E C O N O M I C S B U L L E T I N

Irreversible investment and the value of information gathering

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Abstract

This note develops a model in which a firm has to decide whether to undertake an irreversible investment. The firm has the option to delay it's decision in an effort to observe the actions of other firms. It is shown that a problem, akin to the herding phenomenon also applies, despite the endogenous time framework. In the context of an investment decision this manifests itself as the failure of a positive−payoff project to be undertaken. The most novel finding is that attempts to overcome this difficulty by further information gathering will, as a side effect, generate additional delay which may be enough to offset the gains of any new information.

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1. Introduction

This note develops a model in which a firm, deciding whether to undertake an irreversible investment, has the option to delay it's decision in an effort to observe the actions of other firms. It is shown that a problem, akin to the herding phenomenon in a sequential model also applies despite the endogenous time framework. In the context of an investment decision this manifests itself as the failure of a positive-payoff project to be undertaken. The most novel finding is that attempts to overcome this difficulty by further information gathering will, as a side effect, generate additional delay which may be enough to offset the gains of any new information.[1]

The sequential herding literature was pioneered by Banerjee (1992) and Bikchandhani *et al* (1992), and many of the qualitative features of these early papers carry through to the very different model in this note. For example, once a herd begins (quite possibly a herd on not investing) all will follow the actions of their predecessors (for example, never invest) and new information will cease to become available. Furthermore, since players can decide to act (or never act) at any time a herd will start quickly. The basic model used in this note follows Gale (1996), and is a simplified version of Chamley and Gale (1994), the seminal work on herding in endogenous time. The second part of the note (on information gathering) follows the recent literature on how a firm might manipulate herding behaviour to their advantage, for example Sgroi (2002). Despite this, and the fact that the potential investors are referred to as firms throughout, there is no reason why the findings in this note cannot be applied to any other economic agents facing an irreversible investment decision.

2. A Simple Model

In general we will consider N firms contemplating a potential investment, but initially we will restrict ourselves to $N = 2$. These firms have a decision problem which operates in two dimensions: whether to invest in an *irreversible* project and if so when to invest. The return to this project is w, which is assumed fixed at the beginning of time. Time is indexed by $t \in \mathbb{T}_{++}$ and is therefore discrete and strictly positive. Firm i does not directly observe w, instead it receives a signal μ^i.[2] Signals are independent and identically drawn from the uniform distribution with support $[-1, 1]$. These signals do not change over time, are drawn before the first period, and w is set equal to the sum of all signals, $w = \sum_i \mu^i$. Actions are

[1] This note is based on Sgroi (1998).

[2] In general i will be used to denote the firm whose decision problem we are considering while j will denote the other firm.

defined as: $x^i = 1 \Leftrightarrow$ "invest"; and $x^i = 0 \Leftrightarrow$ "do not invest". A firm can observe it's own signal, but not the signal of the other firm. In each period actions are made simultaneously, so the two firms cannot observe each others' current actions. However, in period 2, i will know the action which j performed in period 1, and through the observed choice of action some information about the nature of the other firm's signal may be revealed. Finally we have payoffs, π_t^i, discounted by $\delta \in (0,1)$:

$$\pi_t^i = \begin{cases} \delta^{t-1} w & \text{if} \quad x^i = 1 \\ 0 & \text{if} \quad x^i = 0 \end{cases}$$

Consider the problem faced by i: whether and if so when to invest. Myopically we might consider a rule based on investing immediately if the expected payoff is positive. However, since firm i will be able to observe x_1^j, and this will provide new information about w, we need to incorporate the *option value* of waiting. This option value comes about because of the possibility that firm i may have invested at $t = 1$ when doing so was foolish given the information available at $t > 1$. We need to balance this benefit of waiting against the cost of delay provided by the discount factor. Following Dixit and Pindyck (1994) and Chamley and Gale (1994) we can define a symmetric signal value $\overline{\mu}$ such that $\mu^i \geq \overline{\mu} > 0 \Leftrightarrow x_1^i = 1$, and we will now attempt to find a symmetric equilibrium based on this decision-rule.[3]

Proposition 1. *There is a symmetric equilibrium in which it is optimal for firm i to invest at time $t = 1$ if and only if $\mu^i \geq \overline{\mu} = \frac{\delta - 2 + 2\left(1+\delta^2-\delta\right)^{\frac{1}{2}}}{3\delta} > 0$.*

