E C O N O M I C S   B U L L E T I N

Unraveling in a dynamic matching market with Nash bargaining

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Abstract

Equilibrium sorting in a finite−horizon, two−sided matching market with heterogeneous agents is considered. It is shown that, if the match production function is additively separable in agent−types and if the division of match output is determined by the Nash bargaining solution, then an unraveling of the market obtains as the unique equilibrium in which all matches are formed in the first period.

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1. Introduction

Since the pioneering work of Becker (1973) on the subject, the study of equilibrium sorting in two-sided matching markets with heterogeneous agents has garnered much attention among economists. In particular, a primary focus of recent research on the topic has been the analysis of equilibrium matching patterns in the presence of search frictions. It has been shown that, in stationary environments with non-transferable utility, equilibria in these markets are characterized by segregationist sorting, whereby agents are partitioned by types into classes, and matching occurs only between agents from the same class (McNamara and Collins, 1990; Burdett and Coles, 1997; Bloch and Ryder, 2000; Smith, 2002). Smith (1995) and Damiano et al (2002) have shown that such segregation by types also obtains in finite-horizon matching markets with no entry of new agents and non-transferable utility.

Here, equilibrium sorting in a finite-horizon market with side-payments is considered.[1] In particular, it is assumed that the division of match production in a partnership is determined by the symmetric Nash bargaining solution. It is shown that, if the match production function is additively separable in agent-types, then an unraveling of the market results as the unique equilibrium outcome in which sorting by types no longer obtains and all matches are consummated in the first period.

2. The Result

The matching market is two-sided, and agents on each side of the market are differentiated by a one-dimensional trait x belonging to the type-space $[a, b] \subset (0, +\infty)$. The distribution of agent-types on either side of the market is given by a twice continuously differentiable function $F(\cdot)$ with density $f(\cdot)$. Without loss of generality, the measure of agents on each side is normalized to be 1.

The market is open for two time periods, with no entry of additional participants in the second period. Matchings are one-to-one, voluntary transactions between agents from each side, and agents exit the market immediately following a match. In both periods, unmatched agents on either side of the market randomly encounter those on the other side, and these meetings are generated by a constant returns-to scale search technology.

The total output produced from a match between an agent of type x and a type-y partner is given by $x + y$. Note that, by suitably redefining the type-space, other additively separable specifications in which match production takes the form $h(x) + h(y)$, where the function $h(\cdot)$ is continuously differentiable with $h(x) > 0$ and $h'(x) > 0$ $\forall x \in [a, b]$, can be reduced to this particular specification. As in Damiano et al (2002), it can be shown that, in equilibrium, only agents of sufficiently high types match in the first period if match utility is

[1] A characterization of equilibrium sorting in infinite-horizon matching markets with Nash bargaining is given in Shimer and Smith (2000).

non-transferable. In contrasts, it is assumed here that the division of output in a match is determined by the (symmetric) Nash bargaining solution, with agents' continuation payoffs serving as the "threat points". A payoff of 0 is received if an agent is unmatched. All agents are risk-neutral and have a common discount factor of 1.

In order to analyze the equilibrium outcome of this market, first consider agents' behavior in the second period. Since the value of remaining unmatched is 0, all agents in the last period will choose to match. Therefore, applying the Nash bargaining solution, the expected payoff to a type-x agent in the second period is $\frac{1}{2}(x+\mu)$, where $\mu \in [a,b]$ is the mean agent-type in that period.

Now, proceed to the first period, and consider the decision problem faced by a type-x agent who meets a type-y agent from the other side of the market. The continuation payoffs to agents x and y of not matching with each other are $\frac{1}{2}(x+\mu)$ and $\frac{1}{2}(y+\mu)$, respectively. Therefore, the total surplus generated by a match between them is

$$S(x,y;\mu) = x + y - \frac{1}{2}(x+\mu) - \frac{1}{2}(y+\mu) = \frac{1}{2}(x+y) - \mu.$$

Since the symmetric Nash bargaining solution allocates match surplus equally between the two agents, the values of matching are $\frac{1}{2}(x+\mu) + \frac{1}{2}S(x,y;\mu)$ and $\frac{1}{2}(y+\mu) + \frac{1}{2}S(x,y;\mu)$ for agents x and y, respectively. Consequently, match formation is optimal if the match surplus $S(x,y;\mu)$ is non-negative[2], i.e. $x+y \geq 2\mu$. Letting $M(x;\mu)$ denote the set of $y \in [a,b]$ for which $S(x,y;\mu) \geq 0$, the optimal matching strategy in the first period as a function of agent-type and μ is given as follows:

