E C O N O M I C S   B U L L E T I N

Detecting changes in persistence in linear time series

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Abstract

The properties of the 'change in persistence' tests developed by Leybourne et al. (2003) are considered in the presence of structural change under the null. Interestingly, it is found that while breaks in drift result in undersizing, breaks in level lead to severe oversizing. The implications of these findings for both empirical research and the development of an alternative approach to the testing of a change in persistence are noted.

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1 Introduction

Examination of the order of integration of time series data has become a familiar feature of applied research in statistics and econometrics. While initial concern focussed upon whether series are better characterised as difference stationary or trend stationary, recent research has considered the possibility that series might experience a change in persistence moving between I(1) and I(0) status, or vice versa. As Leybourne *et al.* (2003), hereafter referred to as LKSN, have noted, the ability to detect such changes and decompose series into their stationary and non-stationary components has clear implications for model building, forecasting and policy implementation. In this paper, the newly developed testing framework proposed by LKSN to detect changes in persistence is examined. The approach of LKSN is of particular interest as unlike other tests of changes in persistence (see Busetti and Taylor 2004; Kim 2000, 2002; Leybourne and Taylor 2004), it operates under the empirically realistic unit root null hypothesis. In the present paper attention is paid to the impact of structural change upon the tests of LKSN. In particular, it is questioned whether the empirical size of the tests might be influenced by structural change under the null.

This paper will proceed as follows. In section [2] the tests proposed by LKSN are outlined. Section [3] examines the properties of the LKSN tests in the presence of unit root processes subject to a break in level, while section [4] provides an analysis of the LKSN tests when breaks in drift are present. Section [5] concludes.

2 Changes in persistence

LKSN consider the possibility that a series experiences a change in persistence from I(1) to I(0), or I(0) to I(1), at an unknown point in the sample period. Drawing upon the notion of reverse regression, LKSN note that a change from I(1) to I(0) at break fraction τ, corresponds to a change from I(0) to I(1) at break fraction $(1-\tau)$. LKSN therefore examine the hypothesis of a change in persistence by testing the unit root hypothesis over a range of breakpoints using an original series and its reversed realisation. To increase the power of the testing approach, local-to-unity detrending via generalised least squares (GLS), as proposed by Elliott *et al.* (1996), is employed with either an intercept or an intercept and trend fitted. The unit root hypothesis is then examined using the minimum resulting GLS-based Dickey-Fuller t-ratio to provide the strongest evidence against the null.

To test the null hypothesis that a series is I(1) throughout, denoted as H^{11}, against the alternative of a change in persistence from I(0) to I(1), denoted as H^{01}, the following approach is adopted. Given a series of interest y_t and deterministic terms z_t, GLS-transformed data are derived as:

$$\begin{aligned} y_{\overline{\alpha}}(\tau) &= \left[y_1, y_2 - \overline{\alpha} y_1, ..., y_{[\tau T]} - \overline{\alpha} y_{[\tau T]-1}\right]' \\ z_{\overline{\alpha}}(\tau) &= \left[z_1, z_2 - \overline{\alpha} z_1, ..., z_{[\tau T]} - \overline{\alpha} z_{[\tau T]-1}\right]' \end{aligned}$$

where $\tau \in (0,1)$ denotes the break fraction, T denotes the sample size and $\overline{\alpha} = 1 + \overline{c} T^{-1}$. LKSN consider all possible breakpoints over the interval $0.2 \leqslant \tau \leqslant 0.8$. The detrended series y_t^d is then derived as $y_t^d = y_t - \widehat{\beta}_0(\tau)$ in the intercept only model, and $y_t^d = y_t - \widehat{\beta}_0(\tau) - \widehat{\beta}_1(\tau)\, t$, $t = 1, 2, ..., \tau T$, in the linear trend case with the $\widehat{\beta}_i(\tau)$ coefficients obtained from the regression of $y_{\overline{\alpha}}(\tau)$ upon $z_{\overline{\alpha}}(\tau)$. A Dickey-Fuller (DF) unit root test without deterministic terms is then performed as follows:[1]

$$\Delta y_t^d = \rho(\tau)\, y_{t-1}^d + \varepsilon_t \qquad t = 1, 2, ..., \tau T \tag{1}$$

with the unit root hypothesis examined using the t-ratio of $\widehat{\rho}$. LKSN denote this statistic as $DF_G^f(\tau)$, with the resulting test employed given by:[2]

$$DF_G^{f\ inf}(\tau) = \underset{\tau}{inf}\ DF_G^f(\tau) \tag{2}$$

$DF_G^{f\ inf}(\tau)$ is referred to as a recursive test given its similarity to the test of Banerjee *et al.* (1992). However, given the use of only the first τT observations, LKSN note the inefficiency of (2), and therefore propose an alternative sequential test based upon the regression below:

$$\Delta y_t^d = \rho(\tau)\, D_t(\tau)\, y_{t-1}^d + \varepsilon_t \qquad t = 1, 2, ..., T \tag{3}$$

where $D_t(\tau) = 1$ for $t \leqslant \tau T$ and 0 otherwise, Δy_t^d is defined as in (1) for $t \leqslant \tau T$, but is defined as $\Delta y_t^d = \Delta y_t - \overline{\Delta y_2}$ when $t > \tau T$ with $\overline{\Delta y_2} = (T - \tau T)^{-1} \sum_{s=\tau T+1}^{T} \Delta y_s$. The t-ratio associated with this regression is then denoted as $\overline{DF}_G^f(\tau)$, with the sequential test given as:

$$\overline{DF}_G^{f\ inf}(\tau) = \underset{\tau}{inf}\ \overline{DF}_G^f(\tau) \tag{4}$$