Proof. See Appendix, part 1. ∎

Proposition 2. *(i). If firm i did not invest at time $t = 1$ it will either invest when $t = 2$ or never invest. (ii). Firm i will only invest at $t = 2$ if j invested at $t = 1$.*

Proof. See Appendix, part 2. ∎

There are numerous features of this model which are very much in keeping with the herding literature. Information is not fully revealed, and there is no direct mapping from signal to action which can be inverted to reveal a firm's signal. Errors are made, and private information may be ignored, in particular even if $\mu^i > 0$ for $i = 1, 2$ neither will invest unless $\mu^i \geq \overline{\mu}(\delta)$ for at least some i. Mistakes which lead to incorrect decisions in turn lead to

[3]The focus on a symmetric equilibrium is justifiable on the grounds that the problem itself is totally symmetric. Gul and Lundholm (1995) make a strong case for the relevance of the symmetric equilibrium in a decision model of this type.

welfare losses. Decisions are made quickly in this model. The final decision of each firm will be made by $t = 2$; beyond this point firms have either invested or will never do so. The addition of further firms would allow the game to continue beyond two periods of interest, but we need at least one investment per period or investment will stop forever, as in the two firm case. This is formally shown to be true in the statement of proposition 3 which extends proposition 2 to the multi-firm case.

Proposition 3. *A single period of no investment will end the prospect of any further investment in a model with $N \in \mathbb{N}_{++}$ firms.*

Proof. See Appendix, part 3. ∎

Gale (1996) provides an intuition for this result, pointing out that in a model of this type there must be a possibility of investment collapse as a necessary condition of equilibrium. This comes about because in order to have any delay there must be a positive option value, and this in turn implies a positive probability that agents do not invest (or no information will be revealed by decisions to invest). Therefore there is a chance that nobody at all invests in the first period, which leads to an investment collapse where potential investors will never invest. We examine this possibility next.

2.1. **Investment Collapse.** We first investigate the nature of the problem caused by the structure of investment in this model which we call investment breakdown.

Definition 1. ***Full investment breakdown*** *is said to occur when a project has positive value but it is not carried out by either firm.*

Close examination of this definition reveals that full investment breakdown is a herd on the wrong action (or in the terminology of Bikchandhani *et al* (1992) a reverse informational cascade). Consider a situation in which $\mu^i < \overline{\mu}$ and $\mu^j < \overline{\mu}$ but where $w > 0$. Let us go through the decision-process in detail. Both firms begin with signals that lie under $\overline{\mu}$. This results in a unilateral decision to delay in order to gain more information. In the second period both observe a failure to invest by the other which results in an updated set of beliefs which suggest to each firm that the other's signal is in expectation $0.5(\overline{\mu} - 1) < 0$. If a firm had reason to delay when the other's signal was in expectation neutral a negative expected signal will reinforce the decision not to invest. Of course there will be no new information forthcoming as neither firm will decide to invest, and so we have a reverse cascade on the action "do not invest". When neither invests at $t = 1$ then they will never invest and the two firms are effectively trapped in an informational cascade on the action "do not invest".

If both signals are positive it is particularly clear that it is the option value of waiting that creates the failure to invest. However, there is an alternative which is also a form of investment collapse.

Definition 2. ***Partial investment breakdown*** *is said to occur when a project has positive value and one firm invests, but the other does not.*

Once again assume that $w = \mu^i + \mu^j > 0$ so the project has positive worth. If $\mu^i > \overline{\mu}$ but $\mu^j < -0.5\,(\overline{\mu} + 1)$ then i will invest at $t = 1$, but j fails to follow as even an observed investment decision by i is not enough to generate a positive expected return from the project. The option value to delay is not responsible for the failure to invest, rather it is j's inability to observe i's signal that leads him to reject investment. The common element in both problems is that they could be overcome if information was shared by both firms, and this is what we examine next.

