E C O N O M I C S   B U L L E T I N

Fractional cointegration between nominal interest rates and inflation: A re−examination of the Fisher relationship in the G7 countries

Sandrine Lardic
University of Paris X − Nanterre − MODEM

Valerie Mignon
University of Paris X − Nanterre − THEMA

Abstract

According to the Fisher hypothesis, the nominal interest rate is equal to the real interest rate, plus expected inflation. Results concerning the empirical validity of this hypothesis are not unanimous. These contradictions may be due to the fact that the usual concept of cointegration is too restrictive. We thus propose here to refer to the concept of fractional cointegration introduced by Granger (1986). We study the Fisher hypothesis by testing for the existence of a fractional cointegration relationship between nominal interest rates and inflation. Our results suggest that, for a large majority of G7 countries, such a relationship exists.

Citation: Lardic, Sandrine and Valerie Mignon, (2003) "Fractional cointegration between nominal interest rates and inflation: A re−examination of the Fisher relationship in the G7 countries." *Economics Bulletin*, Vol. 3, No. 14 pp. 1−10
Submitted: April 2, 2003. **Accepted:** July 2, 2003.
URL: http://www.economicsbulletin.com/2003/volume3/EB−03C20002A.pdf

1 Introduction

According to the Fisher hypothesis, the nominal interest rate is equal to the real interest rate, plus expected inflation. If nominal interest rates and inflation are integrated of order one, i.e. non stationary, it is possible to test the Fisher relationship by testing for the existence of a cointegration relationship between the two variables. In other words, one has to test whether the error term of this long-run relationship is integrated of order zero. Recently, Crowder and Hoffman (1996), using the Johansen procedure, find results consistent with the Fisher hypothesis for the 1952:1 to 1991:4 period on U.S. data. Conversely, using Engle and Granger (1987) cointegration tests, Mishkin (1992) results do not support the Fisher hypothesis on U.S. data. Evans and Lewis (1995), again by applying the Engle and Granger (1987) procedure, show that the Fisher hypothesis is verified only when regime shifts in the expected inflation process are taken into account.

As one can see, results are not unanimous concerning the empirical validity of the Fisher hypothesis. These contradictions may be due to the fact that the usual concept of cointegration is too restrictive. We thus refer here to the concept of fractional cointegration introduced by Granger (1986). This notion is linked to the fractional integration one which is itself linked to the long-term memory property of time series (see Granger and Joyeux (1980) and Hosking (1981)). In these conditions, the integration order of the error correction term is not necessarily 0 or 1, but it can be a real: the error correction term may be fractionally integrated. This allows to obtain more various mean reverting behaviors (see among others Chou and Shih (1997)). More specifically, a fractionally integrated error correction term implies the existence of an equilibrium long-term relationship between nominal interest rates and inflation. Thus, the error correction term needs not to be $I(0)$. Consequently, if the error correction term is fractionally integrated, then it exists a fractional cointegration relationship (that is an equilibrium relationship) between nominal interest rates and inflation, which is consistent with the Fisher hypothesis.

The paper is organized as follows. Section 2 presents fractional cointegration tests. Section 3 reports empirical results for the G7 countries. Section 4 concludes.

2 Tests for fractional cointegration

For the presentation of the tests, we consider two series, x_t and y_t each of which being integrated of order 1. x_t and y_t are fractionally cointegrated if there exists a cointegration relationship:

$$y_t = \alpha + \beta x_t + z_t \tag{1}$$

where z_t is a long-term memory process, such as an $ARFIMA$ process[1]:

$$\Phi(L)(1-L)^d z_t = \Theta(L)\epsilon_t \tag{2}$$

where $\Phi(L)$ and $\Theta(L)$ are autoregressive and moving average polynomials, respectively, ϵ_t is a white noise, L is the lag operator and:

[1] For a presentation of AutoRegressive Fractionally Integrated Moving Average processes, see Granger and Joyeux (1980) and Hosking (1981).

$$(1-L)^d = 1 - dL - \frac{d(1-d)}{2!}L^2 - \frac{d(1-d)(2-d)}{3!}L^3 - .... \quad (3)$$

The following fractional cointegration tests are based on the null hypothesis:

$$H_0 : x_t \text{ and } y_t \text{ are not cointegrated, i.e. } z_t \text{ is } I(1), \text{ for all } \alpha, \beta \in \Re,$$

against the alternative:

$$H_1 : x_t \text{ and } y_t \text{ are cointegrated, i.e. } z_t \text{ is } I(d), \text{ with } d < 1$$

These tests are applied on the estimated residuals $\hat{z}_t$ of the long-term relationship (1).