To test H^{11} against an alternative of a change in persistence from I(1) to I(0) $\left(H^{10}\right)$, LKSN make use of the reversed realisation $\widetilde{y}_t = y_{T-\tau+1}$. Using $\widetilde{y}_t$, a potential change in persistence now occurs at $(1-\tau)\,T$. The GLS transformed version of this series is then derived as previously and is given as:

$$\widetilde{y}_{\overline{\alpha}}(\tau) = \left[\widetilde{y}_1, \widetilde{y}_2 - \overline{\alpha}\widetilde{y}_1, ..., \widetilde{y}_{[(1-\tau)T]} - \overline{\alpha}\widetilde{y}_{[(1-\tau)T]-1}\right]' \tag{5}$$

[1] Note that this testing equation can be augmented by use of Δy_{t-j}^d regressors to overcome problems of serial correlation.

[2] The superscript 'f' denotes the forward series is employed, while the subscript 'G' denotes the use of GLS detrending. The superscript 'r' is used for later tests where the reversed realisation of a series is utilised.

The GLS detrended series $\widetilde{y}_t^d$ is then derived following the approach outlined above, with the test statistic $DF_G^r(\tau)$ given as the t-ratio of ρ in the following regression:

$$\Delta\widetilde{y}_t^d = \rho(\tau)\,\widetilde{y}_{t-1}^d + \varepsilon_t \qquad t = 1, 2, ..., (1-\tau)\,T \tag{6}$$

The recursive statistic for the reverse regression is then $DF_G^{r\ inf}(\tau) = \underset{\tau}{inf}\ DF_G^r(\tau)$. The sequential test for $\widetilde{y}_t^d$ employs the regression:

$$\Delta\widetilde{y}_t^d = \rho(\tau)\,D_t(1-\tau)\,\widetilde{y}_{t-1}^d + \varepsilon_t \qquad t = 1, 2, ..., T \tag{7}$$

where $\Delta\widetilde{y}_t^d$ is defined as in (6) for $t \leqslant (1-\tau)\,T$, and is given as $\Delta\widetilde{y}_t^d = \Delta\widetilde{y}_t - \overline{\Delta\widetilde{y}_2}$ when $t > (1-\tau)\,T$ with $\overline{\Delta\widetilde{y}_2} = (\tau T)^{-1}\sum_{s=(1-\tau)T+1}^{T}\Delta\widetilde{y}_s$. The t-ratio associated with this regression is then denoted as $\overline{DF}_G^r(\tau)$, with the sequential test given as:

$$\overline{DF}_G^{r\ inf}(\tau) = \underset{\tau}{inf}\ \overline{DF}_G^r(\tau) \tag{8}$$

The above recursive and sequential tests are referred to as one-sided by LKSN as they are appropriate for the direction of change in persistence given by the specified alternative hypothesis $\left(H^{01}\text{ or }H^{10}\right)$. As the direction of a potential change in persistence may not be known to the investigator, LKSN suggest joint application of the tests using the forward and reverse realisations of the series, resulting in a joint recursive test given as $min\left(DF_G^{f\ inf}, DF_G^{r\ inf}\right)$ and a corresponding sequential test given as $min\left(\overline{DF}_G^{f\ inf}, \overline{DF}_G^{r\ inf}\right)$. In the following sections, the above tests are considered in the presence of structural change under the null.

3 Breaks in level under the null

3.1 Experimental design

To analyse the behaviour of the above tests of changes in persistence in the presence of level breaks under the null, the following data generation process (DGP) is employed:

$$y_t = \alpha s_t(\lambda) + \xi_t \qquad t = 1, ..., T \tag{9}$$

$$\xi_t = \xi_{t-1} + \eta_t \tag{10}$$

$$\alpha = k\sqrt{T} \tag{11}$$

$$\eta_t \sim i.i.d.\ \mathsf{N}(0,1) \tag{12}$$

$$s_t(\lambda) = \begin{cases} 0 \text{ for } t \leqslant \lambda T \\ 1 \text{ for } t > \lambda T \end{cases} \qquad \lambda \in (0,1) \tag{13}$$

The above DGP draws upon the experimental designs employed by Leybourne *et al.* (1998) and Leybourne and Newbold (2000) to analyse the properties of the DF test and weighted symmetric

