Modeling the French Consumption Function Using SETAR Models

Gilles Dufrenot *University of Paris 12*

Valerie Mignon University of Paris 10 – THEMA

Abstract

We provide new estimations on aggregate consumption series in France using the framework of non-stationary threshold models. Most macroeconomists agree with the idea that, since the beginning of the seventies, the saving ratio has evolved irregularly. Such irregularities are usually interpreted as being caused by mispecification problems or measurement errors. We suggest another explanation that strengthens the role played by structural breaks caused by endogenous factors such as habit formation. In this view, we use threshold models (SETAR) to study both the dynamics of short and long term in order to account for the existence of asymmetric effects in the relationship between consumption and some of its determinants. The estimations and forecasts obtained show that the SETAR error correction model leads to better performance than other specifications such as the usual linear error correction model, the quadratic error correction model and the cubic error correction model.

Citation: Dufrenot, Gilles and Valerie Mignon, (2004) "Modeling the French Consumption Function Using SETAR Models." *Economics Bulletin*, Vol. 3, No. 20 pp. 1–16

Submitted: April 22, 2004. Accepted: June 17, 2004.

URL: http://www.economicsbulletin.com/2004/volume3/EB-04C20018A.pdf

1 Introduction

Over the last two decades the French saving ratio experienced what seemed to be irregular dynamics. From 1970 to 1978, the saving ratio increased, being approximately 19%. From 1979 to 1987, it decreased to 11% and then jumped again between 1988 and 1992. Since then, the saving ratio seems to have remained stable. These observations raised questions about the factors that cause such a seemingly irregularity. With regard to recent empirical surveys, it seems that no clear answer has been provided. There are still some doubts about the relevance of applied works which have been conducted within the framework of the theories that have proven to be key pointers for consumption spending analysis, namely Friedman (1957)'s permanent income theory, Ando-Modigliani (1963)'s life cycle theory, and the approaches that strengthen the role of wealth effects and uncertainty. Many questions have been raised about plausible mispecification problems (for a survey of empirical studies, the reader may refer to Allard (1992) and Bonnet and Dubois (1995)).

In this paper, we propose an approach that highlights the role played by regime-shifts phenomena. The adjustment rates of consumption and savings depend upon how quickly some underlying variables evolve above or under critical or threshold levels that determinate different regimes. State varying consumption dynamics are incorporated, for instance, into habit formation models. Shifting preferences are attributed to a stock variable that can reach several critical levels and this distorts the shape of consumption and savings paths. It has been shown, in the literature, that once consumption is analyzed as a stock dependent variable, habit formation may yield persistent and volatile fluctuations in the time profile of savings (see, among others, Benhabib and Day (1981), Iannaccone (1986), Dockner and Feichtinger (1993), Feichtinger et al. (1994a, 1994b)). Apart from habit formation, there are other variables that are subject to changes in regimes and that may be a source of instability in the dynamics of the saving ratio. Indeed, aggregate consumption is an advanced indicator of macroeconomic variables that are characterized by regime changing dynamics. Among these variables are the following:

- the unemployment rate, characterized by moving equilibria (see Skalin and Teräsvirta (1998)),
- the Gross National Product, characterized by multiple smooth transition dynamics (see Dijk and Franses (1999)),
- the interest rate term structure, reflecting either the changing nature of policy makers' preferences (Floden (2000)) or the asymmetry of monetary policy (see Aftalion (1997) and Dufrénot et al. (2004)).

These variables are present in a consumption equation. So it is plausible that the steepness of the saving functions, i.e., the rate through which people change their saving pattern, is subject to the same kind of dynamics observed for the unemployment rate, the interest rate term structure and so on.

In all the papers cited above, whether one considers habit formation or other factors, it is stressed that a clear way to approach the issue of state-varying dynamics is to use nonlinear models. The form of nonlinearity may be hard to identify. Several suggestions have been made in the literature (Markov switching models, Hamilton's models, smooth transition models, threshold models). In this paper, we explore a class of nonlinear models, already used in many applications, named the self-exciting threshold autoregressive models (henceforth, SETAR).

In these models nonlinearity is handled through piecewise linear equations. Unfortunately, linearity then appears as a "local property", whereas nonlinearity is actually a global property. In SETAR models, the state variables are selected by the data and they evolve between different regimes delimited by threshold values. The reasons that lead us to choose SETAR specifications are the following. Firstly, they have the advantage of simplicity since they are piecewise linear. Secondly, they can be interpreted as reduced forms of theoretical models. This is at least true when the changing shape of the saving rate is due to habit formation. Consequently, even though the econometric approach used in this paper relies upon time series techniques, it does not lack

theoretical foundations. Thirdly, taking into account the exogenous variables in a nonlinear model enhances the question of nonlinear cointegration. Even though there has been a considerable amount of literature recently dedicated to this question (see Dufrénot and Mignon (2002a) for a survey), we rely here on a simple approach: given the property of piecewise linearity that characterizes SETAR models, the testing procedures rely on the traditional ones used in linear models.

The paper is organized as follows. In section 2, we analyze the dynamics of the consumption function using SETAR models. So doing, we privilege the role of lagged consumption. In section 3, we incorporate exogenous variables in our models and examine whether this induces changes in main conclusions. Section 4 concludes the paper.

2 Univariate SETAR models and the dynamics of consumption

2.1 A model with threshold adjustment in consumption

The incidence of SETAR models in determining the dynamics of consumption warrants, first of all, an analysis of their theoretical background. To our knowledge, this is the first attempt in making these models economically meaningful. Indeed, they are usually used in the literature as econometric models without theory (as is the case for many other time series models).

Our theoretical arguments are somewhat similar to suggestions made in earlier papers by Orphanides and Zervog (1995) and Feichtinger $et\ al.$ (1994b). The consumption experience is described by a stock variable s that can be viewed as the customary level of consumption. s evolves according the following dynamics:

$$s_{t+1} = (1 - \delta) s_t + c_t \tag{1}$$

where $\delta \in [0,1[$ and c_t is the current consumption. The consumer utility function is written:

$$U\left(c_{t},s_{t}\right)\tag{2}$$

and satisfies the following properties:

- U is twice continuously differentiable for $c_t, s_t > 0$,
- U'_c is strictly increasing in c_t ,
- U is increasing in c_t and s_t and has strict concavity.