2.1 Fractional cointegration tests based on the estimation of ARFIMA processes

We just recall the main lines of the two techniques employed here: the Geweke and Porter-Hudak (1983) method (GPH) and the exact maximum likelihood procedure.

The application of these procedures on residual series allows us to test the null hypothesis of a unit root ($d = 1$) against the alternative of fractional integration ($d < 1$). This is equivalent to a test of the null $d' = 0$ against $d' < 0$, with $d' = d - 1$ where d is the fractional difference parameter of the series in levels and d' the fractional difference parameter of the series in first differences.

The aim of the Geweke and Porter-Hudak (GPH) method is to estimate the fractional integration parameter $\hat{d}'$ by the following regression:

$$\ln I(\lambda_j) = \hat{\alpha} - 2\hat{d}' \ln\left(2\sin\left(\frac{\lambda_j}{2}\right)\right) + \hat{e}_j \quad (4)$$

where λ_j is the Fourier frequency $\lambda_j = \frac{2\pi j}{T}$, $I(\lambda_j)$ is the periodogram of $\Delta z_t, t = 1, ..., T$, and $j = 1, 2, ..., m$ where m corresponds to the number of periodogram ordinates. Traditionally the number of periodogram ordinates is chosen from the interval $\left[T^{0.45}, T^{0.55}\right]$. However, Hurvich *et al.* (1998) recently showed that the optimal m is of order $O\left(T^{0.8}\right)$. Asymptotic normality of the estimated fractional difference parameter has been proved by Geweke and Porter-Hudak (1983) when $d < 0$ and by Robinson (1990) for $0 < d < 1/2$.

The GPH method is a two step estimating procedure. Indeed, one has to estimate the fractional difference parameter in a first step. In the second step, autoregressive and moving average parameters are estimated using traditional time series methods. There exist however one step estimation procedures, like the exact maximum likelihood (EML) one (see Dahlhaus (1989) and Sowell (1992a,b) for details).

Let $\Delta z_t, t = 1, ..., T$, being a fractionally integrated stationary Gaussian time series. Δz_t follows a normal law with mean zero and covariance matrix Σ. Its density function is given by:

$$f(\Delta z_t, \Sigma) = (2\pi)^{-T/2} |\Sigma|^{-1/2} \exp\left(-\frac{1}{2}\Delta z_t' \Sigma^{-1} \Delta z_t\right) \quad (5)$$

Due to the stationarity property, the covariance matrix has a Toeplitz form: $\Sigma = \left[\gamma_{i-j}\right]$ with $i, j = 1, 2, ..., T$. Moreover, one has:

$$\gamma_s = \frac{1}{2\pi} \int_0^{2\pi} f_{\Delta z}(\lambda) e^{i\lambda s} d\lambda \tag{6}$$

Evaluating the autocovariance function requires calculation of the spectral density of Δz_t. Let $u_t = (1-L)^{d'} \Delta z_t$ and $\Phi(L) u_t = \Theta(L) \varepsilon_t$ where ε_t is a white noise. Let also:

$$\Phi(x) = \prod_{j=1}^{p} \left(1 - \rho_j x\right) \tag{7}$$

with $|\rho_n| < 1$ for $n = 1, 2, ..., p$.