DF test of Park and Fuller (1995) in the presence of breaks under the null. The error series $\{\eta_t\}$ is generated using the RNDNS procedure in the Gauss programming language. All experiments are performed over 10,000 replications using a sample size of 100 observations, again following the simulation analysis of Leybourne *et al.* (1998).[3] Following Leybourne and Newbold (2000), the magnitude of the break is proportional to the sample size and is determined by k, with the values $k \in \{0.5, 1.0\}$ considered. Denoting the break fraction as λ, the break in level is imposed after observation λT with $\lambda = \{0.01, 0.02, ..., 0.99\}$. For each replication, the above $DF_G^{f\ inf}(\tau)$, $DF_G^{r\ inf}(\tau)$, $min\left(DF_G^{f\ inf}(\tau), DF_G^{r\ inf}(\tau)\right)$, $\overline{DF}_G^{f\ inf}(\tau)$, $\overline{DF}_G^{r\ inf}(\tau)$, and $min\left(\overline{DF}_G^{f\ inf}(\tau), \overline{DF}_G^{r\ inf}(\tau)\right)$ tests are performed with an intercept only fitted when undertaking GLS detrending ($z_t = 1$). Following LKSN, the value $\overline{c} = -25$ is employed for all tests. The (false) rejections of the H^{11} null hypothesis are noted at the 5% level of significance using the critical values provided by LKSN.

3.2 Experimental results

To ease interpretation, all experimental results obtained are presented graphically. In addition, given the symmetry of the results obtained for the forward and reversed realisations (the behaviour noted for early (late) breaks using the forward series is replicated under late (early) breaks for the reversed realisation) findings are reported for the $DF_G^{f\ inf}(\tau)$, $min\left(DF_G^{f\ inf}(\tau), DF_G^{r\ inf}(\tau)\right)$, $\overline{DF}_G^{f\ inf}(\tau)$, and $min\left(\overline{DF}_G^{f\ inf}(\tau), \overline{DF}_G^{r\ inf}(\tau)\right)$ tests only. Considering the recursive tests, it is apparent from Figure 1 that $DF_G^{f}(\tau)$ can be subject to severe oversizing when applied to a unit root process which experiences a break in level early in the sample period. For example, when $(k, \lambda) = (1, 0.01)$, an empirical size of 34.6% is observed. The results for the joint test employing $min\left(DF_G^{f\ inf}(\tau), DF_G^{r\ inf}(\tau)\right)$ reported in Figure Two show that as a consequence of considering the reversed realisation of the series, the previously noted size distortion is apparent for both early and late breaks. However, as the critical values for the joint test are more negative than those for $DF_G^{f\ inf}(\tau)$, the oversizing is not as severe. In addition to the noted oversizing, Figures 1 and 2 show some evidence of undersizing over a range of breakpoints. The results for the sequential tests reported in Figures 3 and 4 show further evidence of size distortion. From inspection of Figure 3 it can be seen that $\overline{DF}_G^{f}(\tau)$ exhibits oversizing when breaks occur at the start of the sample and at the start of the grid search procedure employed for the test ($\tau = 0.2$). Again, when considering the associated joint test, $min\left(\overline{DF}_G^{f\ inf}(\tau), \overline{DF}_G^{r\ inf}(\tau)\right)$, Figure 4 shows that the size distortion noted for the forward test is apparent at both ends of the range of breakpoints as a consequence of the use of a reversed realisation. The analysis of level breaks has therefore produced three interesting

[3] Results for a single sample size are reported in the interests of brevity. Further similar results for alternative sample sizes are available from the author upon request.

findings. First, the use of individual tests results in severe distortion (oversizing) if breaks occur at the start (end) of the sample when the forward (reverse) test is employed. Second, the use of a joint test increases the possibility of incorrect inference as both early and late breaks produce size distortion. Third, size distortion is increased by the use of sequential rather than recursive tests. To summarise, in the presence of level breaks, the tests of a change in persistence proposed by LKSN can mistakenly conclude that a series which is I(1) throughout the sample has instead experienced a change of classification to I(0) status.

FIGURES ONE TO FOUR ABOUT HERE

4 Breaks in drift under the null

To analyse the break in drift case, the earlier DGP of (9)-(13) is modified as below:[4]

$$y_t = \alpha s_t(\lambda) + y_{t-1} + \xi_t \qquad t = 1, ..., T \tag{14}$$

$$\xi_t \sim i.i.d. \ \mathsf{N}(0,1) \tag{15}$$

$$\alpha = k/\sqrt{T} \tag{16}$$

$$s_t(\lambda) = \begin{cases} 0 \text{ for } t \leqslant \lambda T \\ 1 \text{ for } t > \lambda T \end{cases} \qquad \lambda \in (0,1) \tag{17}$$

The magnitude of the break imposed is now determined by the values $k \in \{10, 20\}$. For each replication the tests of a change in persistence tests are performed with both an intercept and linear trend included in the GLS detrending procedure.

