E C O N O M I C S B U L L E T I N

Income Tax Evasion and the Penalty Structure

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Abstract

In the Allingham−Sandmo (AS) model of tax evasion, fines are paid on evaded income, whereas in the Yitzhaki (Y) model fines are levied on evaded tax. This note compares the two models. In the Y model, evasion is higher and tax revenue lower than in the AS model. If government seeks to maximize expected tax revenue, it would prefer penalties of the AS type; if it maximizes expected voter welfare, it should choose Y type penalties. A voting model to determine the penalty structure is also considered.

Paper presented at the 2004 European Public Choice Society Conference in Berlin. I would like to thank Laszlo Goerke and Andreas Wagener for helpful comments.
Citation: Borck, Rainald, (2004) "Income Tax Evasion and the Penalty Structure." *Economics Bulletin,* Vol. 8, No. 5 pp. 1−9
Submitted: September 20, 2004. **Accepted:** October 29, 2004.
URL: http://www.economicsbulletin.com/2004/volume8/EB−04H20010A.pdf

1. Introduction

The seminal paper on tax evasion is by Allingham and Sandmo (1972). They assumed fines are assessed on the amount of income evaded by the taxpayer. It turns out that the effect of increasing the tax rate on evasion is unclear. The reason is that there is a substitution effect (higher marginal tax rate makes evasion more attractive) and an income effect (higher tax rate lowers net income) which offset each other in the case of decreasing absolute risk aversion (since the lower income makes the taxpayer less willing to take risks and hence evade income).

In a comment on that paper, Yitzhaki (1974) showed that the substitution effect disappears when fines are assessed on the level of taxes evaded.[1] The intuition is simple: increasing the tax rate not only increases the marginal benefit from evasion, i.e., taxes saved, but also the marginal cost in the form of higher expected fines. At the taxpayer's optimum, the two effects exactly balance each other. Hence, only the income effect operates in the Yitzhaki model and a higher marginal tax rate leads taxpayers to evade less, assuming decreasing absolute risk aversion.

Interestingly, while Yitzhaki's comment has generated a sizeable literature on fine schedules,[2] seemingly obvious questions have not been addressed, most notably the effects on evasion, tax revenue and taxpayers' welfare. Economic analysis of tax evasion has proceeded to analyze the choice of audit strategies by revenue maximizing governments.[3] However, surprisingly, it has so far not attempted to explain why penalty functions should have one form or the other. An exception is Balassone and Jones (1998) who argue that using evaded tax instead of evaded income as basis for the penalty schedule reduces the excess burden of taxation because the substitution effect induced by an increasing tax rate is removed.

In this note, I consider a convex combination of the Allingham-Sandmo (A-S) and Yitzhaki (Y) models and study its properties. The penalty schedule is equal to the penalty rate times a weighted average of evaded income (the A-S model) and evaded tax (the Y model). I study the effect of varying the penalty structure on evasion, epexted tax revenue,

[1]See also Balassone and Jones (1998) for the effects of the penalty structure on how evasion changes with tax rates.

[2]See, e.g., Koskela (1983), Balassone and Jones (1998), Goerke (2003) and Richter and Boadway (2003). For a survey, see Andreoni et al. (1998) or Slemrod and Yitzhaki (2002).

[3]See, e.g., Andreoni et al. (1998) and Slemrod and Yitzhaki (2002) and the references therein.

and taxpayer welfare. I then endogenize the choice of the penalty structure in two models with a government sector.[4]

The next section presents the model. Section 3 studies the choice of a government which maximizes a weighted sum of welfare and expected tax revenue. Section 4 presents a model where the penalty structure is chosen by majority voting. The last section concludes the paper.

2. The Model

Individuals have utility $u(c)$ defined over consumption, with $u' > 0 > u''$, so taxpayers are risk averse. The coefficient of absolute risk aversion is denoted $\rho \equiv -u''/u'$. I will assume decreasing absolute risk aversion (DARA), i.e., $\rho' < 0$ for all c. An individual has gross income y which is subject to a linear income tax at rate t. Individuals may choose to evade income; evasion is denoted e. The individual is audited with exogenous probability p, in which case her income becomes known to the tax authority and she has to pay a penalty. The penalty schedule is

$$S = (1 - \alpha + \alpha t)se, \quad \alpha \in [0, 1].$$

With $\alpha = 0$, the penalty is levied on the amount evaded, which corresponds to the A-S model; with $\alpha = 1$ on the other hand, fines are levied on evaded tax, which corresponds to the Y model. Given these assumptions, the individual problem is

$$\begin{aligned} EU &\equiv \max_e pu(c_d) + (1-p)u(c_n) \\ \text{s.t. } c_d &= (1-t)y - (1-\alpha+\alpha t)se \\ c_n &= (1-t)y + te, \end{aligned}$$

and the first order condition for an interior maximum

$$(1-p)tu'_n - p(1-\alpha+\alpha t)su'_d = 0, \tag{1}$$

where $u'_i \equiv u'(c_i)$ for $i = d, n$. Let $r \equiv (1-p)t - p(1-\alpha+\alpha t)s$ be the expected return to a dollar evaded. Assume $r > 0$, so that taxpayers will always evade some income.