Sowell (1992) showed that the spectral density of a stationary time series generated by an $ARFIMA(p, d', q)$, where the roots of the autoregressive polynomial are simple, is given by:

$$f_{\Delta z}(\lambda) = \sigma^2 \sum_{l=-q}^{q} \sum_{j=1}^{p} \Psi(l) \xi_j \left[\frac{\rho_j^{2p}}{1 - \rho_j \omega} - \frac{1}{1 - \rho_j^{-1} \omega} \right] (1-\omega)^{-d'} \left(1 - \omega^{-1}\right)^{-d'} \omega^{p+l} \tag{8}$$

where $\omega = e^{i\lambda}$ and:

$$\xi_j = \left[\rho_j \prod_{i=1}^{p} \left(1 - \rho_i \rho_j\right) \prod_{m \neq j} \left(\rho_j - \rho_m\right) \right]^{-1} \tag{9}$$

$$\Psi(l) = \sum_{s=\max[0,l]}^{\min[q,q-l]} \theta_s \theta_{s-l} \tag{10}$$

Substituting the value of $f_{\Delta z}(\lambda)$ in (6) gives (see Sowell (1992), p.173):

$$\gamma_s = \sigma^2 \sum_{l=-q}^{q} \sum_{j=1}^{p} \Psi(l) \xi_j C\left(d', p+l-s, \rho_j\right) \tag{11}$$

where:

$$\begin{aligned} C\left(d', h, \rho\right) &= \frac{1}{2\pi} \int_0^{2\pi} \left[\frac{\rho^{2p}}{(1 - \rho e^{-i\lambda})} - \frac{1}{(1 - \rho^{-1} e^{-i\lambda})} \right] \\ &\times \left(1 - e^{-i\lambda}\right)^{-d'} \left(1 - e^{i\lambda}\right)^{-d'} e^{-i\lambda h} d\lambda \end{aligned} \tag{12}$$

with $h = p + l - s$.
For empirical purposes, one uses the following form:

$$C\left(d',h,\rho\right) = \frac{\Gamma\left(1-2d'\right)\Gamma\left(d'+h\right)}{\Gamma\left(1-d'+h\right)\Gamma\left(1-d'\right)\Gamma\left(d'\right)} \times \left[\rho^{2p}F\left(d'+h,1;1-d'+h;\rho\right) + F\left(d'-h,1;1-d'-h;\rho\right) - 1\right] \tag{13}$$

where $F(a,b;c;x)$ is the hypergeometric function:

$$F\left(a,b;c;x\right) = \sum_{n} \frac{\Gamma\left(a+n\right)\Gamma\left(b+n\right)\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)\Gamma\left(c+n\right)\Gamma\left(n+1\right)} x^n \tag{14}$$

This method has the interest of using all information relative to the short and long-term behavior of the series since it estimates simultaneously all the parameters of the $ARFIMA(p,d',q)$ representation.

2.2 The modified R/S analysis

Lo (1991) derived a test, called the modified R/S statistic, of the null hypothesis of short-range dependence which is invariant to a general class of short-term memory processes. The modified R/S statistic, denoted as Q_{mT}, is given by:

$$Q_{mT} = R/s_T\left(q\right) = \frac{1}{s_T\left(q\right)}\left[\underset{1\le k\le T}{Max}\sum_{j=1}^{k}\left(\Delta\hat{z}_j - \overline{\Delta\hat{z}}\right) - \underset{1\le k\le T}{Min}\sum_{j=1}^{k}\left(\Delta\hat{z}_j - \overline{\Delta\hat{z}}\right)\right] \tag{15}$$

where

$$s_T^2\left(q\right) = \frac{1}{T}\sum_{j=1}^{T}\left(\Delta\hat{z}_j - \overline{\Delta\hat{z}}\right)^2 + \frac{2}{T}\sum_{j=1}^{q}\omega_j\left(q\right)\left[\sum_{i=j+1}^{T}\left(\Delta\hat{z}_j - \overline{\Delta\hat{z}}\right)\left(\Delta\hat{z}_{i-j} - \overline{\Delta\hat{z}}\right)\right] \tag{16}$$

with

$$\omega_j\left(q\right) = 1 - \frac{j}{q+1}, q < T \tag{17}$$

We see that the autocovariances are weighted according to lags q (see Andrews (1991) for the choice of q and Newey and West (1987) for the weights $\omega_j\left(q\right)$). The limiting distribution of the modified R/S statistic is known (see Lo, 1991) and it is thus possible to test the null hypothesis of short-term memory against the alternative of long-term memory (fractional integration) of the error term by comparing the statistic $V = Q_{mT}/\sqrt{T}$ to critical values.