3. Information Gathering

Consider a joint venture by firms specifically designed to improve information, perhaps because of a fear of a reverse cascade and the profit-damaging implications this entails. A reasonable reaction would be to pool information, especially when there is little strategic conflict between the firms. Alternatively both firms might be considering the purchase of some new technology and the developer of the new technology might wish to provide unbiased credible information to the firms which rule out a reverse cascade. Finally, we might imagine that the government is sponsoring an investment opportunity which it wishes to see undertaken by private sector firms, but fearing investment breakdown it decides to provide definite positive information about the project's value. In all cases the issue is whether such information revelation is worthwhile given that it would undoubtedly entail some expense, take some time, and as shown in this section, quite possibly add further delay to the firms' decision-making.

3.1. Complete Revelation of the True State.

We start with a definition.

Definition 3. *Complete revelation of the true state of the world* $\{w_t, t \in \mathbb{T}_{++}\}$ *at some predetermined point in the future* $t = \tau^*$*, where* $\tau^* \in \mathbb{T}_{++}$ *is common knowledge to all potential investors, is said to occur when the true value of the state of the world* w_{τ^*} *at time* $t = \tau^*$ *is revealed to all potential investors.*

We model this by introducing a third period. The first two periods are as previously; however, in the third period we allow the firms considering the investment opportunity to know the true state of the world. Therefore, in period 3 the firms are left with:

$$\pi_3^i = \begin{cases} \delta^2 w_3 & if \quad x_3^i = 1 \\ 0 & if \quad x_3^i = 0 \end{cases}$$

Proposition 4. *If $\tau^* > 2$ then by time $t = \tau^*$ there will be a decision to invest or never invest, where τ^* is the time of complete revelation. In particular, if $\tau^* = 3$, then the decision will be made by period 3 with a decision to invest or never invest.*

Proof. See Appendix, part 4. ∎

Put simply this proposition establishes that the quick decisions of proposition 2 may be slowed by the potential for further information. Since more information is forthcoming we would not necessarily expect the same threshold value to hold force under complete revelation. We denote the new threshold for period 1 as $\overline{\overline{\mu}}$. As shown in part 5 of the appendix:

$$\overline{\overline{\mu}} = \frac{2-\delta-2\left[1-\delta+\delta^2-\delta^3\right]^{\frac{1}{2}}}{4\delta^2-3\delta} > \overline{\mu} \text{ for } \delta \in (0,1)$$

Therefore we have:

Proposition 5. *Complete revelation generates additional delay when $\delta \in (0,1)$.*

3.2. **The Usefulness of New Information.** New information can slow decision-making, but it also enables firms to avoid a failure to invest in profitable projects. We will now examine precisely this trade-off.

Proposition 6. *Complete revelation will only be of any benefit in the fraction of cases given by:*

$$f\left(\overline{\mu}\left(\delta\right)\right) \equiv f\left(\delta\right) = \tfrac{1}{2}\overline{\mu}^2 + \tfrac{1}{16}\left(1-\overline{\mu}\right)^2$$

Proof. See Appendix, part 6. ∎

Note that since $\frac{\partial f(\overline{\mu})}{\partial \overline{\mu}} > 0$ and $\frac{\partial \overline{\mu}(\delta)}{\partial \delta} > 0$ we have $\frac{\partial f(\delta)}{\partial \delta} > 0$. Therefore $\max_\delta f\left(\delta\right) = f\left(1\right) = 8\frac{1}{3}\%$ which tells us that with maximum patience complete revelation will be of use in $8\frac{1}{3}\%$ of cases. For a more reasonable patience level, say 0.5, we get $f\left(0.5\right) \approx 0.057$, so gathering extra information would be worthwhile in under 6% of cases. We see that extra information is indeed useful but in only a surprisingly small fraction of cases, and this is when such extra information is assumed to be costlessly obtained.

Proposition 7 gives a necessary condition for undertaking complete revelation when information gathering has a cost.

Proposition 7. *With a cost of gathering information $C_g > 0$ an ex ante (prior to the realization of signal values) necessary condition for welfare-improving complete revelation when revelation occurs at $t = \tau^*$, is:*

$$C_g < \delta^{\tau^*} \left[\overline{\mu}^3 + \frac{1}{32} (1 - \overline{\mu})^3 \right]$$

Proof. See Appendix, part 7. ∎

It should be stressed that this is a necessary condition and a very weak one, based on maximum possible signal values throughout, and it will rarely be sufficient. This implies that the information will only be of any use in a small number of cases. To give some idea of the magnitudes involved consider the following example.