The second order condition is

$$D = (1-p)t^2u''_n + p[(1-\alpha+\alpha t)s]^2u''_d < 0,$$

[4] For models where individuals vote over tax rates, see Borck (2003, 2004).

which is fulfilled due to the concavity of the utility function.

The comparative statics effects are mostly straightforward (Allingham and Sandmo, 1972). In particular, individuals will evade less when p or s increase. An increase of income leads to more evasion with DARA. Furthermore, the fraction of income evaded increases in the case of income under decreasing relative risk aversion (DRRA) and decreases under increasing (IRRA) relative risk aversion.

The effect of the tax rate on evasion depends on the penalty structure, as shown by Yitzhaki (1974). Differentiating (1) with respect to t gives:

$$\begin{aligned}\frac{\partial e}{\partial t} &= -\frac{1}{D}[(1-p)u_n' - p\alpha s u_d' - y((1-p)tu_n'' - p(1-\alpha+\alpha t)su_d''))] \quad (2)\\ &= -\frac{1}{D}[(1-p)u_n' - p\alpha s u_d' + y(1-p)tu_n'(\rho_n - \rho_d)], \quad (3)\end{aligned}$$

where $\rho_i \equiv \rho(c_i)$. For $\alpha = 0$, the substitution effect (the first term in (3)) and the income effect (the last term in (3)) work in opposite directions under DARA. For $\alpha = 1$, (1) implies that $(1-p)u_n' - psu_d' = 0$, and hence, $\partial e/\partial t < 0$ with DARA. By continuity, there exists an $\tilde{\alpha} < 1$ such that for $\alpha > \tilde{\alpha}$ evasion decreases with the tax rate under DARA.

How does evasion vary with the penalty structure? Differentiation of (1) with respect to α gives the following:

$$\frac{\partial e}{\partial \alpha} = -\frac{1}{D}[p(1-t)su_d' - p(1-\alpha+\alpha t)s^2(1-t)eu_d''] > 0.$$

Hence, an increase in α increases tax evasion. The intuition for this result is that there are two effects of increasing α from the individual's point of view. Increasing α reduces the marginal penalty per evaded dollar, for positive tax rates. This is the substitution effect: evading taxes becomes more profitable, at the margin. There is also an income effect, since the taxpayer's net-of-penalty income when caught increases. Given that the taxpayer is risk averse, the marginal benefit of evasion increases. Both effects lead to higher evasion.

Note that when α changes for constant s, the marginal penalty rate and therefore the total penalty (for given evasion) also changes. Therefore, evasion increases with α since the marginal penalty rate falls. Suppose instead that when varying α, the penalty rate s is adjusted such that total tax revenue stays constant. Since expected tax revenue is given by $ET = ty + (p(1-\alpha+\alpha t)s - (1-p)t)e$, differentiating gives

$$\left.\frac{ds}{d\alpha}\right|_{ET=const.} = \frac{(1-t)spe - (p(1-\alpha+\alpha t)s - (1-p)t)e_\alpha}{(1-(1-t)\alpha)pe + (p(1-\alpha+\alpha t)s - (1-p)t)e_s}, \quad (4)$$

which is clearly positive since $e_\alpha, e_s > 0$ and $p(1-\alpha+\alpha t)s-(1-p)t<0$ by assumption.

Since

$$\left.\frac{de}{d\alpha}\right|_{ET=const.} = e_\alpha + e_s \left.\frac{ds}{d\alpha}\right|_{ET=const.}, \tag{5}$$

using (4) gives

$$\left.\frac{de}{d\alpha}\right|_{ET=const.} = -\frac{(1-s)(1-t)e^2ps(1-(1-t)\alpha)u''_d}{De+(p(1-\alpha+\alpha t)s-(1-p)t)(u'_d-(1-\alpha+\alpha t)eu''_d)}. \tag{6}$$

The denominator is negative since $r>0$. Therefore, as long as $s<1$, evasion increases with α, even if expected revenue is held constant.