The budget constraint is written:

$$c_t + p_t s_t = R_t \tag{3}$$

We normalize the price of consumption to unity. p_t is the price of current stock of consumption and R_t is the income earned during period t.

The consumer faces the following decision problem:

$$\underset{\{c_t\}}{Max} \sum_{t=0}^{\infty} \beta^t U(c_t, s_t) \tag{4}$$

subject to the following constraints:

 $c_t + p_t s_t = R_t, c_t \ge 0, s_t \ge 0, s_{t+1} = (1 - \delta) s_t + c_t, s_0$ given.

The problem yields a Bellman equation:

$$V\left(s_{t}\right) = \underset{\left\{c_{t}\right\}}{Max} \left[\omega\left(s_{t}\right) + \beta V\left(s_{t+1}\right)\right], \omega\left(s_{t}\right) = U\left(R_{t} - p_{t}s_{t}, s_{t}\right)$$

$$(5)$$

Assume that there exists a value function f such that:

$$c_t = f\left(s_t\right) \tag{6}$$

The exact form of the function f depends upon how habit formation is modelized. We assume time non-separability: the way an individual values his current consumption is influenced by the level of consumption reached in the past. Note as H the habit formation function. Our assumption implies:

$$U_c' = H\left(c_t, s_t\right) \ \forall t \tag{7}$$

H can be interpreted as a learning rule that begins immediately after initial consumption and that causes the consumer to adjust consumption to a desired level for period t. The consumption target is endogenously determined as a result of a "signal" extraction in the learning by consuming experience with equation (1) as the updating rule. Such a behavior is consistent with empirical observations made by psychologists (see Ainslie (1975), Lang (1983)). Unless one supposes that the consumers are psychotic, the adjustment of consumption takes time and occurs intermittently. As suggested in other papers, there are critical levels (or threshold levels) in the habit formation mechanism that induce some moving in current marginal utility and thereby in current consumption (see Feichtinger $et\ al.\ (1994)$).

A very simple model allowing threshold adjustment is as follows:

$$U'_{c} = \begin{cases} 1U'_{c} & \text{if } s_{t} < b_{1} \\ 2U'_{c} & \text{if } b_{1} \leq s_{t} < b_{2} \\ \dots \\ nU'_{c} & \text{if } b_{n-1} \leq s_{t} < b_{n} \end{cases}$$
(8)

This equation says that the way an individual reacts to past consumption depends upon the levels reached in past consumptions. The latter are summarized by the stock variables s_t . The threshold levels $b_1, b_2, ..., b_{n-1}, b_n$ may be thought of as being endogenously determined and vary among consumers. It is seen that this yields several regimes in consumption. Note also that the number of regimes varies between individuals. (8) is a particular case of the specification envisaged in Feichtinger et al. (1994) to account for an s-shaped consumption function.

Assume for instance a logarithmic utility function:

$$U(c_t, s_t) = \begin{cases} \phi_1 \ln c_t + \theta_1 \ln s_t & \text{if } s_t < b_1\\ \phi_2 \ln c_t + \theta_2 \ln s_t & \text{if } b_1 \le s_t < b_2\\ \dots\\ \phi_n \ln c_t + \theta_n \ln s_t & \text{if } b_{n-1} \le s_t < b_n \end{cases}$$
(9)

The maximization problem yields the following piecewise linear dynamic model:

$$c_{t} = \begin{cases} \alpha_{1}c_{t-1} + \gamma_{1}s_{t} & \text{if } s_{t} < b_{1} \\ \alpha_{2}c_{t-1} + \gamma_{2}s_{t} & \text{if } b_{1} \leq s_{t} < b_{2} \\ \dots \\ \alpha_{n}c_{t-1} + \gamma_{n}s_{t} & \text{if } b_{n-1} \leq s_{t} < b_{n} \end{cases}$$

$$(10)$$

The parameters in (10) are functions of $\beta, \phi_1, ..., \phi_n, \theta_1, ..., \theta_n, \delta$. The econometric counterpart of equation (10), which includes error terms for estimation purposes, is known in the literature as a $SETAR(n, p_1, ..., p_n)$ model:

$$c_{t} = \begin{cases} \alpha_{1}c_{t-1} + \gamma_{1}s_{t} + \epsilon_{t}^{1} & \text{if } s_{t} < b_{1} \\ \alpha_{2}c_{t-1} + \gamma_{2}s_{t} + \epsilon_{t}^{2} & \text{if } b_{1} \leq s_{t} < b_{2} \\ \dots \\ \alpha_{n}c_{t-1} + \gamma_{n}s_{t} + \epsilon_{t}^{n} & \text{if } b_{n-1} \leq s_{t} < b_{n} \end{cases}$$

$$(11)$$

where the vector $(\epsilon_t^1, ..., \epsilon_t^n)$ is a collection of iid processes with a variance-covariance matrix Ω . n is the number of regimes. $p_1, ..., p_n$ are the maximum lags within each regime. Note that consumption stock may be viewed as a combination of past consumption levels:

$$s_t = \sum_{i=1}^p \mu_i c_{t-i} \tag{12}$$

so that c_t is a function of $c_{t-1}, c_{t-2}, ..., c_{t-p}$. Also, it is possible that the variable s_t that enters in the "if condition" is a partition of the variable s_t in the main relationship. So, to be rigorous, we must write a relation like:

$$c_{t} = \begin{cases} \alpha_{1}c_{t-1} + \gamma_{1}s_{t} + \epsilon_{t}^{1} & \text{if } \tilde{s}_{t} < b_{1} \\ \alpha_{2}c_{t-1} + \gamma_{2}s_{t} + \epsilon_{t}^{2} & \text{if } b_{1} \leq \tilde{s}_{t} < b_{2} \\ \dots \\ \alpha_{n}c_{t-1} + \gamma_{n}s_{t} + \epsilon_{t}^{n} & \text{if } b_{n-1} \leq \tilde{s}_{t} < b_{n} \end{cases}$$

$$(13)$$

where $\tilde{s}_t \subseteq s_t$.

2.2 The econometric analysis

In this section, we briefly explain econometric methodology, present the data and then comment our results.