Example 1. *All approximations are to three significant figures. For $\delta = 0.5$, $\tau^* = 3$ we have $\overline{\mu}(\delta) \approx 0.155$ and $\overline{\overline{\mu}}(\delta) \approx 0.162$. So the prospect of extra information raises the threshold value by a little over 4.5% of the original value. Furthermore $f(\delta) \approx 0.056$, so by proposition 6 complete revelation is only useful in 5.6% of cases. Using proposition 7 we have as a necessary condition that the cost of public information gathering C_g must be below 0.0054. To put this into a reasonable metric, the maximum possible ex ante project value is 2, so a necessary condition for complete revelation in the case when $\delta = 0.5$ is that the cost of information gathering not exceed 0.27% of the maximum value of a given project, so for a project with a maximum possible value of \$1million, the cost of information gathering should not exceed \$2700. Even with $\delta = 1$ (so $\overline{\mu}$ is at its maximium value of 0.333) the cost of information gathering should not exceed 0.76% of the maximum value of a given project, or \$7600 on a \$1million project.*

Note that a low value of δ makes the necessary condition even stricter. A final qualifier for the usefulness of complete revelation is that it should not occur too late.

Proposition 8. *Assume As $\tau^* \to \infty$ complete revelation will have a falling effect on the threshold value, with the effect disappearing completely in the limit. Therefore, a necessary condition for complete revelation to be weakly profit-improving as $\tau^* \to \infty$ is that it has zero cost.*

Proof. See Appendix, part 8. ∎

4. Conclusion

Fully revealing the value of a potential investment at some point in time will certainly overcome the dangers of a failure to invest when a project is worthwhile, and, if costless, will *ex ante* raise payoffs. However, it will also shift out the minimum signal value required to produce immediate investment. This may reduce the speed of decision-making and so add to discounting, which will reduce the benefit of any extra information. In practice gathering new information, pooling information or otherwise revealing the true worth of the project will not be costless and when a direct cost is combined with the extra delay produced the net result is a requirement for costs to be very low for extra information to be useful.

The message is simple: when reverse cascades are a serious danger firms should not be too quick to attempt to overcome these dangers through further information gathering unless additional information can be obtained at very low cost (in the order of well under 1% of the total potential value of the project).

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Appendix

1. Proof of Proposition 1:

The cost of delay is the expected payoff at time 1 minus the expected payoff at time 2, $(1-\delta)\,\mu^i$. To find the benefit in delay we need to consider the possibility that an investment made at time 1 seems less sensible when information made available at time 2 is revealed. Information of this sort comes about if it is observed that firm j did not invest at time 1, therefore revealing that $\mu^j < \overline{\mu}$ which provides some evidence that the state of the world is less likely to merit investment. This can be avoided if firm i waits and so this provides an *option value* of waiting which occurs with probability $\Pr\left[\mu^j < \overline{\mu}\right]$. The option value can therefore be defined as the expected loss avoided by firm i by not investing at $t = 1$ in the event that firm j does not invest at $t = 1$:

$$-\delta \Pr\left[\mu^j < \overline{\mu}\right]\left\{\mu^i + E\left[\mu^j \mid \mu^j < \overline{\mu}\right]\right\}$$

We have a condition which leaves the marginal decision-maker indifferent when deciding to invest at time 1: indifference occurs when the option value exactly offsets the delay cost; this is none other than the standard value matching condition for a dynamic programming problem which implicitly defines the value of $\overline{\mu}$. Using the properties of the uniform distribution:

$$\begin{aligned}
(1-\delta)\overline{\mu} &= -\delta \Pr\left[\mu^j < \overline{\mu}\right]\left\{\overline{\mu} + E\left[\mu^j \mid \mu^j < \overline{\mu}\right]\right\} \\
&\Rightarrow (1-\delta)\overline{\mu} = -\delta\tfrac{\overline{\mu}+1}{2}\left(\overline{\mu} + \tfrac{\overline{\mu}-1}{2}\right) \\
&\Rightarrow (4-4\delta)\overline{\mu} = -2\delta\overline{\mu}\left(\overline{\mu}+1\right) - \delta\left(\overline{\mu}+1\right)\left(\overline{\mu}-1\right) \\
&\Rightarrow 3\delta\overline{\mu}^2 + (4-2\delta)\overline{\mu} - \delta = 0 \\
&\Rightarrow \overline{\mu} = \tfrac{-(4-2\delta)\pm\left[(4-2\delta)^2+12\delta^2\right]^{\frac{1}{2}}}{6\delta}
\end{aligned}$$