If tax evasion is higher with penalties of the Y kind than with AS penalties, why would governments want to levy the former? This question is addressed in the following section. The answer will lie in the way that government trades off voter welfare and tax revenue.[5]

3. Government Behavior

Suppose that individuals are identical, and the population is normalized to one. Consider the following sequence of events: government first sets the penalty rate structure, and individuals then decide on how much income to declare.

In order to study rational government behavior, one must specify its objective function. Several assumptions are possible. For instance, government could be benevolent or a self interested Leviathan. Without studying the details of government behaviour, I assume that government maximizes a weighted sum of expected tax revenue and expected voter welfare. This assumption implies that while the government cares about a large budget, it also has to take account of voter backlash in case of excessive taxation. The government's problem is:

$$\Psi(p,s,t,\alpha) = \beta EU(\cdot) + (1-\beta)ET(\cdot),$$

where β is the weight attributed by the government to voter welfare versus tax revenue.

Suppose that p, s, and t are given, and the government maximizes Ψ by choosing α.

[5] See also Balassone and Jones (1998) who show that Y-type penalties, by eliminating the substitution effect of higher tax rates, cause a lower excess burden than AS-type penalties.

Using the envelope theorem, the first order condition is:

$$\beta\frac{\partial(EU)}{\partial\alpha} + (1-\beta)\frac{\partial(ET)}{\partial\alpha} = 0, \tag{7}$$

$$\text{where } \frac{\partial(EU)}{\partial\alpha} = p(1-t)seu'_d > 0, \tag{8}$$

$$\frac{\partial(ET)}{\partial\alpha} = -p(1-t)se + (p(1-\alpha+\alpha t)s - (1-p)t)e_\alpha < 0, \tag{9}$$

where the last inequality follows from $r > 0$. Assume that the second order condition is fulfilled. While tax revenue decreases with a larger α, voter welfare increases, creating a tradeoff for government. At the optimum, the welfare gain of voters from increased 'Yness' of the penalty structure (weighted by β) just equals the expected loss of tax revenue (weighted by $1-\beta$).

Obviously, the higher the weight on voter welfare in the government objective function, the larger is α at the optimum. If one interprets β as a measure of democracy, the result would imply that perfectly democratic countries ($\beta = 1$) should have Y-type penalty functions whereas autocratic governments ($\beta = 0$) should have AS-type penalty schedules. More generally, more autocratic countries should have penalties which are more tightly tied to evaded income than evaded tax. Empirically, the evidence seems to be that in most western democracies α is close to one. Taken at face value, this would be consistent with the model. It would also be good news in the sense that one could infer that the penalty structure is that preferred by voters, not bureaucrats.

However, individuals are not identical, and democracy exists to resolve conflicts among heterogeneous individuals. If individuals differ, the choice of penalty structure will have distributional consequences. In the next section I analyze how this issue is resolved under majority voting.

4. Voting on α

Suppose α is determined in a majority vote. Taxes and fines are redistributed lump sum to the individuals, that is, individuals each receive a per capita grant g. Individuals differ by income, which is distributed according to some distribution function $F(y)$ with density $f(y)$. Average income and evasion are denoted by $\bar{y} = \int y dF(y)$ and $\bar{e} = \int e(y,\cdot)dF(y)$. While each voter benefits from a higher α (see (8)), voters with different incomes benefit to different degrees.

The individual problem is to maximize utility, subject to the government budget constraint (GBC):

$$\max_{e} \quad pu((1-t)y-(1-\alpha+\alpha t)se+g)+(1-p)u((1-t)y+te+g) \tag{10}$$

$$\text{s.t. } g = t\bar{y}+(p(1-\alpha+\alpha t)s-(1-p)t)\bar{e}. \tag{11}$$

What is the equilibrium α chosen under majority vote? For an individual voter, the optimum α is that where her indifference curve in (g,α) space is just tangent to the budget constraint (see figure 1). Differentiating (11) gives the slope of the government budget constraint:[6]

$$\left.\frac{dg}{d\alpha}\right|_{GBC} = \frac{-p(1-t)s\bar{e}+(p(1-\alpha+\alpha t)s-(1-p)t)\bar{e}_{\alpha}}{((1-p)t-p(1-\alpha+\alpha t)s)\bar{e}_{g}} < 0. \tag{12}$$

It can be shown that an equilibrium exists if the single crossing condition holds. This condition says that voter preferences over g and α can be ordered independently of the policy. In the present context, single crossing holds if the slope of an indifference curve in (g,α) space is monotonic in y.[7] Differentiating (10), gives the slope of an indifference curve:

$$\sigma = \left.\frac{\partial g}{\partial \alpha}\right|_{EU=const.} = -\frac{\partial(EU)/\partial\alpha}{\partial(EU)/\partial g} = -\frac{(1-t)ste}{(1-(1-t)\alpha)s+t} < 0, \tag{13}$$

use having been made of (1).