2.2.1 The econometric methodology

The model (13) is estimated using the first differences of the consumption variables. We do this to avoid spurious regression problems due to the non stationarity of the raw data¹. The estimation requires several steps. To avoid extensive technical details, we shall summarize the main aspects of the techniques used and refer the reader to other papers for more details. The methodology used is as follows.

Step 1. We first need to test for threshold nonlinearity, in order to see whether the SETAR specification is a good way to capture the nonlinearities induced by habit formation in the consumption decisions. This is done by applying the linearity test suggested by Tsay (1989). The basic idea is to test the null hypothesis that the consumption variable follows a linear model against the alternative hypothesis that there exist switching regimes by changing points in the data. Thus, the test heavily relies upon recursive regressions. Further, since the presence of regimes in the SETAR models are due to threshold variables (in our case an element of s_t), it is useful to order the data according to the increasing values of the threshold variable. Thereby, to compute the test statistic, we proceed as follows.

- An ARMA(P,Q) model is fitted to the rearranged data in order to select the optimal values of the maximum lags P and Q.
- Then, given the selected values for P and Q, a CUSUM regression is conducted and the corresponding standardized residuals are computed. Let us note $\{\hat{e}_t\}$ the vector of the standardized residuals.
- One does the regression of the \hat{e}_t 's on the lags of $\{\Delta c_t\}$ and computes the residuals $\{\hat{\epsilon}_t\}$.
- The statistic that allows testing for linearity is computed as:

$$\hat{F} = \frac{\left(\sum \hat{e}_t^2 - \sum \hat{\epsilon}_t^2\right)/nd1}{\sum \hat{\epsilon}_t^2/nd2}$$
(14)

¹Results of non stationarity tests (Augmented Dickey-Fuller, Phillips-Perron, and Kwiatkowski, Phillips, Schmidt and Shin) are not reported here but are available upon request to authors.

and follows an F distribution F(nd1, nd2). nd2 is the number of degrees of freedom corresponding to the second regression and nd1 is the difference between the degrees of freedom corresponding respectively to the first and second regressions. This linearity test is done for a set of different plausible variables.

Step 2. This step allows us to locate the threshold values (the coefficients $b_1, ..., b_n$) corresponding to the threshold variables selected in the preceding step. This can be done by plotting the standardized residuals or the t-ratios of the coefficients of recursive regression versus selected threshold variables. When there are no regime shifts, the standardized residuals and the t-ratios smoothly converge to a given value as recursion occurs. The presence of regimes in the data is thus identified by the fact that the t-ratios and residuals change direction at the threshold value.

Step 3. Once the threshold variables and the corresponding threshold values are identified, the SETAR model can be estimated using standard regression methods. The estimation is done on the original data $\{\Delta c_t\}$ but the model includes dummy variables that allow us to separate the different regimes according to the threshold variable and the threshold values.

Step 4. A residual analysis is conducted and it is interesting to draw the distribution of the forecasts corresponding to the regression in step 3. Very often, the presence of different regimes in the dynamics of a variable induces multimodel selection.

Step 5. This last step amounts to comparing the SETAR model with linear models.

2.2.2 Data and results

We give here the data selected for consumption. Explanatory variables are described in the next section where we discuss issues concerning cointegration problems in SETAR models when the consumption/savings decisions depend upon exogenous variables.

We use the French quarterly statistical series from the O.E.C.D. database. The data cover the period from 1970:1 to 2000:4. The estimation period ranges from 1970:1 to 1996:4 and the rest of the data is kept for forecast analysis. For the consumption series, we chose household consumption in the sense of national accounts. The series is seasonally adjusted and deflated by the consumer price index. Real consumption is then transformed in logarithm. As indicated above, unit root tests on the logarithmic series show the presence of non stationary dynamics. So, we work with first differences of the series. In what follows, Δc_t will denote the first differences of the logarithm of real consumption².

Our best ARMA model on Δc_t is an ARMA(3,4) according to the AIC criteria and the usual residual tests. The results concerning the linearity test are reported in table 1. We indicate both the computed statistics and their corresponding p-values. It should be noted that among the different candidates for transition variables, the highest value of the Fisher statistic corresponds to Δc_{t-6} . This suggests that learning by consuming behavior influences current consumption with regard to the level of consumption achieved during the last six quarters. Using the recursive plot methods, we identify a threshold value of 0.77%.

Table 1. The nonlinearity test for Δc_t . Period 1970:1 - 1996:4

Candidates for transition variables	Fisher statistic	p-value
Δc_{t-2}	5.98069	$0.173.10^{-4}$
Δc_{t-3}	4.95298	$0.132.10^{-3}$
Δc_{t-4}	6.36006	$0.979.10^{-4}$
Δc_{t-5}	26.5616	0
Δc_{t-6}	38.8281	0
Δc_{t-7}	15.8970	0
Δc_{t-8}	15.209	0

²To avoid an overabundance of tables and figures, we do not report intermediate results concerning the unit root tests, the estimated ARMA model, and plots of the standardized residuals and t-ratios against the threshold value. These are available upon request to authors.

Given these results, we estimate the following SETARMA model:

$$\Delta c_t = \left[\mu_1 + \sum_{i=1}^{p_1} \phi_i^1 \Delta c_{t-i} \right] I_t^1 + \left[\mu_2 + \sum_{i=1}^{p_2} \phi_i^2 \Delta c_{t-i} \right] I_t^2 + \sum_{i=1}^{q} \theta_i \epsilon_{t-i} + \epsilon_t$$
 (15)

where
$$\epsilon_t \sim N\left(0,\sigma_\epsilon^2\right)$$
, $I_t^1 = \left\{ \begin{array}{ll} 1 & \text{if } \Delta c_{t-6} \leq 0.0077 \\ 0 & \text{if } \Delta c_{t-6} > 0.0077 \end{array} \right.$, $I_t^2 = \left\{ \begin{array}{ll} 1 & \text{if } \Delta c_{t-6} > 0.0077 \\ 0 & \text{if } \Delta c_{t-6} \leq 0.0077 \end{array} \right.$ A moving average component is added to the SETAR model with regard to possible autocor-

A moving average component is added to the SETAR model with regard to possible autocorrelated residuals. Using the information criteria (AIC, SIC), the best model yields $p_1 = 3$, $p_2 = 1$ and Q = 4. The estimated coefficients are given in table 2.