For $\delta \in (0,1)$ and $\overline{\mu} \in [-1,1]$ we can rule out one of these two results, eliminating:

$$\overline{\mu} = \tfrac{-(4-2\delta)-\left[(4-2\delta)^2+12\delta^2\right]^{\frac{1}{2}}}{6\delta} \notin [-1,1] \text{ for } \delta \in (0,1)$$

Simplifying this leaves the value of $\overline{\mu}$ as:

$$\overline{\mu} = \tfrac{\delta-2+2\left(1+\delta^2-\delta\right)^{\frac{1}{2}}}{3\delta}$$

This expression is well defined for $\delta \in (0,1)$ and gives a range of values for $\overline{\mu}$ of $\overline{\mu} \in \left(0, \frac{1}{3}\right]$.[4]

It has been shown that there is a unique value of $\overline{\mu}$ such that if $\mu^i > \overline{\mu}$ the cost of delay strictly offsets the option value of waiting. We have the "$>$" relation since the cost of delay is rising in μ^i (and falling in δ) which therefore defines the optimal decision rule for firm i at

[4]The expression for $\overline{\mu}$ can be approximated by the linear function $\overline{\mu} = \frac{1}{3}\delta$ over the relevant range of values of δ.

time 1. The fact that $\overline{\mu} > 0$ shows that in equilibrium there will be a positive option value to delay for the marginal investor.

2. Proof of Proposition 2:

In (i) we are given that firm i did not invest at time $t = 1$. If this was so we know from proposition 1 that in equilibrium $\mu^i < \overline{\mu}$. In order to rationally delay, it must therefore be the case that for firm i, $E[w] = \mu^i + E[\mu^j|\mu^j < \overline{\mu}] < 0$. There are now two possibilities. Firstly, if firm j does not invest at $t = 1$, then firm i will not invest at $t = 2$, and by symmetry neither with firm j, proving part (ii). Secondly, if firm j does invest at $t = 1$, then depending upon the value of μ_i firm i may invest at $t = 2$. This covers both cases, and since information sets beyond $t = 2$ remain the same, but payoffs are falling by a factor of δ per period there will never be any investment beyond $t = 2$, proving part (i).

3. Proof of Proposition 3:

We need to show that in an n firm game, if firm i decides not to invest at $t = \tau$, then the expected payoff of investing at $t = \tau+1$, conditional on no other firm having invested at $t = \tau$ is negative. Delay by firm i implies $E_\tau [w] = \mu^i + E[\sum_{j \neq i} \mu^j | \forall \mu^j < \overline{\mu}] < 0$ or firm i would have invested in period τ. Observing no investment by any other firm at $t = \tau$ implies $\forall \mu^j < \overline{\mu}$ and so yields $E_{\tau+1} [w] = \mu^i + E[\sum_{j \neq i} \mu^j | \forall \mu^j < \overline{\mu}] < 0$ and so $E_{\tau+1} \left[\pi^i_{\tau+1}\right] = \delta^\tau E_{\tau+1} [w] < 0$ so investment will not occur at $t = \tau + 1$.

4. Proof of Proposition 4:

Immediate from the fact that while there is no option value at time $t = \tau^*$, so there is no further benefit to delaying beyond τ^*, there is a positive cost to delay in terms of discounting.

5. Calculation of the New Signal Threshold:

Once again we assume a candidate symmetric equilibrium such that firm i invests at $t = 1$ if and only if $\mu^i \geq \overline{\overline{\mu}} > 0$, where $\overline{\overline{\mu}}$ denotes the new threshold value and is once again positive on the assumption of a positive option value to delay. Note that this in no way presupposes that $\overline{\overline{\mu}} \neq \overline{\mu}$. We will consider $\tau^* = 3$, and therefore allow firm i three potential periods in which it might be optimal to invest. If $\mu^i \geq \overline{\overline{\mu}}$ firm i will invest when $t = 1$. If $\mu^i < \overline{\overline{\mu}}$, firm i will delay but may invest at $t = 2$ if firm j invested at $t = 1$, or firm i may invest at $t = 3$ if at that time w is revealed to be positive. To solve for $\overline{\overline{\mu}}$ we simply take the marginal decision to invest at $t = 1$ which sets $\mu^i = \overline{\overline{\mu}}$. This is set equal to the payoff resulting from the certain decision to invest at $t = 2$ if firm j has invested (since then $w \geq 0$ is certain as