Note that σ can be interpreted as a voter's willingness to pay for redistribution in terms of the penalty structure. Since $\sigma < 0$, a voter has to be compensated for a penalty structure which is 'more of the AS type' by a higher transfer.

Differentiating (13) gives

$$\frac{\partial\sigma}{\partial y} = -\frac{(1-t)st}{(1-(1-t)\alpha)s+t}\frac{\partial e}{\partial y}. \tag{14}$$

Since $\partial e/\partial y > 0$ with DARA, richer voters would have steeper indifference curves, which implies a higher marginal preference for α. The rough intuition for this is that the marginal benefit of increasing a is proportional to evasion, and evasion increases with

[6] The sign restriction in (12) follows since with DARA evasion increases with g and by assumption, $(1-p)t-(p(1-\alpha+\alpha t)s > 0$.

[7] See Gans and Smart (1996) for a general discussion of the single crossing approach. Borck (2003) presents a model where the tax rate is chosen by majority vote in the presence of tax evasion.

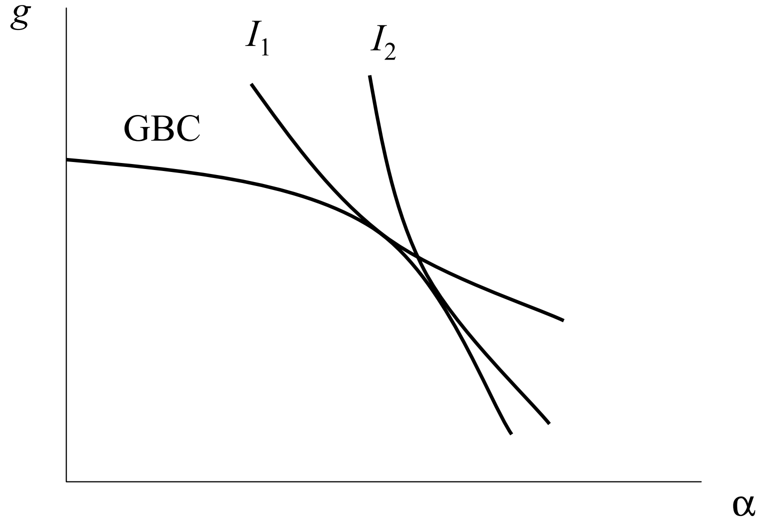

Figure 1: Voting on the penalty structure.

income under DARA. Hence, richer voters would be more willing to finance redistribution out of Y type penalties.

Note that since σ is monotonic in income, a voting equilibrium exists. To prove this, consider Figure 1. The government budget constraint is the curve labelled GBC, whereas the curves labeled I_1 and I_2 are the indifference curves for two voters with different income levels. With DARA, (14) implies $\sigma_y < 0$ and hence $y_2 > y_1$, where y_i is the income of the voter with indifference curve I_i. To prove existence of a voting equilibrium, note that if y_1, say, is the median income, then $\sigma_y < 0$ implies that half of the population (those with $y > y_m$) have steeper indifference curves and half (those with $y < y_m$) flatter indifference curves than y_1. This implies that there is no feasible point on the GBC which would beat y_1's optimum.

In the empirically relevant case of DARA, the implication of the model is that the richer a jurisdiction's median voter is relative to the mean, the more likely it would be that the penalty structure in that jurisdiction would be of the Yitzhaki type. An empirical hypothesis derived from the model would then be that the more egalitarian a country's income distribution (in the sense of a high ratio of median to mean income), the more of the Y type should its penalty structure be.

5. Conclusion

In this note, I have sketched simple models to address some neglected issues regarding penalties on tax evasion. In particular, when fines are levied on evaded tax, evasion is higher, tax revenue lower and voter welfare higher than when fines are assessed on evaded income. This presents a natural starting point to study the choice of penalty structure. When government maximizes a weighted sum of voter welfare and expected tax revenue, the part of the fine levied on evaded tax should be higher, the larger the weight on voter welfare. When voters differ by income, fines redistribute between voters. The richer the decisive voter, the more likely it will be that the fine will be levied on evaded tax, assuming decreasing absolute risk aversion.

Some extensions and modifications suggest themselves. One would be to allow for nonlinear penalty functions. The other would be to study the choice of penalties and auditing jointly in a principal agent framework. This would complement studies which have looked at the tax authorities audit strategy for given penalties (see Andreoni et al., 1998, for a survey). A third route would be the joint determination of taxes and penalties. However, it turns out that voting on tax rates does not necessarily guarantee existence of equilibrium with tax evasion (Borck, 2003) . This problem would be aggravated with multidimensional issues.

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