Table 2. Univariate SETARMA(2,3,1,4) model³

μ_1	ϕ_1^1	ϕ_1^2	ϕ_2^1	ϕ_3^1	θ_4	
$0.001496 \ (2.6213)$	$0.814 \ (7.138)$	$0.832 \atop (9.97)$	$ \begin{array}{c} -0.321 \\ (-2.23) \end{array} $	0.249 (2.192)	$0.0099 \ (2.18)$	
St	Statistics (p-values are in parenthesis)					
DW	LB F S K JE					
2.02	0.2411	47.797	0.703	4.663	$90.92 \ (0.00)$	
DH(lag1) = 68.669(0), DH(lag2) = 69.115(0)						
DH(lag3) = 68.86(0)						

For the estimated coefficients, t-ratios are in parenthesis. \overline{DW} : Durbin Watson statistic. LB: Ljung-Box statistic for autocorrelation. F is the Fisher statistic of the test $\phi_1^1 = \phi_1^2$. S is the skewness of the residuals. K is the kurtosis of residuals. DB is the Jarque-Bera statistic for normality. DB is the Doornik-Hansen multivariate normality test.

As one may observe from table 2, the estimated SETARMA gives rise to two lag structures in the dynamics of consumption, depending upon the observed regime. The SETAR specification is adequate in comparison to a linear model, as suggested by the Fisher test. Heteroskedasticity is a key feature of SETAR models, as residual variances are different across the regimes. This is also the case here where the Jarque-Bera test yields to reject the null hypothesis of normality. This conclusion is due to a high kurtosis. Also, we note that the DH test rejects the null hypothesis of a Gaussian distribution for the residuals of our model. This can be easily understood with regard to the plot of the distribution of the residuals of the SETARMA model which indicates that the presence of two regimes gives rise to a bimodal distribution.

From the estimated coefficients, we see that habit formation does not necessarily make consumption smoother. Notably, in regime 2, the coefficients of the autoregressive term alternate between positive and negative values. As suggested in other papers dealing with habit formation, the changing pattern in aggregate consumption may illustrate the fact that the usual self-preferences of an individual reveal time inconsistency instead when consuming induces a learning process (see Winston (1980), Schelling (1984)). In such conditions, the consumption path might be cyclical: the consumption experience either raises or lowers future utility and thus future consumption. This is illustrated by the changing signs.

For comparison purposes, table 3 reports the root mean squared error (RMSE) of the static and dynamic forecasts corresponding to several models. All the models are estimated over the period 1970:1 - 1996:4. and the forecasts are computed over the period 1997:1 - 2000:4.

Table 3. RMSE corresponding to out-of-sample forecasts (1997:1 - 2000:4)

	Static forecasts	Dynamic forecasts
SETARMA $(2,3,1,4)$	$1.3742.10^{-3}$	$1.6037.10^{-3}$
ARMA(1,1)	0.0212	0.0267
AR(1)	$3.122.10^{-3}$	$7.885.10^{-3}$
Random walk	0.04	0.219

³We only report the coefficients that are statistically significant.

It is clear that the forecasts of the SETARMA model outperform the forecasts of the linear models. However, it is possible that other nonlinear models could be used to account for habit formation. Even though threshold effects may characterize the dynamics of consumption at a micro level, it is not clear with regard to heterogenous behaviors whether such thresholds would still be present in aggregate behavior. SETAR models may thus be considered as an approximation of other nonlinear models.

3 A SETAR model with exogenous variables: Threshold adjustments to equilibrium

3.1 Economic motivation

So far, we have considered that threshold effects and regime shifts were endogenous to consumer preferences. The changing shape of consumption may, however, come from an unstable environment. Attention will now be turned to the role played by the income, the inflation rate, the unemployment rate, the slope of the term structure of interest rates. The important question concerns the way threshold effects can be introduced in presence of these exogenous variables.

Recent papers have suggested that consumption decisions, and more generally all economic decisions, could be interpreted as a consequence of intertemporal optimizing decisions where consumption, or the endogenous variable, is subject to partial adjustments to the long run equilibrium (see Escribano and Pfann (1998), Peel and Davidson (1998)). For consumption, some basic arguments are the following. People fix target levels of consumption. These targets reflect different situations. For instance, for credit constrained French households, the ratio of debt over consumption is legally fixed at 30%. In the presence of a shock (an increased inflation rate), this ratio goes above this limit and households may try to return to this level by reducing consumption. A positive shock (a higher inflation rate) that induces a decrease in the real interest rate, yields the opposite behavior. In this case, adjustment to equilibrium is likely to be asymmetric because it is easier to increase one standard of living, than to reduce it. Another example is Friedman's permanent income theory. People have fixed spendability ratios for different assets. A shock (such as a modification in the behavior of monetary policy-makers) implies adjustment toward the desirable ratios, by either increasing or decreasing consumption. Also in this case, the adjustment costs of changing can be asymmetric because small modifications in the monetary policy alter individual purchases more than large modifications do. Another cause of asymmetry in the consumption adjustment over time is the following. The way consumers react to variations of the inflation and unemployment rates may vary with the consumption over income ratio. Consumers are not identical in their spending habits. Threshold models may be used to distinguish between those who spend an important share of their earnings in consumption and those who are less "spend-thrifty". Some previous studies on French data suggest that people with a relatively important share of income in consumption are prudent during periods of dull markets, and conversely, increase their spending when the economy keeps going. They are very sensitive to variations of the unemployment rate over the business cycle. Consumers whose consumption ratio is relatively low are more sensitive to the inflation rate. Heterogeneity in spending thereby creates a dichotomy between the effects of the unemployment rate and the inflation rate on the propensity to consume (and thus the propensity to save). Structural breaks may characterize the dynamics of consumption depending upon the group whose behavior is dominant over the business cycle.

With regard to the possible asymmetric adjustment toward long term equilibrium, it has been suggested to use nonlinear error correction models through nonlinear error term mechanism. Several specifications can be used to account for the long run nonlinear relationship between consumption and its main determinants. Peel and Davidson (1998) estimate a bilinear error mechanism model on real expenditure data in the U.K. and suggest that such a model accounts for plausible abrupt changes in adjustment speeds. Here, we shall rely on a SETAR formulation and compare our results to other nonlinear formulations: quadratic and cubic error correction models.

