E C O N O M I C S   B U L L E T I N

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FRACTIONAL COINTEGRATION IN THE CONSUMPTION AND INCOME RELATIONSHIP USING SEMIPARAMETRIC TECHNIQUES

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ABSTRACT

This paper deals with the issue of the Permanent Income Hypothesis (PIH) and we show that consumption and income may be fractionally cointegrated. We use a semiparametric frequency domain procedure of Robinson (1995a), and the results show that the UK and the Japanese consumption and income are related in the long run throughout a fractional model.

Keywords: Permanent Income Hypothesis; Fractional cointegration.

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1. Introduction

The relationship between consumption and income is arguably one of the most important in macroeconomics. The most influential and perhaps most widely tested view of this relationship is the Permanent Income Hypothesis (PIH) (see, e.g. Hall, 1978, 1989). According to this theory, consumption and income should be related in the long run and standard econometric techniques have been used to test this hypothesis using cointegrated models. These two variables were analysed from an error correction point of view in Davidson, Hendry, Srba and Yeo, (DHSY, 1978) and from a time series viewpoint in Hall (1978) and others. In the first of these studies, evidence was presented for the error correction model of consumption behaviour from both theoretical and empirical points of view: consumers make plans which may be frustrated; they adjust next period's plans to recoup a portion of the error between consumption and income. Hall (1978) found evidence that US consumption was a random walk and that past values of income had no explanatory power, which implied that income and consumption were not cointegrated. Neither of these studies modelled income itself and it was taken as exogenous in DHSY (1978). Engle and Granger (1987) performed first tests of Fuller (1976) and Dickey and Fuller (1979) to check if both individual variables were in fact I(1). Then, they performed several cointegration tests, concluding that they were cointegrated, though income may be exogenous in view of the error correction representation. Using the same dataset, DeJong (1992) used a Bayesian approach to analyse the cointegration inference in these two variables and his conclusions were mixed.

All these previous works test the PIH by means of classical cointegrating techniques, in the sense that they implicitly assume (or test in an a priori step) that the individual series are I(1), while the cointegrating relationship is I(0) stationary. In this paper, we claim that consumption and income may be fractionally cointegrated. In other words, the error correction term might exhibit long memory, so that deviations from equilibrium are highly persistent. Under these circumstances, a fractional cointegrating relationship provides a much better understanding of the dynamic behaviour of the series and may overcome the mixed evidence found when using classical techniques. The outline of the paper is as follows: Section 2 briefly describes Robinson's (1995a) semiparametric procedure. In Section 3, this method is applied to the UK and the Japanese consumption and income relationship, while Section 4 contains some concluding comments.

2. The semiparametric procedure

We define an I(0) process, $\{u_t, t = 0, \pm 1, ...\}$, as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. In this context, we say that x_t is I(d) if:

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ..., \tag{1}$$

where L is the lag operator ($Lx_t = x_{t-1}$) and d can be any real number. Clearly, the unit root corresponds to d = 1. If d > 0 in (1), x_t is said to be a long memory process, so-called because of the strong degree of association between observations widely separated in time. If $d \in (0, 0.5)$, x_t is covariance stationary, having autocovariances which decay much more slowly than those of an ARMA process, in fact, so slowly as to be non-summable; if $d \in [0.5, 1)$, x_t is no longer covariance stationary but it is still mean reverting, with the effects of the shocks dying away in the long run. Finally, if d ≥ 1, the series is nonstationary and non-mean-reverting.

There exist many approaches for estimating and testing the fractional differencing parameter. Some of them are parametric, in which the model is specified up to a finite number of parameters, (e.g. Fox and Taqqu, 1986; Dahlhaus, 1989; Sowell, 1992; Robinson, 1994a; etc.). However, on estimating with parametric approaches, the correct choice of the model is important. If it is misspecified, the estimates are liable to be inconsistent. In fact, misspecification of the short-run components of the series may invalidate the estimation of the long run parameter. Thus, there may be some advantages on estimating d with semiparametric techniques. In this article, we use a procedure of Robinson (1995a), that we are now to describe.