4.1 Experimental results

Figure 5 presents the empirical rejection frequencies for $DF_G^f(\tau)$ in the presence of a break in drift. It is apparent that the oversizing observed for level breaks is now replaced by undersizing, particularly when a break occurs around $\lambda = 0.15$. As the break is imposed later in the sample, the empirical rejection frequency returns to the nominal value of 0.05. Considering the results for the joint test in Figure 6, the previously noted symmetry is apparent again with undersizing evident when breaks occur at either end of the range considered. As would be expected, the larger break ($k = 20$) produces greater distortion than the smaller break ($k = 10$). Turning to the results for the sequential tests in Figures 7 and 8, the undersizing observed in Figures 5 and 6 is magnified. It can therefore be seen that as noted for level breaks, the use of sequential tests increases the size distortion observed for recursive tests.

[4] The treatment of initial conditions, method of random number generation, sample size, and number of replications and discards for the break in drift experiments are the same as for the earlier level break experiments.

FIGURES FIVE TO EIGHT ABOUT HERE

5 Conclusion

In this paper the tests of a change in persistence proposed by Leybourne *et al.* (2003) have been considered in the presence of a structural change under the null. It has been shown that the size properties of the tests differ dramatically depending upon whether the break occurs in the level of a unit root process or its drift parameter. While substantial undersizing can occur in the presence of breaks in drift, breaks in level have been shown to generate oversizing, particularly when the sequential forms of the LKSN test statistics are considered. The findings of the present study therefore suggest that empirical evidence resulting from the application of these tests should be treated with caution as apparent changes in persistence may be spurious. In addition, the results presented suggest that an alternative approach might be considered when constructing a tests of change in persistence given the sensitivity of the GLS-based Dickey-Fuller test to structural change. In particular, given the robustness of the weighted symmetric Dickey-Fuller test to breaks in level and drift (see Leybourne and Newbold 2000), this high-powered test may be considered as a basis for the construction of an alternative testing procedure.[5] This possibility is the subject of further research.

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[5] The use of the weighted symmetric Dickey-Fuller test is also of interest as, in comparison to the GLS-based Dickey-Fuller test, its observed high power is not so heavily dependent upon the treatment of initial conditions (see Müller and Elliott 2003).

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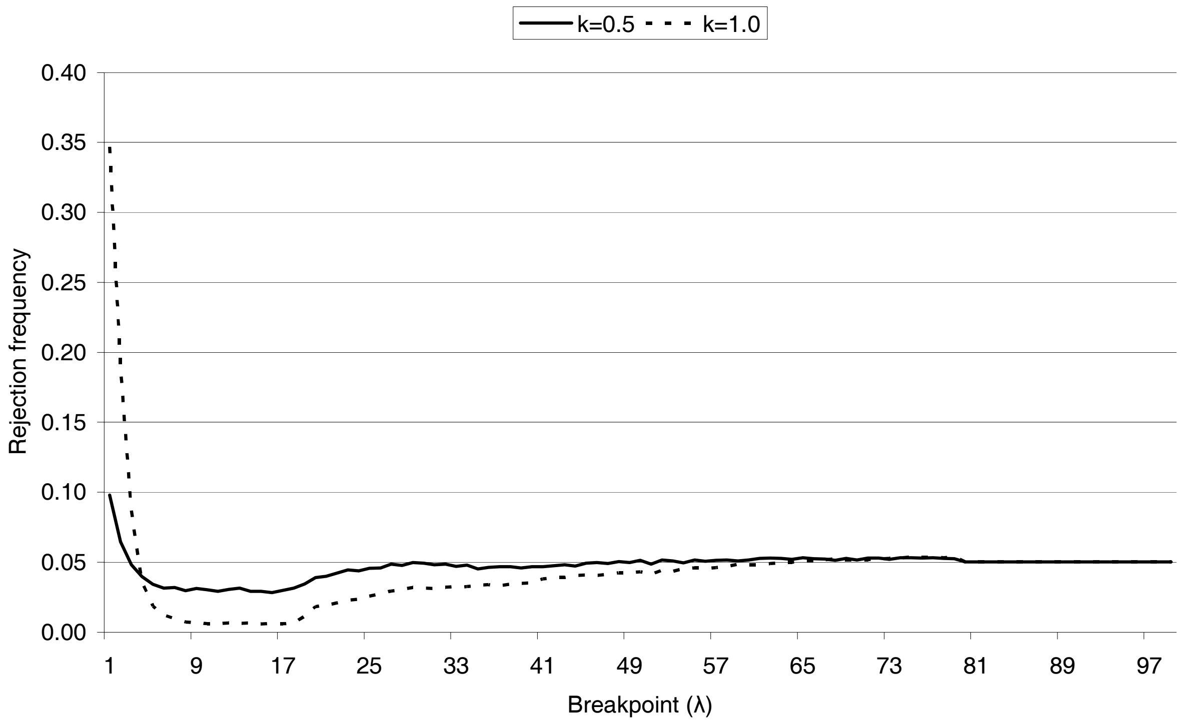

Figure 1: The empirical size of $DF_G^{f\ inf}(\tau)$ in the presence of a level break.