3.2 Error correction models with SETAR errors

In this section, we briefly expose the model that is being estimated, as well as the econometric procedures used to estimate the equations.

3.2.1 The SETAR - Error Correction Model (SETAR-ECM)

We privilege the following interpretation of a nonlinear ECM. Consider two variables x and y, where x is endogenous. Further, let $z_t = x_t - \alpha y_t$ and F a nonlinear function. The ECM we consider is as follows:

$$\Delta x_{t} = \sum_{i=1}^{p} \mu_{i}^{1} \Delta x_{t-i} + \sum_{i=1}^{q} \mu_{i}^{2} \Delta y_{t-i} + \rho_{1} z_{t-1} + \rho_{2} F(z_{t-1}) + \epsilon_{t}$$
(16)

where $\epsilon_t \sim iid(0, \sigma_{\epsilon}^2)$.

Nonlinearities characterize the mechanisms of adjustment toward equilibrium and the latter may exhibit rigidities or irreversibilities implying a time-dependent path. The strength of attraction to the long-term attractor thus implies a nonlinear cointegration, providing that $\{z_t\}$ is stationary. As one can see, the definition of the error term is unchanged as compared to the linear case, but the distance from the long-term equilibrium evolves nonlinearly. There is a vast literature on nonlinear cointegration (for an overview of different interpretations and econometric procedures, see Dufrénot and Mignon (2002a, 2002b)).

In what follows, F will be a SETAR model. For instance, the hypothesis of SETAR cointegration between consumption and its exogenous determinants can be tested as follows:

• Firstly, one estimates a SETAR model:

$$c_{t} = \begin{cases} \sum_{i=1}^{5} \alpha_{i}^{1} X_{it} + \beta_{1} + v_{t}^{1} & \text{if } \Delta Y_{t} \leq a \\ \sum_{i=1}^{5} \alpha_{i}^{2} X_{it} + \beta_{2} + v_{t}^{2} & \text{if } \Delta Y_{t} > a \end{cases}$$

$$(17)$$

where X_{it} denotes the exogenous variables (i = 1, ..., 5), ΔY_t is the transition variable and a is the threshold value.

• Secondly, the null hypothesis of stationarity is tested on:

$$z_{t} = \begin{cases} \mu_{1} + \rho_{1} z_{t-1} + \omega_{t}^{1} & \text{if } z_{t-1} \leq \theta \\ \mu_{2} + \rho_{2} z_{t-1} + \omega_{t}^{2} & \text{if } z_{t-1} > \theta \end{cases}$$
(18)

where:

$$z_{t} = \begin{cases} c_{t} - \sum_{i=1}^{5} \hat{\alpha}_{i}^{1} X_{it} + \hat{\beta}_{1} = \hat{v}_{t}^{1} & \text{if } \Delta Y_{t} \leq a \\ c_{t} - \sum_{i=1}^{5} \hat{\alpha}_{i}^{2} X_{it} + \hat{\beta}_{2} = \hat{v}_{t}^{2} & \text{if } \Delta Y_{t} > a \end{cases}$$

$$(19)$$

There are several economic motivations for considering specifications such as (17). Evidently, most of the exogenous variables that we consider are influenced by activity and business cycle. Many empirical studies have shown that business cycles in industrial countries are asymmetric and characterized by regime-switching properties. The dynamics of the business cycle is captured in the above example by ΔY_t and parameter a is a threshold that separates two regimes (for instance, expansion and depression). As suggested by Escribano and Pfann (1998), regime-shifts in the business cycle may have some influence on the way variables converge to their long term target in ECM. Indeed, adjustment is not proportional to the equilibrium reached in the preceding

periods. It is noteworthy that differing regimes might induce a type of heterogeneity in long term consumption decisions. Notably, coefficients may differ significantly implying that substitution and income effects, wealth effects, attitudes with regard to the uncertainty of future income vary between regimes.

3.2.2 Estimation and testing procedures

Step 1. We apply the linearity test and the recursive plot methods exposed in section 2 in order to choose a transition variable (among the explanatory variables) and to select its threshold value. This is done on a static relationship between c_t and all the exogenous variables.

Step 2. We estimate a SETAR model such as (17) in which we include the whole set of exogenous variables. We note \tilde{z}_t the corresponding error term.

Step 3. The stationarity of \tilde{z}_t is then tested through a model like equation (18). A few comments are in order here. As suggested by Balke and Fomby (1995), since SETAR models are piecewise models, the traditional tests of non stationarity can be applied. The authors draw Monte Carlo simulations that show the power of the different tests. Here, we follow a procedure very similar to theirs. The main difference is that we apply a set of encompassed tests that amount to check the following ergodicity conditions for SETAR(2,1,1) models. Details concerning the testing procedures are provided in Dufrénot and Mignon (2000). These amount to testing the ergodicity conditions of the following model:

$$\tilde{z}_{t} = \begin{cases} \tilde{\mu}_{1} + \tilde{\rho}_{1} \tilde{z}_{t-1} + \tilde{\omega}_{t}^{1} & \text{if } \tilde{z}_{t-d} \leq \theta \\ \tilde{\mu}_{2} + \tilde{\rho}_{2} \tilde{z}_{t-1} + \tilde{\omega}_{t}^{2} & \text{if } \tilde{z}_{t-d} > \theta \end{cases}$$

$$(20)$$

where d is a delay parameter. (20) is ergodic if one of the following condition holds (see Chan et al. (1985) for proofs):

- (i) $\rho_2 < 1$, $\rho_1 < 1$ and $\rho_1 \rho_2 < 1$
- (ii) $\rho_2 = 1$, $\rho_1 < 1$ and $\mu_2 > 0$
- (iii) $\rho_2 < 1$, $\rho_1 = 1$ and $\mu_1 < 0$
- (iv) $\rho_1 = \rho_2 = 1$ and $\mu_1 < 0 < \mu_2$
- (v) $\rho_2 \ \rho_1 = 1 \text{ and } \rho_2 < 0, \ \mu_1 + \rho_1 \mu_2 > 0$

Step 4. An ECM is estimated using either the two steps Engle - Granger approach, or the Phillips - Loretan method.