$\mu^i = \overline{\overline{\mu}} > 0$ and $\mu^j \geq \overline{\overline{\mu}} > 0$), and otherwise to invest at $t = 3$ if w is revealed to be positive:[5]

$$\begin{aligned} \overline{\overline{\mu}} &= \delta \left\{ \Pr\left[\mu^j \geq \overline{\overline{\mu}}\right] \left(\overline{\overline{\mu}} + E\left[\mu^j \mid \mu^j \geq \overline{\overline{\mu}}\right]\right)\right\} \\ &\quad + \delta^2 \left\{ \Pr\left[\mu^j < \overline{\overline{\mu}}\right] \Pr\left[\mu^j \geq -\overline{\overline{\mu}} \mid \mu^j < \overline{\overline{\mu}}\right] \left(\overline{\overline{\mu}} + E\left[\mu^j \mid -\overline{\overline{\mu}} < \mu^j < \overline{\overline{\mu}}\right]\right)\right\} \end{aligned}$$

Simplifying we get:

$$\begin{aligned} \overline{\overline{\mu}} &= \delta \left\{ \tfrac{1-\overline{\overline{\mu}}}{2} \left(\overline{\overline{\mu}} + \tfrac{1+\overline{\overline{\mu}}}{2} \right) \right\} + \delta^2 \left\{ \tfrac{1+\overline{\overline{\mu}}}{2} \tfrac{2\overline{\overline{\mu}}}{1+\overline{\overline{\mu}}} \left(\overline{\overline{\mu}} + 0 \right) \right\} \\ &= \delta \tfrac{1+2\overline{\overline{\mu}}-3\overline{\overline{\mu}}^2}{4} + \delta^2 \overline{\overline{\mu}}^2 \\ &= \left(\delta^2 - \tfrac{3}{4}\delta\right) \overline{\overline{\mu}}^2 + \tfrac{1}{2}\delta\overline{\overline{\mu}} + \tfrac{1}{4}\delta \\ &\Rightarrow \left(\delta^2 - \tfrac{3}{4}\delta\right) \overline{\overline{\mu}}^2 + \left(\tfrac{1}{2}\delta - 1\right) \overline{\overline{\mu}} + \tfrac{1}{4}\delta = 0 \\ &\Rightarrow \overline{\overline{\mu}} = \frac{-\left(\frac{1}{2}\delta - 1\right) \pm \left[\left(\frac{1}{2}\delta - 1\right)^2 - 4\left(\delta^2 - \frac{3}{4}\delta\right)\frac{1}{4}\delta\right]^{\frac{1}{2}}}{2\left(\delta^2 - \frac{3}{4}\delta\right)} \\ &\Rightarrow \overline{\overline{\mu}} = \frac{1 - \frac{1}{2}\delta \pm \left[1 - \delta + \delta^2 - \delta^3\right]^{\frac{1}{2}}}{2\delta^2 - \frac{3}{2}\delta} \end{aligned}$$

We can rule out one of these roots since

$$\overline{\overline{\mu}} = \frac{1 - \frac{1}{2}\delta + \left[1 - \delta + \delta^2 - \delta^3\right]^{\frac{1}{2}}}{2\delta^2 - \frac{3}{2}\delta} \notin [-1, 1] \text{ for } \delta \in (0, 1)$$

Which gives the value of $\overline{\overline{\mu}}$ as:

$$\overline{\overline{\mu}} = \frac{2 - \delta - 2\left[1 - \delta + \delta^2 - \delta^3\right]^{\frac{1}{2}}}{4\delta^2 - 3\delta}$$

Comparing this with the value for $\overline{\mu}$, we have $\overline{\overline{\mu}} > \overline{\mu}$ for $\delta \in (0, 1)$ as expected.