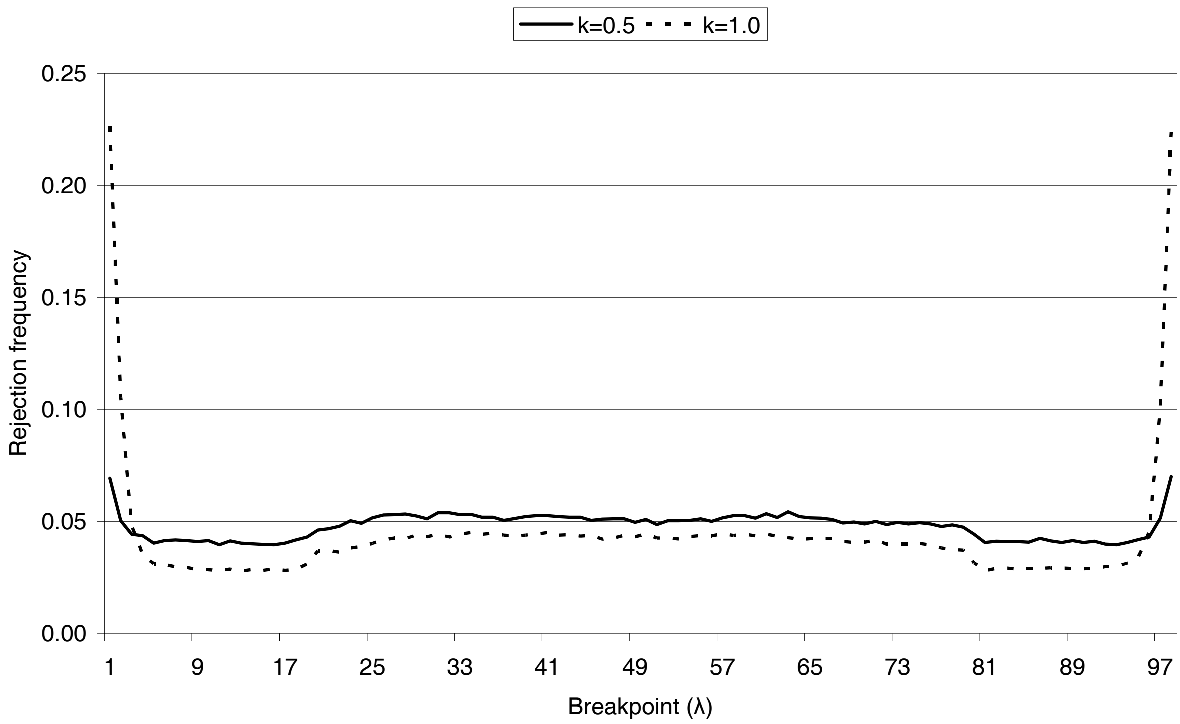

Figure 2: The empirical size of $min\left(DF_G^{f\ inf}(\tau), DF_G^{r\ inf}(\tau)\right)$ in the presence of a level break.

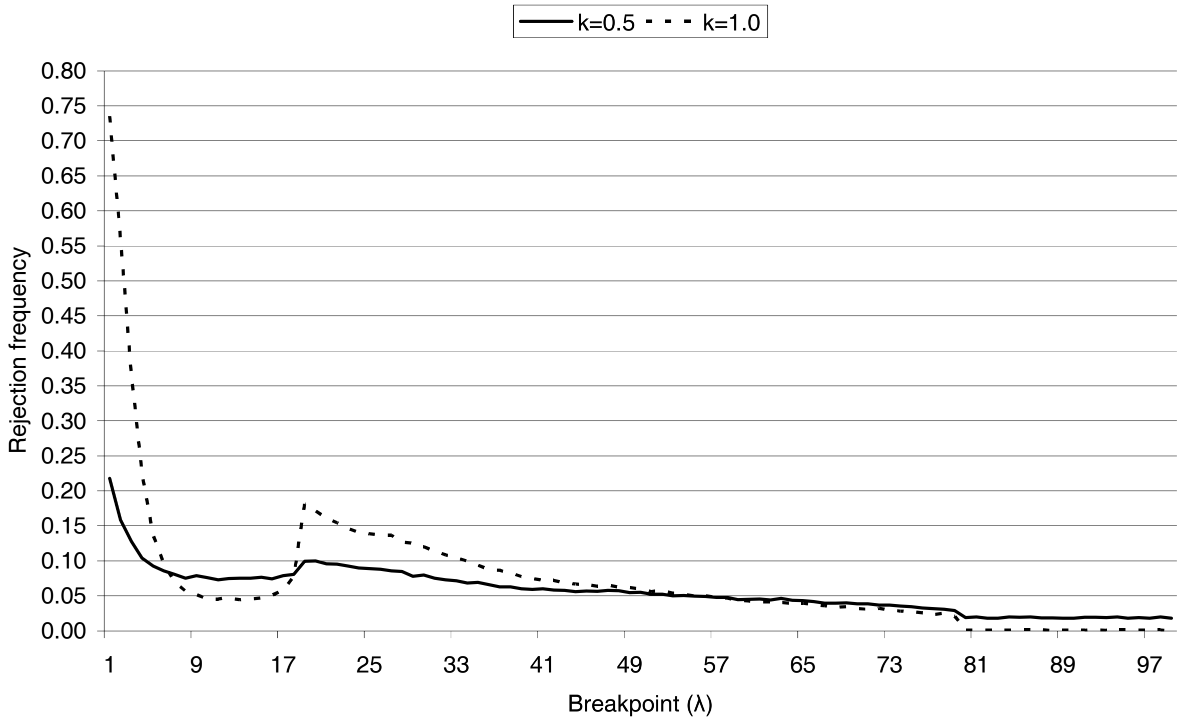

Figure 3: The empirical size of $\overline{DF}_G^{f\ inf}(\tau)$ in the presence of a level break.