2.3 The Lobato-Robinson test

Lobato and Robinson (1998) have proposed an LM-type test of the null hypothesis of $I(0)$ against the alternative $I(d')$ with $d' \neq 0$. The test statistic is given by:

Table 1: Unit root tests on interest rates series

		ADF	*PP*	*KPSS* l_4	*KPSS* l_{12}
Germany	Level	-1.24 (1)	-1.23 (1)	0.59	0.27*
	Variation	-15.74* (1)	-16.27* (1)	0.05*	0.03*
France	Level	-1.11 (1)	-1.14 (1)	1.64	0.72
	Variation	-22.11* (1)	-22.10* (1)	0.04*	0.05*
U.K.	Level	-2.23 (2)	-2.25 (1)	1.31	0.06
	Variation	-17.10* (1)	-17.13* (1)	0.58	0.06*
Canada	Level	-0.93 (1)	-0.94 (1)	1.36	0.58
	Variation	-18.01* (1)	-18.05* (1)	0.07*	0.07*
Italy	Level	-2.75 (2)	-2.41 (1)	1.08	0.47
	Variation	-15.64* (1)	-15.39* (1)	0.11*	0.15*
Japan	Level	-2.11 (1)	-2.82 (1)	2.88	1.19
	Variation	-10.17* (1)	-10.15* (1)	0.14*	0.10*
U.S.	Level	-1.28 (1)	-1.16 (1)	1.22	0.52
	Variation	-15.06* (1)	-14.75* (1)	0.05*	0.04*

(1): Model without constant, nor deterministic trend. (2): Model with constant and without trend.
*: stationary series at the 1% significance level.

$$LR = -m^{1/2} \frac{\sum_{j=1}^{m} v_j I(\lambda_j)}{\sum_{j=1}^{m} I(\lambda_j)} \tag{18}$$

with $v_j = \log j - m^{-1} \sum_{i=1}^{m} \log i$ and $I(\lambda_j)$ is the periodogram of $\Delta \hat{z}_t$. If the test is applied to the true error term, then its asymptotic distribution is standard normal.

3 Empirical results

We consider monthly series of three-month nominal interest rates and inflation rates for the G7 countries on the 1970:01 to 2001:03 period. We first test for the (non)stationarity of these series.

Results of usual unit root tests (Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski *et al.* (KPSS) tests) are reported in tables 1 and 2[2]. According to the three tests, all interest rates series are integrated of order one. For inflation rates series, results are more mitigate. According to the KPSS test, the U.S. series is stationary. However, according to the non stationarity tests (ADF and PP), this series is integrated of order one. The Germany series is stationary according to ADF and PP tests and $I(1)$ according to the KPSS test. Despite these two contradictions, we retain globally that all series are integrated of order one.

[2] For the KPSS test, we use the values recommended by Schwert (1989) for the truncation parameter : $l_4 = int\left[4\left(\frac{T}{100}\right)^{1/4}\right]$ and $l_{12} = int\left[12\left(\frac{T}{100}\right)^{1/4}\right]$.

Table 2: Unit root tests on inflation rates series

		ADF	*PP*	*KPSS*	
				l_4	l_{12}
Germany	Level	-5.64* (3)	-8.19* (1)	0.17 (t)	0.15 (t)
	Variation			0.01*	0.03*
France	Level	-1.09 (1)	-2.04 (1)	3.54	1.48
	Variation	-11.97* (1)	-23.27* (1)	0.05*	0.10*
U.K.	Level	-2.15 (1)	-3.88* (1)	2.64	1.27
	Variation	-20.70* (1)		0.01*	0.03*
Canada	Level	-1.53 (1)	-3.34 (3)	2.75	1.22
	Variation	-20.80* (1)	-24.02* (1)	0.03*	0.07*
Italy	Level	-1.40 (2)	-2.21 (1)	2.82	1.24
	Variation	-11.94* (1)	-17.56* (1)	0.03*	0.06*
Japan	Level	-2.06 (1)	-2.22 (1)	0.94	0.61
	Variation	-25.88* (1)	-21.99* (1)	0.02*	0.05*
U.S.	Level	-1.42 (1)	-2.71 (1)	0.11*	0.08*
	Variation	-15.24* (1)	-17.94* (1)		