The Gaussian semiparametric estimate of Robinson (1995a) is basically a 'Whittle estimate' in the frequency domain, considering a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$\hat{d} = \arg min_d \left(\log \overline{C(d)} - 2\, d\, \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right), \qquad (2)$$

$$\overline{C(d)} = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j)\, \lambda_j^{2d}, \qquad \lambda_j = \frac{2\pi j}{T}, \qquad \frac{m}{T} \to 0,$$

where m is a bandwidth parameter number, $I(\lambda_j)$ is the periodogram of the raw time series, and $d \in (-0.5, 0.5)$.[1] Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proved that:

$$\sqrt{m}\,(\hat{d} - d_o) \to_d N(0, 1/4) \quad as\ T \to \infty,$$

where d_o is the true value of d and with the only additional requirement that $m \to \infty$ slower than T. Robinson (1995a) showed that m must be smaller than T/2 to avoid aliasing effects. A multivariate extension of this estimation procedure can be found in Lobato (1999). There also exist other semiparametric procedures for estimating the fractional differencing parameter, for example, the Log-Periodogram regression Estimate (LPE), initially proposed by Geweke and Porter-Hudak (1983) and modified later by Künsch (1986) and Robinson (1995b) and the Averaged Periodogram Estimate (APE) of Robinson (1994b). However, we have decided to use in this article the Whittle estimate because of its computational simplicity: Using the Whittle method, we do not need to employ any additional user-chosen numbers in the estimation (as is the case with the LPE and the APE). Also, we do not have to assume Gaussianity in order to obtain an asymptotic normal distribution, the Whittle method being more efficient than the LPE.

[1] Velasco (1999a, b) has recently showed that the fractional differencing parameter can also be consistently semiparametrically estimated in nonstationary contexts by means of tapering.

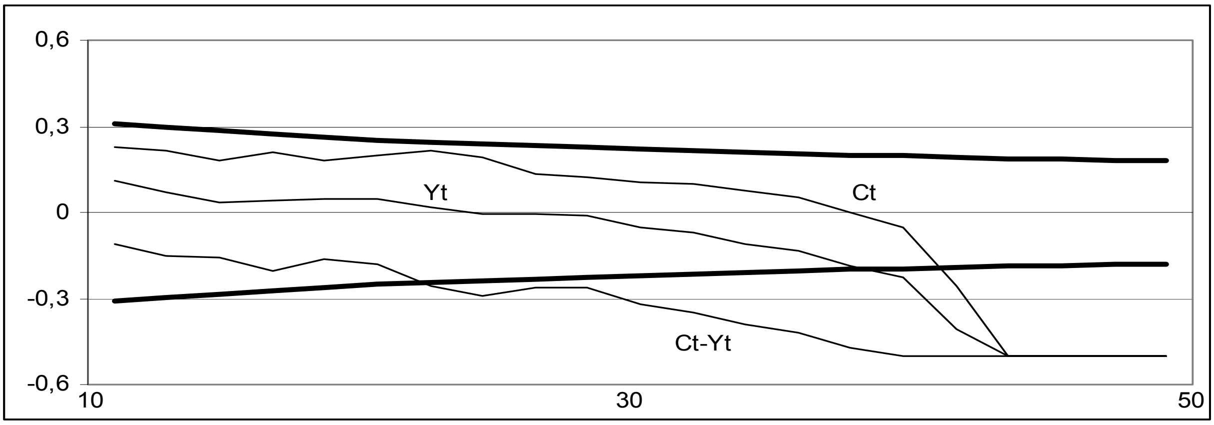
3. An application to the UK and the Japanese consumption and income

The time series data analysed in this section correspond to the logarithmic transformations of the quarterly, seasonally unadjusted, consumption expenditure on non-durables (C_t), and personal disposable income (Y_t), in the UK and Japan. The time period for the UK is 1955q1–1984q4, and for Japan 1961q1-1987q4.