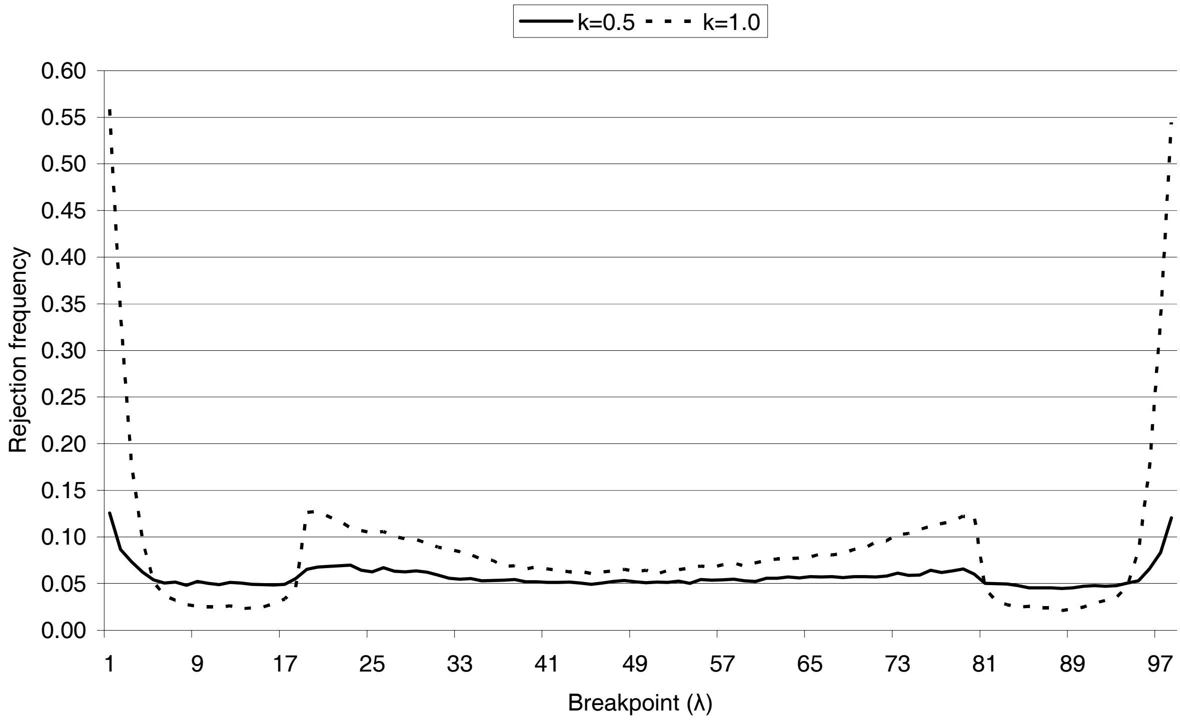

Figure 4: The empirical size of $min\left(\overline{DF}_G^{f\ inf}(\tau), \overline{DF}_G^{r\ inf}(\tau)\right)$ in the presence of a level break.