Step 5. The ECM is compared to other nonlinear specifications. The latter include quadratic and cubic terms of the lagged linear error correction term.

3.2.3 Data and results

As for the consumption variable, our data concerning the exogenous variables are chosen from the O.E.C.D. database and extend over the period 1970:1 to 2000:4. The periodicity of observations is quarterly. The variables are the following:

- REV: household's real disposable income in the sense of national accounts, e.g., the nominal disposable income deflated by the consumer price index.
- IPC: the consumer price index
- i_t^C : the short term nominal interest rate (3-month interbank loan rate)
- i_t^L : the long term interest rate (yield to maturity on long-term government bonds, 10 years)
- \bullet UNR: the unemployment rate

• *INTB* : the borrowing interest rate.

The following transformations are applied to original data. From IPC, we compute the inflation rate (INFL) as the first difference of ln(IPC). Both interest rates are used to compute an indicator of the term structure of interest rates $(STRUC = i_t^L - i_t^C)$. The borrowing rate is measured in real terms, e.g., we consider the difference $INTB_t - INFL_t$.

Unit root tests (not reported here) show that the first differences of STRUC and INFL are both I(0). REV, INTB and UNR are I(1).

Table 4 reports the results of the nonlinearity test. The highest Fisher statistic corresponds to ΔREV_{t-2} . The threshold value identified from the CUSUM regression and plots of the standardized residuals is approximately 0.01.

Table 4. Linearity test using the exogenous variables

Threshold variable	Fisher statistic
ΔREV_t	27.552
$\Delta STRUC_t$	22.483
$\Delta INTB_t$	22.842
ΔUNR_t	18.779
$INFL_t$	24.536
ΔREV_{t-1}	34.873
ΔREV_{t-2}	35.898
ΔREV_{t-3}	25.186
ΔREV_{t-4}	27.757

All the p-values were under 5% and approximately equal to 0. The null hypothesis of linearity is thus rejected.

A SETAR static model is thus estimated using ΔREV_{t-2} as the threshold variable. This yields a vector of residuals $\{\tilde{z}_t\}_{t=1}^T$. To estimate a model such as (20), we need first to identify d and θ . This is done by applying again the linearity test and by plotting the standardized recursive residuals of the CUSUM regression against the selected transition variable. Table 5 shows that \tilde{z}_{t-2} is selected as the transition variable. We identified the threshold value as being approximately 0.0011.

Table 5. Linearity test on \tilde{z}_{t-d}

	0 00
Threshold variable	Fisher statistic
\tilde{z}_{t-1}	31.6649
\tilde{z}_{t-2}	48.9961
$ ilde{z}_{t-3}$	13.1416
\widetilde{z}_{t-4}	31.6641
$ ilde{z}_{t-5}$	17.7987
\tilde{z}_{t-6}	10.9990

All the p-values were under 5% and approximately equal to 0. The null hypothesis is thus rejected.

Table 6 documents the results concerning the cointegration tests where regimes 1 et 2 respectively correspond to $\tilde{z}_{t-2} \leq 0.0011$ and $\tilde{z}_{t-2} > 0.0011$. Application of both Augmented Dickey-Fuller and Shin cointegration tests lead us to reject conditions (ii) to (v) but to accept condition (i).

Table 6. Cointegration tests

	ADF	Conclusion	Shin	Conclusion
Regime 1 $\tilde{z}_{t-2} \leq 0.0011$	-7.1747	(ii)-(v) rejected	$9.03.10^{-2}$	(i) accepted
Regime 2 $\tilde{z}_{t-2} > 0.0011$	-4.8249	(ii)-(v) rejected	$6.74.10^{-2}$	(i) accepted

The ADF statistics are compared to the values tabulated by Engle and Yoo (1987) for spurious static regressions. For this test, the null hypothesis that there exists a unit root either in regime 1 or in regime 2 is rejected. To see whether (i) applies, we need a test with stationarity as the null hypothesis. The Shin test is similar to the Kwiatkowski *et al.* (1992)'s test on the cointegration relationship. Comparison of the estimated values with critical values leads us to accept the null hypothesis that \tilde{z}_t is stationary at the 5% level.

We now report our estimates on the 1970:1-1996:4 period. Equation 1 gives the results of the SETAR-ECM model. Equation 2 concerns the linear ECM formulation while equations 3 and 4 respectively correspond to the quadratic and cubic formulation of the ECM model. Equation 5 gives the results of the SETAR model when we impose an equality constraint on the long run adjustment coefficients. For each model, we give the R^2 , the sum of squared residuals (SSR), the value of the log-likelihood at optimum (LL), and the values of Akaike (AIC) and Schwarz (SIC) information criteria. RESCO are the estimated residuals of the static long run relationship between consumption, income, unemployment rate, inflation and the term structure of interest rates. RES1 and RES2 are the estimated residuals of the SETAR model in, respectively, regime 1 and regime 2. RES are the estimated residuals of the SETAR model without making any distinction between the two regimes.

Equation 1: SETAR-ECM

Equation 2: Linear ECM

$$\begin{array}{lll} \Delta c_t & = & 0.0001 + 0.6984 \; \Delta c_{t-1} - 0.3321 \; \Delta c_{t-2} + 0.2865 \; \Delta c_{t-3} + 0.1754 \; \Delta REV_t \\ & & - 0.2338 \; INFL_t + 0.2481 \; INFL_{t-1} + 0.0005 \; STRUC_t - 0.0740 \; RESCO_{t-1} \\ & & - (2.69) \; & (2.89) \; & (2.76) \; & (2.76) \end{array}$$

Equation 3: Quadratic ECM

$$\begin{array}{lll} \Delta c_t & = & 0.0002 + 0.6998 \; \Delta c_{t-1} - 0.3243 \; \Delta c_{t-2} + 0.2954 \; \Delta c_{t-3} + 0.1748 \; \Delta REV_t \\ & & - 0.2196 \; INFL_t + 0.2217 \; INFL_{t-1} + 0.0005 \; STRUC_t - 0.0677 \; RESCO_{t-1} \\ & & - (-2.54) \; & (2.57) \; & (2.88) \; & (-2.2$$