(1): Model without constant, nor deterministic trend. (2): Model with constant and without trend. (3): Model with constant and trend. *: stationary series at the 1% significance level.

Table 3: Unit root tests on residual series

	ADF	*PP*
Germany	-3.73	-4.01
France	-3.72	-4.04
U.K.	-3.38	-3.54
Canada	-3.17	-4.31
Italy	-3.59	-3.88
Japan	-1.93	-1.98
U.S.	-2.45	-3.77

It seems thus interesting to test for the existence of a stable long-term relationship between nominal interest rates and inflation series. Results of the application of traditional cointegration tests (ADF and PP tests) on residuals are reported in table 3. These results show that the error term is non stationary at the 1% significance level, suggesting that the Fisher hypothesis does not hold.

However, this last result may be due to the fact that the usual concept of cointegration is too restrictive. We thus apply fractional cointegration tests. Since the tests should be applied on stationary series, they have been run on residuals in first differences. Results are reported in table 4 for the modified R/S analysis, in table 5 for the GPH and Lobato-Robinson tests and in table 6 for the exact maximum likelihood procedure.

It is important to note that the properties of these tests, such as the asymptotic distribution under the null hypothesis, are known only if the true equilibrium errors z_t are observable. However this is not the case here since fractional cointegration tests are applied to estimated residuals. Consequently, the error correction term tends to be biased in favor of the stationarity hypothesis leading to too many rejections of the null hypothesis of no cointegration.

Table 4: Modified R/S analysis

	d'	V
Germany	-0.0627	0.6929
France	-0.0449	0.7661
U.K.	-0.0699	0.6609
Canada	-0.0649	0.6806
Italy	-0.0022	0.9867
Japan	-0.0739	0.6656
U.S.	-0.0605	0.6987

Table 5: GPH and Lobato-Robinson tests

	GPH		LR
	d'	$t_{d'}$	
Germany	-0.2334	-3.5844	-0.5560
France	-0.3024	-4.8109	-4.8133
U.K.	-0.1679	-2.6178	-2.8725
Canada	-0.3154	-5.0188	-3.1400
Italy	-0.2532	-4.0281	-7.5048
Japan	-0.2741	-3.5750	-1.7201
U.S.	-0.3042	-4.8402	-1.5566

$t_{d'}$ is the t-ratio of the estimated d'.

Table 6: Exact maximum likelihood procedure

	$AICc$		SIC	
	$ARFIMA(p, d', q)$	LL	$ARFIMA(p, d', q)$	LL
Germany	$(1, -0.4145, 0)$	-490.02	$(1, -0.4145, 0)$	-490.02
	$t_{d'} = -3.6287$		$t_{d'} = -3.6287$	
France	$(0, -0, 2801, 0)$	-636.44	$(1, -0.3973, 0)$	-633.67
	$t_{d'} = -6.2554$		$t_{d'} = -5.8117$	
U.K.	$(1, -0.4279, 0)$	-588.32	$(1, -0.4279, 0)$	-588.32
	$t_{d'} = -5.4416$		$t_{d'} = -5.4416$	
Canada	$(0, -0.2825, 0)$	-649.74	$(0, -0.2825, 0)$	-649.74
	$t_{d'} = -6.0695$		$t_{d'} = -6.0695$	
Italy	$(0, -0.1289, 0)$	-786.06	$(0, -0.1289, 0)$	-786.06
	$t_{d'} = -2.0134$		$t_{d'} = -2.0134$	
Japan	$(0, -0.2716, 0)$	-297.59	$(0, -0.2716, 0)$	-297.59
	$t_{d'} = -1.9027$		$t_{d'} = -1.9027$	
U.S.	$(1, -0.4296, 0)$	-549.12	$(1, -0.4296, 0)$	-549.12
	$t_{d'} = -5.8888$		$t_{d'} = -5.8888$	

$t_{d'}$ is the t-ratio of the estimated d'. $AICc$: Akaike information criterion corrected by Hurvich and Tsai (1989). SIC: Schwarz information criterion. LL: log-likelihood at optimum.