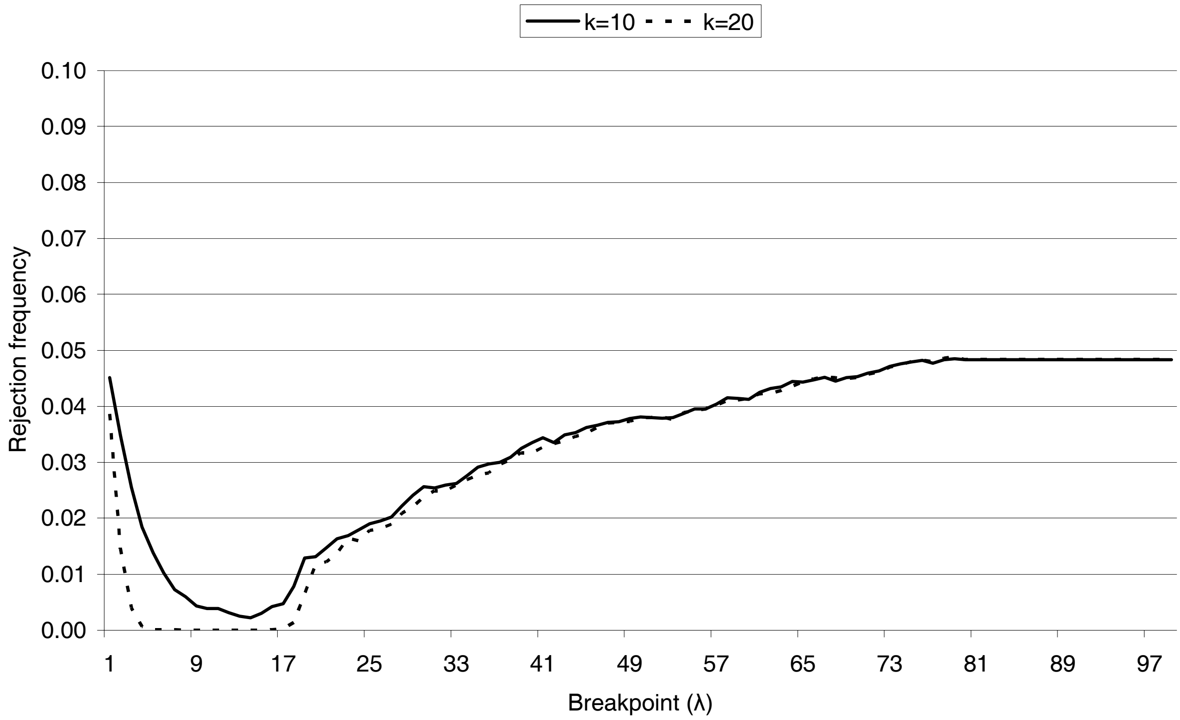

Figure 5: The empirical size of $DF_G^{f\ inf}(\tau)$ in the presence of a break in drift.

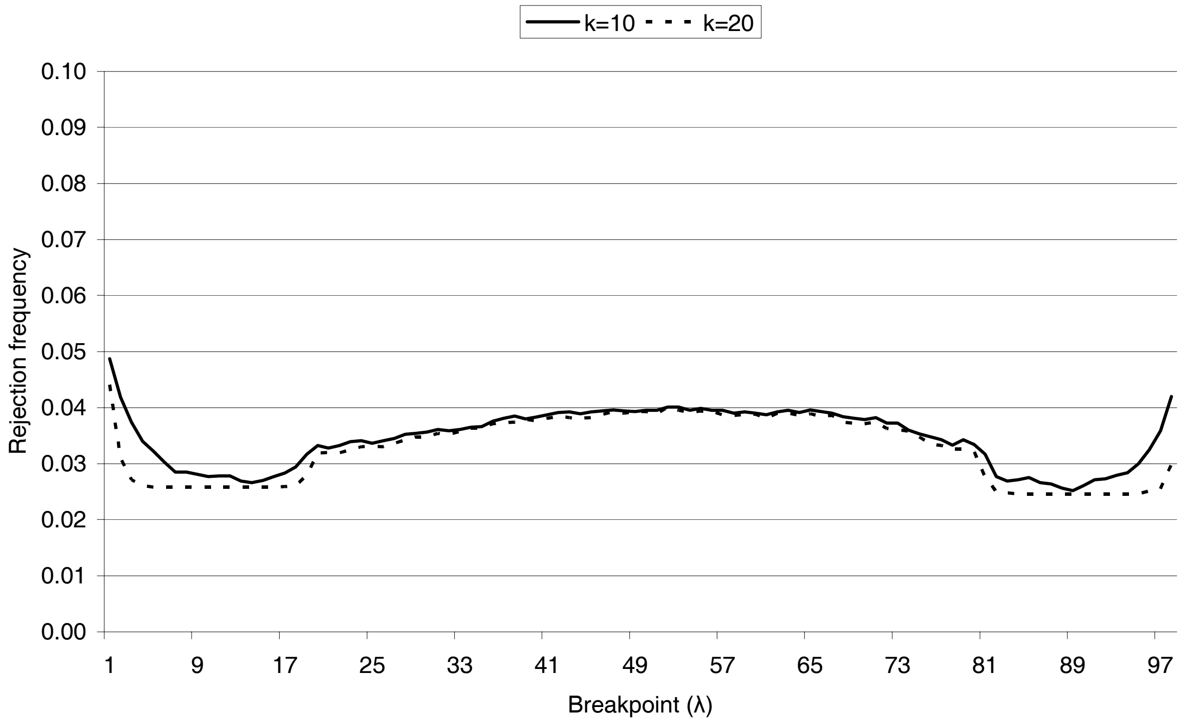

Figure 6: The empirical size of $min\left(DF_G^{f\ inf}(\tau), DF_G^{r\ inf}(\tau)\right)$ in the presence of a break in drift.