Equation 4: Cubic ECM

$$\Delta c_t = 0.00003 + 0.6768 \Delta c_{t-1} - 0.3219 \Delta c_{t-2} + 0.2964 \Delta c_{t-3} + 0.1741 \Delta REV_t$$

$$- 0.2198 INFL_t + 0.2217 INFL_{t-1} + 0.0005 STRUC_t - 0.0634 RESCO_{t-1}$$

$$+ 3.9921 RESCO_{t-1}^2 - 22.5828 RESCO_{t-1}^3$$

$$R^2 = 0.7411, SSR = 0.0005, LL = 481.9904, AIC = -9.145445, SIC = -8.864066$$

Equation 5: Constraint SETAR-ECM

$$\begin{array}{lll} \Delta c_{t} & = & 0.0001 + 0.6905 \; \Delta c_{t-1} - \; 0.3497 \; \Delta c_{t-2} + \; 0.3057 \; \Delta c_{t-3} + \; 0.1758 \; \Delta REV_{t} \\ & - \; 0.2420 \; INFL_{t} + \; 0.2583 \; INFL_{t-1} + \; 0.0005 \; STRUC_{t} - \; 0.0673 \; RESCO_{t-1} \\ & - \; 0.0449 \; RES_{t-1} \end{array}$$

 $R^2 = 0.7158, \, SSR = 0.0005, \, LL = 460.7593, \, AIC = -9.106248, \, SIC = -8.844114$

	Table 7. Residual tests					
	Eq. 1	Eq. 2	Eq. 3	Eq. 4	Eq. 5	
DW	2.07	2.04	2.07	2.07	2.03	
LB(4)	2.12 (0.71)	2.31 (0.68)	2.75 (0.60)	2.79 (0.59)	2.23 (0.69)	
LB(8)	10.77 (0.21)	9.90 (0.27)	10.51 (0.23)	10.69 (0.22)	9.18 (0.32)	
TR^2	0.54 (0.46)	$0.23 \ (0.62)$	0.006 (0.93)	$0.02\ (0.89)$	$0.29 \ (0.58)$	
S	0.14	0.35	0.18	0.18	0.27	
K	4.22	5.07	4.42	4.40	4.74	
IB	6.70 (0.03)	20.56 (0)	9.26 (0.009)	8 99 (0 01)	13 57 (0 001)	

Table 8. Static and dynamic forecasts (1997:1 - 2000:4)

	Eq. 1	Eq. 2	Eq. 3	Eq. 4	Eq. 5
RMSE	0.003235	0.003259	0.003238	0.003241	0.003264
TUVISE	0.002649	0.002728	0.002601	0.002594	0.002777
MAPE1	0.002504	0.002509	0.002491	0.002491	0.002526
MAI DI	0.002209	0.002241	0.002123	0.002117	0.002325
MAPE2	127.6471	128.4220	127.1998	127.2390	131.7032
MAI EZ	133.0150	133.0722	123.1762	123.4278	130.9700
Theil ineq. coef.	0.217314	0.218632	0.217535	0.217720	0.228160
	0.191423	0.197275	0.187574	0.187019	0.210291
Bias prop.	0.000006	0.000003	0.000013	0.000014	0.000001
	0.000317	0.000232	0.000447	0.000461	0.000177
Variance prop.	0.046172	0.049398	0.045131	0.044814	0.050800
	0.140156	0.137823	0.124994	0.124980	0.134374

Eq. 1: SETAR-ECM. Eq. 2: linear ECM. Eq. 3: quadratic ECM. Eq. 4: cubic ECM. Eq. 5: constraint SETAR-ECM. RMSE: root mean squared error. MAPE1: mean absolute percent error. MAPE2: mean absolute percentage error. For each criterium, the first row corresponds to static forecasts and the second row concerns dynamic forecasts. Theil ineq. coef.: Theil inequality coefficient. Bias prop.: bias proportion. Variance prop.: variance proportion.

Estimates in equation 1 suggest that the speed of adjustment towards the long run equilibrium differs between the two regimes. It is also observed that only one error correction term is statistically significant (at the 10% level), corresponding to regime 2. Thus, exploiting the hypothesis of an asymmetric long run generating mechanism yields a result that encompasses these usually obtained in the literature on French consumption data. With regard to regime 1, we would conclude that household consumption is only caused in the short term. A model expressed in differences is sufficient to account for its dynamics. Such a result is in line with some of the conclusions obtained by Bonnet and Dubois (1995). Conversely, a long run equilibrium exists for regime 2, thus indicating that an error correction mechanism is recommended to model the French

consumption function. The coefficients in equation 1 are rightly signed and in line with what is usually found in the literature. Notably, the coefficient corresponding to the current inflation rate is negative, thereby suggesting the predominance of substitution effects over income effects. We also see that the coefficient corresponding to the term structure of interest rates is positively signed, as expected.

In order to check the importance of asymmetric dynamics in the long run, we estimated a model with no distinction between regimes (equation 5). This is a linear ECM where all exogenous variables enter into the long term relationship. It is found that the ECM term is not statistically significant. We estimate two other nonlinear models that include quadratic and cubic terms of the linear ECM component. We do this because the SETAR-ECM might appear as an approximation of other nonlinear long run adjustment mechanisms. Notably, cubic functions are used in studies dealing with asymmetric long run equilibrium when a possibility of multiple equilibria exists (see among others Escribano (1997)).

According to the different residual tests, the five models have been well specified : absence of autocorrelation and no ARCH effect.

According to the R^2 , the log likelihood and the sum of squared residuals, the "best" model is the SETAR-ECM. The model which minimizes AIC and SIC criteria is the quadratic one. We can reject the cubic model since error terms are not statistically significant at conventional levels. To summarize, two models seem to be satisfactory: the SETAR-ECM and the quadratic model. In the SETAR-ECM, coefficients of error correction terms are negative, as expected. However, in the quadratic model, the quadratic error correction term is positive, which is somewhat difficult to interpret from an economic point of view. Thus, according to an adequate economic interpretation, we conclude that the SETAR-ECM is the "best".