6. Proof of Proposition 6:

Complete revelation is only of any use if, in the world before the prospect of revelation, the firms' signal values were such that full or partial investment breakdown would have occurred.

[5]For completeness, note that investing in period 2 even if the other firm does not invest in period 1 can be ruled out by the simple argument that if this was feasible it would be better to invest in period 1 and avoid a period of discounting. We can also rule out an alternative candidate symmetric equilibrium based around a threshold value $\widehat{\mu}$, defined such that the firm invested if and only if $\mu^i \geq \widehat{\mu} > 0$ and did not invest until period 3 even if firm j invests in period 1. This is immediate since, in period 2, having observed a decision to invest by firm j, the marginal firm with $\mu^j = \widehat{\mu}$ will know for certain that $w > 0$, and will therefore wish to invest immediately to avoid further discounting.

For investment breakdown this requires that μ^i and μ^j are both in the region $[-1, \overline{\mu}(\delta)]$ and that $w = \mu^i + \mu^j > 0$. The distribution of the value of the project at t, below the threshold value, is the sum of two uniform distributions with support $[-1, \overline{\mu}]$ and is therefore symmetric triangular with support $[-2, 2\overline{\mu}]$. Denote the probability of investment breakdown as $g(\overline{\mu}(\delta)) \equiv g(\delta)$. Using the properties of the triangular distribution $g(\delta)$ is given by:

$$\begin{aligned} g(\delta) &= \Pr\left[w > 0 \mid \mu^i < \overline{\mu} \;\&\; \mu^j < \overline{\mu}\right] \Pr\left[\mu^i < \overline{\mu} \;\&\; \mu^j < \overline{\mu}\right] \\ &= 2\left(\tfrac{\overline{\mu}}{\overline{\mu}+1}\right)^2 \left(\tfrac{\overline{\mu}+1}{2}\right)^2 \\ &= \tfrac{1}{2}\overline{\mu}^2 \end{aligned}$$

Denote the probability of partial investment breakdown as $h(\overline{\mu}(\delta)) \equiv h(\delta)$, noting that since this could impact on either firm we double the probability that partial investment breakdown affects each firm.

$$\begin{aligned} h(\delta) &= 2\Pr\left[\mu^i > \overline{\mu}, \mu^j < -\tfrac{(\overline{\mu}+1)}{2}\right] \Pr\left[\mu^i + \mu^j > 0 \mid \mu^i > \overline{\mu}, \mu^j < -\tfrac{(\overline{\mu}+1)}{2}\right] \\ &= 2\Pr\left[\mu^i > \overline{\mu}, \mu^j < -\tfrac{(\overline{\mu}+1)}{2}\right] \tfrac{1}{2}\Pr\left[\mu^i > \tfrac{(\overline{\mu}+1)}{2} \mid \mu^i > \overline{\mu}\right] \\ &= 2\left[\tfrac{1-\overline{\mu}}{2}\left(\tfrac{-\frac{(\overline{\mu}+1)}{2}-(-1)}{2}\right)\right]\left[\tfrac{1}{2}\tfrac{1-\frac{(\overline{\mu}+1)}{2}}{1-\overline{\mu}}\right] \\ &= 2\left[\tfrac{1}{8}(1-\overline{\mu})^2\right]\tfrac{1}{4} = \tfrac{1}{16}(1-\overline{\mu})^2 \end{aligned}$$

Now since $f(\delta) = g(\delta) + h(\delta)$, combining the expressions for $g(\delta)$ and $h(\alpha, \delta)$ yields:

$$f(\delta) = \tfrac{1}{2}\overline{\mu}^2 + \tfrac{1}{16}(1-\overline{\mu})^2$$

Which is the required fraction of time when complete revelation is useful.