One should thus use other critical values than those calculated on the basis of the true observed series. The reader is referred for example to Septhon (1993), Cheung and Lai (1993), Barkoulas *et al.* (1997) and Dittmann (2000) for critical values of fractional cointegration tests.

According to the modified R/S analysis, all residual series are fractionally integrated, except for the French case, since the statistic V does not range in its confidence interval given by Dittmann (2000). According to the GPH procedure, all residual series are fractionally integrated since the fractional difference parameter appears to be significantly different from zero. The Lobato-Robinson test indicates that only one series is not characterized by fractional integration: the German error term.

The application of the exact maximum likelihood procedure requires the choice of initial values for the parameters of the $ARFIMA(p, d', q)$ representation. This choice is fundamental since the log-likelihood is not globally concave. There exists two possibilities for choosing these initial values:

- We first estimate the $\hat{d}'$ value of the fractional integration parameter by modified R/S analysis or GPH procedure. Then, we apply the $(1-L)^{\hat{d}'}$ filter to the considered series Δz_t. Finally, one has to choose for the initial values of autoregressive and moving average coefficients the parameters estimated on the $(1-L)^{\hat{d}'} \Delta z_t$ series.
- Given an arbitrary set of d' values, we calculate, for each value of d', the $(1-L)^{d'} \Delta z_t$ series. One estimates, by usual time series methods, the ARMA parameters and the white noise variance. The initial values are then given by those associated to the estimated model with the lowest white noise variance.

We apply these two procedures in order to be sure that the maximum has been reached. Then, the log-likelihood is maximized according to all the parameters of the $ARFIMA(p, d', q)$ representation. Finally, we retain the model which minimizes the Akaike information criterion corrected by Hurvich and Tsai (1989) and the Schwarz information criterion. Results in table 6 indicate that all residual series are fractionally integrated since the fractional difference parameter appears to be significantly different from zero.

4 Conclusion

According to the simulations made by Dittmann (2000), the Lobato-Robinson and the GPH tests exhibit the highest power among the fractional cointegration tests considered here but they have poor size properties. On the other hand, the modified R/S analysis exhibits only moderate size distortion but is less powerful than the other tests. If we privilege the power aspect, we thus conclude that all error terms, except the German one, are fractionally integrated. These results support the Fisher hypothesis since they imply the existence of a fractional cointegration relationship, that is a stable long-term equilibrium relationship, between nominal interest rates and inflation. This conclusion illustrates the interest of fractional cointegration since such a long-term relationship between the two variables does not exist according to usual cointegration tests.

References

[1] Andrews, D. (1991) "Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation" *Econometrica* **59**, 817-858.

[2] Barkoulas, J.T., Baum, C.F. and G.S. Oguz (1997) "Fractional Cointegration Analysis of Long Term International Interest Rates" *International Journal of Finance* **9**, 586-606.

[3] Cheung, Y-W. and K.S. Lai (1993) "A Fractional Cointegration Analysis of Purchasing Power Parity" *Journal of Business and Economic Statistics* **11**, 103-112.

[4] Chou, W. and Y. Shih (1997) "Long-run Purchasing Power Parity and Long-term Memory: Evidence from Asian Newly Industrialized Countries" *Applied Economics Letters* **4**, 575-578.

[5] Crowder, W.J. and D.L. Hoffman (1996) "The Long-run relationship Between Nominal Interest rates and Inflation: The Fisher Equation Revisited" *Journal of Money, Credit and Banking* **28**, 102-118.