We have also conducted static and dynamic forecasts over the 1997:1 to 2000:4 period for each model (table 8). For static predictions, the model which minimizes the root mean squared error and the Theil inequality coefficient is the SETAR-ECM. In terms of dynamic predictions, results are more in favor of the quadratic model. However, since the quadratic error term of this last model is wrongly signed, we finally retain the SETAR-ECM.

4 Conclusion

Two main conclusions may be drawn from the present study. First, SETAR models seem useful to account for the seemingly irregularity that characterized the aggregate saving ratio in France over the last two decades. As we have seen, such models capture the role played by habit formation that renders household's tastes endogenous. The fact that consumption is described by two regimes illustrates the presence of heterogenous behaviors among consumers. Secondly, when exogenous variables and an ECM term are added to account for the long term dynamics of consumption, SETAR-ECM appear to be a tidy way of illustrating the asymmetric dynamics that characterized the long term relationship between consumption and its determinants. We must emphasize that our SETAR cointegration models lead to better forecasts than linear ECM, quadratic ECM, cubic ECM and constraint SETAR-ECM. A further step in this study would consist in comparing the results obtained here for France with other analyses of O.E.C.D. countries.

References

- [1] Aftalion (1997), "The Asymmetry of Monetary Policies and STAR Model Estimations", International Conference on Forecasting Financial Markets, London.
- [2] Ainslie G. (1975), "Specious Reward: A Behavioral Theory of Impulsiveness and Impulse Control", *Psychological Bulletin*, 82, 463-496.
- [3] Allard P. (1992), "La modélisation de la consommation des ménages en France", Revue d'Economie Politique, 102, 727-765.

- [4] Ando A. and Modigliani F. (1963), "The Life Cycle Hypothesis of Saving: Aggregate Implications and Tests", American Economic Review, 53, 55-84.
- [5] Balke N.S. and Fomby T.B. (1995), "Threshold Cointegration", Review of Economics and Statistics, 5, 629-645.
- [6] Benhabib J. and Day R.H. (1981), "Rational Choice and Erratic Behavior", Review of Economic Studies, 48, 459-471.
- [7] Bonnet X. et Dubois E. (1995), "Peut-on comprendre la hausse imprévue du taux d'épargne des ménages depuis 1990?", Economie et Prévision, 5, 39-58.
- [8] Chan K.S., Petrucelli J.D., Tong H. and Woolford S.W. (1985), "A Multiple Threshold AR(1) Model", Journal of Applied Probability, 22, 267-279.
- [9] van Dijk D. and Franses P.H. (1999), "Modelling Multiple Regimes in the Business Cycle", Macroeconomic Dynamics, 3, 311-340.
- [10] Dockner E.J. and Feichtinger G. (1993), "Cyclical Consumption Patterns and Rational Addiction", American Economic Review, 83, 256-263.
- [11] Dufrénot G. and Mignon V. (2000), "Using Non-Stationary Threshold Models to Study the Dynamics of Aggregate Consumption: The Case of France", Working Paper ERUDITE, University of Paris 12, and Working Paper MODEM, University of Paris 10.
- [12] Dufrénot G. and Mignon V. (2002a), Nonlinear cointegration with Applications to Macroe-conomics and Finance, Kluwer Academic Press.Dufrénot G. and Mignon V. (2002b), "La cointégration non linéaire: une note méthodologique", Economie et Prévision, 155, 117-137.
- [13] Dufrénot G., Mignon V. and Peguin-Feissolle A. (2001), "Business Cycles Asymmetry and Monetary Policy: A Further Investigation using MRSTAR Models", *Economic Modelling*, 21(1), 37-71.
- [14] Engle R.F. and Yoo B.S. (1987), "Forecasting and Testing in Cointegrated Systems", *Journal of Econometrics*, 35, 143-159.
- [15] Escribano A. and Pfann G.A. (1998), "Nonlinear Error Correction, Asymmetric Adjustment and Cointegration", *Economic Modelling*, 15, 197-216.
- [16] Feichtinger G. and Novak A. (1994a), "Persistent Oscillations and Chaos in a Threshold Adjustment Model", W.P., Institute for Econometrics, University of technology, Vienna.
- [17] Feichtinger G., Hommes C. and Millik A. (1994b), "Chaotic Consumption Patterns in a Simple 2-D Addiction Model", W.P., Institute for Econometrics, University of technology, Vienna.
- [18] Floden M. (2000), "Endogenous Monetary Policy and the Business Cycle", European Economic Review, 44, 1409-1429.
- [19] Friedman M. (1957), A Theory of the Consumption Function, Princeton University Press.
- [20] Iannaccone L.R. (1986), "Addiction and Statiation", Economics Letters, 21, 95-99.
- [21] Kwiatkowski D., Phillips P.C.B., Schmidt P. and Shin Y. (1992), "Testing the Null Hypothesis of Stationarity Against the Alternative of a Unit Root", Journal of Econometrics, 54, 159-178.
- [22] Lang A.R. (1983), "Addictive Personality: A Viable Construct?", in *Commonalities in Sustance Abuse and Habitual Behavior*, P.K. Levison and D.G. Gerstein (eds), Lexington.
- [23] Orphanides A. and Zervog D. (1995), "Rational Addiction with Learning and Regret", Journal of Political Economy, 103, 739-779.

- [24] Peel D. and Davidson J. (1998), "A Nonlinear Error Correction Mechanism Based on the Bilinear Model", *Economics Letters*, 58, 165-170.
- [25] Schelling T.C. (1984), "Self-Command, in Practice, in Policy and in a Theory of Rational Choice", American Economic Review, 74, 1-11.
- [26] Shin Y. (1994), "A Residual Based Test of the Null of Cointegration Against the Alternative of Non Cointegration", *Economic Theory*, 10, 91-115.
- [27] Skalin J. and Teräsvirta T. (1998), "Modelling Asymmetries and Moving Equilibria in Unemployment Rates", Working Paper Series in Economics and Finance, 262, Stockholm School of Economics.
- [28] Tsay R.S. (1989), "Testing and Modelling Threshold Autoregressive Processes", Journal of the American Statistical Association, 84, 231-240.
- [29] Winston G.L. (1980), "Addiction and Backsliding: A Theory of Compulsive Consumption", Journal of Economic Behavior and Organization, december, 295-324.