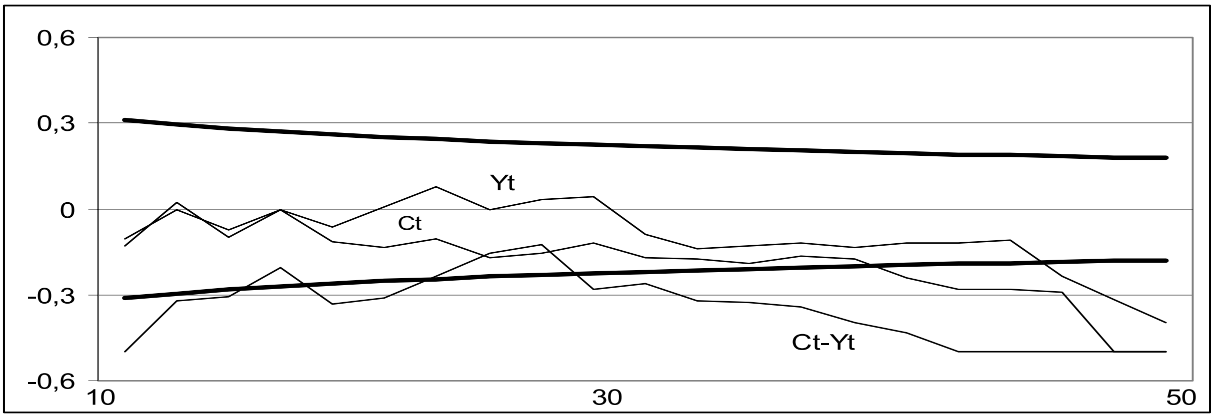
FIGURE 1

Estimates of d based on Robinson (1995a) for Ct, Yt and Ct - Yt

JAPAN



UNITED KINGDOM



The horizontal axe refers to the bandwidth parameter number m, while the vertical one corresponds to the estimated values of d.

Figure 1 displays the estimates based on Robinson (1995a), i.e., $\hat{d}$ given by (2), for a range of values of m from 10 to 50.[2] Since the time series are clearly nonstationary, the analysis will be carried out based on the first differenced data, adding then 1 to the estimated values of d to obtain the proper orders of integration of the series. We also display in the figure the

[2] Some attempts to calculate the optimal bandwidth numbers have been examined in Delgado and Robinson (1996) and Robinson and Henry (1996). However, in the case of the Whittle estimator, the use of optimal values has not been theoretically justified.

confidence intervals corresponding to the null hypothesis: d = 0, (i.e., d = 1 in the original series). Starting with Japan, we see that the estimates for C_t and Y_t are within the unit root interval for most of the values of m. However, if we look at its difference, (C_t - Y_t), the estimates are always smaller, being below that interval for most of the values of m, especially if m > 30. Similarly for the UK, the estimates for the individual series are practically all within the unit-root interval, while those corresponding to the differenced series are below 1, with the estimates of d fluctuating in all cases around –0.3 (i.e., 0.7 in the original series). We can summarise the results in this figure by saying that the UK and the Japanese consumption and income series are all I(1) variables, while their differences are I(d) with d < 1, implying that there exists a fractionally cointegrated relationship between consumption and income according to a simplistic version of the PIH (DHSY, 1978).

4. Concluding comments

In this paper we have shown that the UK and the Japanese consumption and income are fractionally cointegrated. We have used a semiparametric frequency domain procedure of Robinson (1995a). The results show that, in both countries, the individual series are I(1) while the differences appear to be I(d) with d < 1, showing thus mean reversion in the long run equilibrium relationship.

The procedure implemented in this article can also be used to estimate and test the order of integration on the residuals from the cointegrating regression. In other words, it can be performed in a similar way as in Engle and Granger (1987), testing the null hypothesis of no cointegration against the alternative of (fractional) cointegration. In fact, Caporale and Gil-Alana (2002) use this approach on real exchange rates. However, a problem with this procedure appears in that the residuals used are not actually observed but obtained from minimizing the residual variance of the cointegrating regression and, in finite samples, the residual series might be biased towards stationarity. In that respect, we have preferred to use the procedure based on observed data and test for cointegration, imposing the cointegrating vector (1, -1). Other more elaborated techniques of fractional cointegration have been developed by Dueker and Startz (1998), Martin (2001), Robinson and Hualde (2002, 2003) and others, and they should also be performed on these series. There also exists a reduced-rank procedure, suggested by Robinson and Yajima (2002). However, it is not directly applicable here, since that method assumes I(d) stationarity (d < 0.5) for the individual series while we have shown that they are in fact I(1) nonstationary processes.

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