between the strategic effect and the replacement effect. The former uses a deep purse argument in favour of monopoly, while the latter speaks in favour of (more) competitive industries. In the present model only monopoly is considered; hence, strictly speaking, one could not refer to the Schumpeter-Arrow debate here. However, the presence of demand uncertainty gives rise to a setting where price and quantity behaviour on the part of the monopolist entails different R&D incentives *for a given industry structure*, because expected gross profits differ across cases. That is, a plausible way of rephrasing the respective positions of Schumpeter and Arrow consists in asking how higher or lower expected profits may shape the intensity of innovative activities, taking as given the fact that the industry is a monopoly.[3] The case of increasing marginal costs coupled with additive demand shocks has a definite Arrowian flavour, as lower profits generated by price-setting call for more intense R&D efforts. In the case of constant marginal costs and multiplicative demand shocks, instead, the interpretation of the result is Schumpeterian, with higher R&D efforts being observed when expected gross profits are larger, which happens under quantity-setting.

3 Concluding remarks

I have modelled R&D efforts for process innovation in a monopoly with uncertain demand. Two different models have been considered: one where an additive shock on the reservation price couples with an increasing marginal production cost, and the other with a multiplicative shock on the slope of demand and a constant marginal production cost. In either case, price-setting behaviour generates a larger R&D investment than quantity-setting behaviour. The reason for this result is that process R&D provides the firm with an insurance policy against uncertainty in the first model, while it cannot do so in the second model.

Extending the above analysis to oligopoly models may represent a productive perspective, which is left to future research.

[3] This perspective has been already used in the literature (although in models that differ from the one I consider), to compare the incentives of a given number of oligopolists when switching from price- to quantity-setting behaviour (see, e.g., Delbono and Denicolò, 1990).

References

[1] Arrow, K.J. (1962), "Economic Welfare and the Allocation of Resources for Invention", in R. Nelson (ed.), *The Rate and Direction of Industrial Activity*, Princeton, NJ, Princeton University Press.

[2] Delbono, F. e V. Denicolò (1990), "R&D Investment in a Symmetric and Homogeneous Oligopoly: Bertrand vs Cournot", *International Journal of Industrial Organization*, **8**, 297-313.

[3] Klemperer, P. an M. Meyer (1986), "Price Competition vs Quantity Competition: The Role of Uncertainty", *RAND Journal of Economics*, **17**, 618-38.

[4] Leland, H. (1972), "Theory of the Firm Facing Uncertain Demand", *American Economic Review*, **62**, 278-91.

[5] Martin, S. (2001), *Advanced Industrial Economics. Second Edition*, Oxford, Blackwell.

[6] Reinganum, J. (1989), "The Timing of Innovation: Research, Development and Diffusion", in Schmalensee, R. and R. Willig (eds.), *Handbook of Industrial Organization*, vol. 1, Amsterdam, North-Holland.

[7] Sandmo, A. (1971), "On the Theory of the Firm under Price Uncertainty", *American Economic Review*, **61**, 65-73.

[8] Schumpeter, J.A. (1942), *Capitalism, Socialism and Democracy*, New York, Harper.

[9] Weitzman, M. (1974), "Prices vs Quantities", *Review of Economic Studies*, **41**, 477-91.