(i) if $\mu \in \left[a, \frac{1}{2}(a+b)\right]$, then

$$M(x;\mu) = \begin{cases} [2\mu - x, b], & \text{if } x \in [a, 2\mu - a) \\ [a,b], & \text{if } x \in [2\mu - a, b] \end{cases}; \tag{1}$$

(ii) if $\mu \in \left(\frac{1}{2}(a+b), b\right]$, then

$$M(x;\mu) = \begin{cases} \emptyset, & \text{if } x \in [a, 2\mu - b) \\ [2\mu - x, b], & \text{if } x \in [2\mu - b, b] \end{cases}. \tag{2}$$

Thus, for fixed μ, the optimal strategy in period 1 is a reservation rule whereby matching obtains as long as partner-type is above a threshold. Moreover, this lower bound for acceptable partner-type is decreasing in agent-type, so that agents of lower types are more selective in terms of whom they choose to match with. Note in particular that $M(x;a) = [a,b]\ \forall x \in [a,b]$.

Using the above characterization of first period action, the expected agent-type in period 2, μ, can be derived. Taking $\mu \in [a,b]$ as given for now, (1) and

[2]Since the distribution of agent-type is atomless, it can be assumed without loss of generality that agents choose to match in the case of indifference.

(2) imply that the measure of (unmatched) agents in the second round is given by

$$R(\mu) = \begin{cases} \int_a^{2\mu-a} f(x)\,F(2\mu-x)\,dx, & \text{if } \mu \in \left[a, \frac{1}{2}(a+b)\right] \\ F(2\mu-b) + \int_{2\mu-b}^{b} f(x)\,F(2\mu-x)\,dx, & \text{if } \mu \in \left(\frac{1}{2}(a+b), b\right] \end{cases}. \quad (3)$$

Therefore, the distribution of agent-type in the second period as a function of the given μ, $G(\cdot;\mu)$, satisfies:

(i) if $\mu \in \left[a, \frac{1}{2}(a+b)\right]$, then

$$G(y;\mu)\,R(\mu) = \begin{cases} \int_a^{y} f(x)\,F(2\mu-x)\,dx, & \text{if } y \in [a, 2\mu-a) \\ R(\mu), & \text{if } y \in [2\mu-a, b] \end{cases}; \quad (4)$$

(ii) if $\mu \in \left(\frac{1}{2}(a+b), b\right]$, then

$$G(y;\mu)\,R(\mu) = \begin{cases} F(y), & \text{if } y \in [a, 2\mu-b) \\ F(2\mu-b) + \int_{2\mu-b}^{y} f(x)\,F(2\mu-x)\,dx, & \text{if } y \in [2\mu-b, b] \end{cases}. \quad (5)$$

An equilibrium of the market is thus given by a value $\mu^* \in [a,b]$ such that

$$\mu^* = \int_a^b x\,dG(x;\mu^*),$$

where

$$\int_a^b x\,dG(x;a) \equiv \lim_{\mu\to a}\int_a^b x\,dG(x;\mu).$$

The proposition below states that, for all distribution functions $F(\cdot)$, there exists a unique equilibrium in which all agents are matched in the first period.

Proposition *A unique equilibrium exists, with* $\mu^* = a$.

Proof. The proof proceeds as follows. It will be shown that, firstly, $a = \lim_{\mu\to a}\int_a^b x\,dG(x;\mu)$, establishing the existence of the equilibrium $\mu^* = a$, and, secondly, $\mu > \int_a^b x\,dG(x;\mu)$ for all $\mu \in (a,b]$.

Now, for $\mu \in \left(a, \frac{1}{2}(a+b)\right]$, (3) and (4) give

$$\int_a^b x\,dG(x;\mu) = \frac{\int_a^{2\mu-a} x f(x)\,F(2\mu-x)\,dx}{\int_a^{2\mu-a} f(x)\,F(2\mu-x)\,dx}. \quad (6)$$

L'Hôpital's Rule then yields

$$\begin{aligned}
& \lim_{\mu \to a} \int_a^b x dG(x;\mu) \\
= & \lim_{\mu \to a} \frac{\int_a^{2\mu-a} xf(x) F(2\mu - x) dx}{\int_a^{2\mu-a} f(x) F(2\mu - x) dx} \\
= & \lim_{\mu \to a} \frac{2\int_a^{2\mu-a} xf(x) f(2\mu - x) dx}{2\int_a^{2\mu-a} f(x) f(2\mu - x) dx} \\
= & \lim_{\mu \to a} \frac{4\left[(2\mu - a) f(2\mu - a) f(a) + \int_a^{2\mu-a} xf(x) f'(2\mu - x) dx\right]}{4\left[f(2\mu - a) f(a) + \int_a^{2\mu-a} f(x) f'(2\mu - x) dx\right]} \\
= & \; a.
\end{aligned}$$

To show that $\mu > \int_a^b x dG(x;\mu)\ \forall \mu \in (a, b]$, consider the following two cases.