7. Proof of Proposition 7:

The extra delay in investment causes a small but strictly positive loss in joint payoffs denoted by $\kappa > 0$. From proposition 6 complete revelation will be useful in countering full investment breakdown for the fraction of cases $\frac{1}{2}\overline{\mu}^2$. In these cases the maximum potential gain in profit is the sum of the two highest signal values which still lie in the investment breakdown signal region, i.e. the two highest signals for which $\mu^i \in [0, \overline{\mu}(\delta)]$, $\mu^j \in [0, \overline{\mu}(\delta)]$ and $\mu^i + \mu^j > 0$. This produces the maximum possible combined signal value of $2\overline{\mu}$. Furthermore, the payoff will only occur at the point of complete revelation, therefore the payoffs must be discounted up to that point. Hence we have a maximum possible gain from countering full investment breakdown of $2\delta^{\tau^*}\overline{\mu}\left(\frac{1}{2}\overline{\mu}^2\right)$. Complete revelation is also useful in countering partial investment breakdown which occurs in the fraction of cases $\frac{1}{16}(1-\overline{\mu})^2$. In these cases the maximum possible gain is $1-\frac{1}{2}(\overline{\mu}+1)$, which must again be discounted up to the point of complete revelation, and so we have a maximum possible gain from countering partial investment breakdown of $\delta^{\tau^*}\frac{1}{2}(1-\overline{\mu})\frac{1}{16}(1-\overline{\mu})^2$. Combining all of this we have a necessary condition on the cost of information gathering:

$$C_g \leq 2\delta^{\tau^*}\overline{\mu}\left(\tfrac{1}{2}\overline{\mu}^2\right)+\delta^{\tau^*}\tfrac{1}{2}(1-\overline{\mu})\tfrac{1}{16}(1-\overline{\mu})^2-\kappa$$

So we have as a weaker necessary condition:

$$C_g < \delta^{\tau^*}\left[\overline{\mu}^3+\tfrac{1}{32}(1-\overline{\mu})^3\right]$$

8. Proof of Proposition 8:

As in Part 5 of the Appendix, to solve for $\overline{\overline{\mu}}$ we simply take the marginal decision to invest at $t=1$ which sets $\mu^i=\overline{\overline{\mu}}$. This is set equal to the payoff resulting from the decision to invest at $t=2$ if firm j has invested, and otherwise to invest at $t=\tau^*$ if w is revealed to be positive:

$$\begin{aligned}\overline{\overline{\mu}} &= \delta\left\{\Pr\left[\mu^j\geq\overline{\overline{\mu}}\right]\left(\overline{\overline{\mu}}+E\left[\mu^j\mid\mu^j\geq\overline{\overline{\mu}}\right]\right)\right\}\\ &\quad+\delta^{\tau^*-1}\left\{\Pr\left[\mu^j<\overline{\overline{\mu}}\right]\Pr\left[\mu^j\geq-\overline{\overline{\mu}}\mid\mu^j<\overline{\overline{\mu}}\right]\left(\overline{\overline{\mu}}+E\left[\mu^j\mid-\overline{\overline{\mu}}<\mu^j<\overline{\overline{\mu}}\right]\right)\right\}\end{aligned}$$

Now we let $\tau^*\to\infty$, so we have:

$$\begin{aligned}\lim\{\overline{\overline{\mu}}\}_{\tau^*\to\infty} &= \delta\Pr\left[\mu^j\geq\overline{\overline{\mu}}\right]\left(\overline{\overline{\mu}}+E\left[\mu^j\mid\mu^j\geq\overline{\overline{\mu}}\right]\right)\\ &= \delta\left\{\tfrac{1-\overline{\overline{\mu}}}{2}\left(\overline{\overline{\mu}}+\tfrac{1+\overline{\overline{\mu}}}{2}\right)\right\}\\ &\Rightarrow 3\delta\overline{\overline{\mu}}^2+(4-2\delta)\overline{\overline{\mu}}-\delta=0\\ &\Rightarrow \overline{\overline{\mu}}=\frac{-(4-2\delta)\pm\left[(4-2\delta)^2+4(3\delta)\delta\right]^{\frac{1}{2}}}{2(3\delta)}\end{aligned}$$

We can rule out one of these roots since:

$$\overline{\overline{\mu}}=\frac{-(4-2\delta)-\left[(4-2\delta)^2+12\delta^2\right]^{\frac{1}{2}}}{6\delta}\notin[-1,1]\ \text{ for }\delta\in(0,1)$$

Which gives the value of $\overline{\overline{\mu}}$ as:

$$\overline{\overline{\mu}}=\frac{\delta-2+2\left(1+\delta^2-\delta\right)^{\frac{1}{2}}}{3\delta}=\overline{\mu}$$

Therefore, complete revelation at $\tau^*\to\infty$ has no impact on the threshold value, though it will reduce net profits if it entails a positive cost.