[6] Dahlhaus, R. (1989) "Efficient Parameter Estimation for Self-Similar Processes" *Annals of Statistics* **17**, 1749-1766.

[7] Dickey, D.A. and W.A. Fuller (1979) "Distribution of the Estimators for Autoregressive Time Series with a Unit Root" *Journal of the American Statistical Association* **74**, 427-431.

[8] Dickey, D.A. and W.A. Fuller (1981) "The Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root" *Econometrica* **49**, 1057-1072.

[9] Dittmann, I. (2000) "Residual-Based Tests for Fractional Cointegration: A Monte Carlo Study" *Journal of Time Series Analysis* **6**, 615-647.

[10] Engle, R.F. and C.W.J. Granger (1987) "Cointegration and Error Correction: Representation, Estimation, and Testing" *Econometrica* **55**, 251-276.

[11] Evans, M. and K. Lewis (1995) "Do Expected Shifts in Inflation Affect Estimates of the Long-run Fisher Relation?" *Journal of Finance* **50**, 225-253.

[12] Geweke, J. and S. Porter-Hudak (1983) "The Estimation and Application of Long Memory Time Series Models" *Journal of Time Series Analysis* **4**, 221-238.

[13] Granger, C.W.J. (1986) "Developments in the Study of Cointegrated Economic Variables" *Oxford Bulletin of Economics and Statistics* **48**, 231-228.

[14] Granger, C.W.J. and R. Joyeux (1980) "An Introduction to Long-Memory Time Series Models and Fractional Differencing" *Journal of Time Series Analysis* **1**, 15-39.

[15] Hosking, J.R.M. (1981) "Fractional Differencing" *Biometrika* **68**, 165-176.

[16] Hurvich, C.M., R.S. Deo and J. Brodsky (1998) "The Mean Squared Error of Geweke and Porter-Hudak's Estimator of the Memory Parameter of a Long-memory Time Series" *Journal of Time Series Analysis* **19**, 19-46.

[17] Hurvich, C.M. and C.L. Tsai (1989) "Regression and Time Series Model Selection in Small Samples" *Biometrika* **76**, 297-307.

[18] Johansen, S., 1998, Statistical Analysis of Cointegrated Vectors. *Journal of Economic Dynamics and Control,* 12, 231-254.

[19] Kwiatkowski, D., P. Phillips, P. Schmidt, and Y. Shin (1992) "Testing the Null Hypothesis of Stationary Against the Alternative of a Unit Root: How Sure Are We that Economic Time Series Have a Unit Root ?" *Journal of Econometrics* **54**, 159-178.

[20] Lo, A.W. (1991) "Long-Term Memory in Stock Market Prices" *Econometrica* **5**, 1279-1313.

[21] Lobato, I.N. and P.M. Robinson (1998) "A Nonparametric Test for I(0)" *Review of Economic Studies* **65**, 475-495.

[22] Mishkin, F. (1992) "Is the Fisher Effect for Real? A Reexamination of the Relationship Between Inflation and Interest Rates" *Journal of Monetary Economics* **30**, 195-215.

[23] Newey, W. and K. West (1987) "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix" *Econometrica* **55**, 703-708.

[24] Phillips, P.C.B. and P. Perron (1988) "Testing for a Unit Root in Time Series Regression" *Biometrika*, 335-346.

[25] Robinson, P.M. (1990) "Time Series with Strong Dependence" *Advances in Econometrics*, 6th World Congress, Cambridge University Press.

[26] Schwert, G.W. (1989) "Tests for Unit Roots: A Monte Carlo Investigation" *Journal of Business and Economic Statistics* **7**, 147-159.

[27] Septhon, P. (1993) "Fractional Cointegration: Monte Carlo Estimates of Critical Values with an Application" *Working Paper*, University of New Brunswick.

[28] Sowell, F. (1992a) "Maximum Likelihood Estimation of Stationary Univariate Fractionally Integrated Time Series Models" *Journal of Econometrics*, 165-188.

[29] Sowell, F. (1992b) "Modeling Long-Run Behavior with the Fractional ARIMA Model" *Journal of Monetary Economics* **29**, 277-302.