- *Case 1*: $\mu \in \left(a, \frac{1}{2}(a + b)\right]$

  In this case, (6) gives

$$\mu - \int_a^b x dG(x;\mu) = \mu - \frac{\int_a^{2\mu-a} xf(x) F(2\mu - x) dx}{\int_a^{2\mu-a} f(x) F(2\mu - x) dx}.$$

  To establish that this term is strictly positive, it suffices to show that

$$\int_a^{2\mu-a} (\mu - x) f(x) F(2\mu - x) dx > 0.$$

  Now,

$$\begin{aligned}
& \int_a^{2\mu-a} (\mu - x) f(x) F(2\mu - x) dx \\
= & \int_a^{\mu} (\mu - x) f(x) F(2\mu - x) dx + \int_{\mu}^{2\mu-a} (\mu - y) f(y) F(2\mu - y) dy \\
= & \int_a^{\mu} (\mu - x) f(x) F(2\mu - x) dx + \int_a^{\mu} (x - \mu) f(2\mu - x) F(x) dx \\
= & \int_a^{\mu} (\mu - x) \left[\frac{d(F(x) F(2\mu - x))}{dx}\right] dx \\
= & \int_a^{\mu} F(x) F(2\mu - x) dx > 0,
\end{aligned}$$

  where the second equality follows from the change-of-variable $x = 2\mu - y$, and the last equality obtains using integration-by-parts. This is the desired result.

- *Case 2*: $\mu \in \left(\frac{1}{2}(a+b), b\right]$

  In this case, (3) and (5) give

$$\mu - \int_a^b x dG(x;\mu) = \mu - \left[\frac{\int_a^{2\mu-b} x f(x)\, dx + \int_{2\mu-b}^b x f(x) F(2\mu - x)\, dx}{\int_a^{2\mu-b} f(x)\, dx + \int_{2\mu-b}^b f(x) F(2\mu - x)\, dx} \right],$$

which is strictly positive if it can be shown that

$$\int_a^{2\mu-b} (\mu - x) f(x)\, dx + \int_{2\mu-b}^b (\mu - x) f(x) F(2\mu - x)\, dx > 0.$$

Now, using the same steps as in the previous case,

$$\begin{aligned} \int_{2\mu-b}^b (\mu - x) f(x) F(2\mu - x)\, dx &= -(b-\mu) F(2\mu - b) \\ &\quad + \int_\mu^b F(x) F(2\mu - x)\, dx. \end{aligned}$$

In addition,

$$\int_a^{2\mu-b} (\mu - x) f(x)\, dx = (b-\mu) F(2\mu - b) + \int_a^{2\mu-b} F(x)\, dx,$$

which follows from integration-by-parts. Therefore,

$$\begin{aligned} &\int_a^{2\mu-b} (\mu - x) f(x)\, dx + \int_{2\mu-b}^b (\mu - x) f(x) F(2\mu - x)\, dx \\ = &\int_a^{2\mu-b} F(x)\, dx + \int_\mu^b F(x) F(2\mu - x)\, dx > 0. \end{aligned}$$

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If the market is open for any finite number of rounds, then it follows from the stated result and backward induction that a unique equilibrium obtains in which all agents are matched in the first period. Therefore, equilibrium matching in these markets is random, and all agents behave as if the market is open for a single round.

3. Concluding Remarks

Although the focus here on the separable production function is admittedly restrictive, this specification nevertheless provides some insights into equilibrium matching behavior in finite-horizon search environments. In particular, the result given here demonstrates how allowing for bargaining can dramatically alter the pattern of equilibrium sorting, and it shows that the introduction of side-payments leads to an unraveling of the market. Note that this unraveling mechanism differs from that analyzed in Li and Rosen (1998) and Li and Suen (2000), both of which focus on production uncertainty and agents' risk-aversion as the driving force behind early contracting and unraveling in labor markets. In contrast, the analysis here indicates that unraveling can obtain as an equilibrium outcome even with risk-neutral agents when utility is transferable. Finding general conditions on the match production function that lead to unraveling of markets with *ex ante* heterogeneous agents is a task that can be tackled in future research.

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