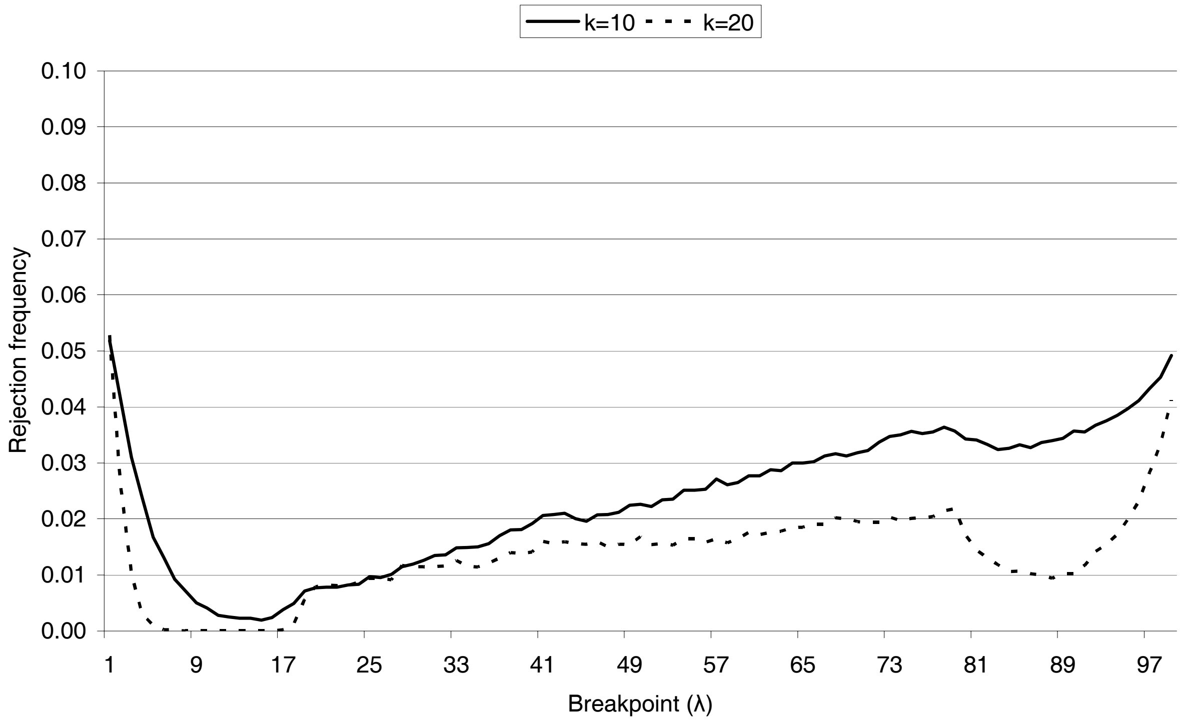

Figure 7: The empirical size of $\overline{DF}_G^{f\ inf}(\tau)$ in the presence of a break in drift.

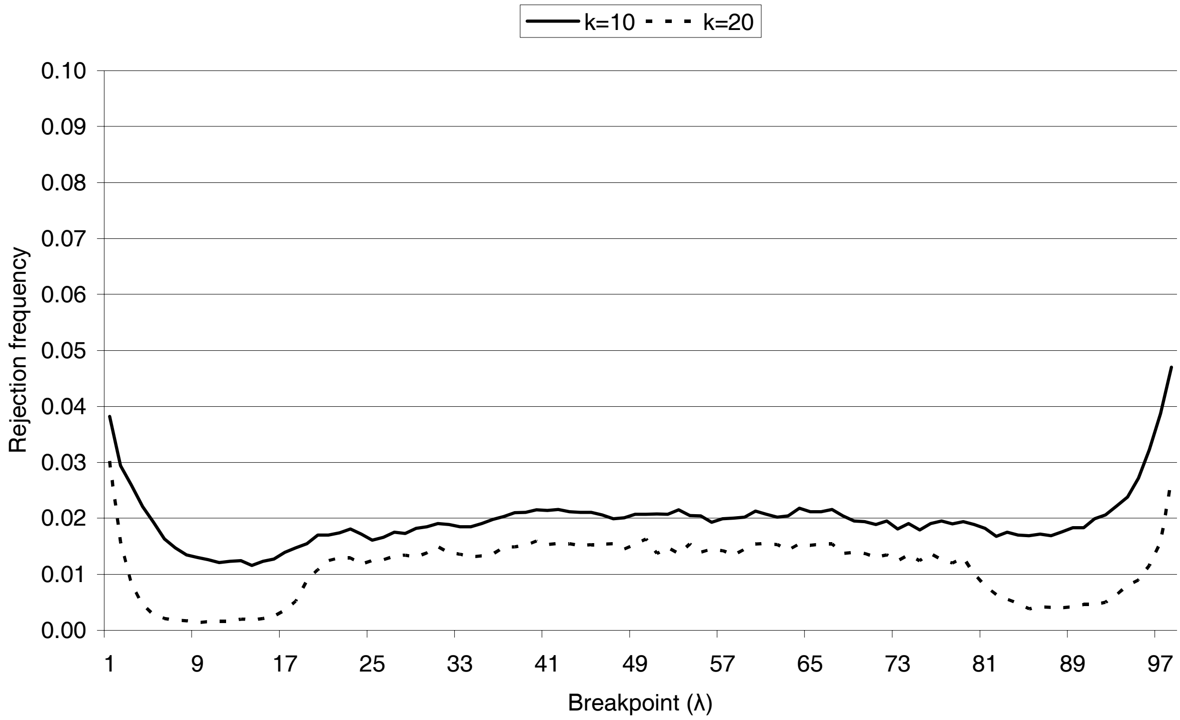

Figure 8: The empirical size of $min\left(\overline{DF}_G^{f\ inf}(\tau), \overline{DF}_G^{r\ inf}(\tau)\right)$ in the presence of a break in drift.