E C O N O M I C S B U L L E T I N

On the subsidization of local monopolies

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Abstract

Firms compete in model of horizontal differentiation a la Hotelling. They use two−part tariffs but the market is such that, in equilibrium, it is not fully covered (firms are then local monopolies). The question we adressed in this paper is to determine what kind of subsidies are the best instruments to increase the coverage at the lowets cost. This paper shows first that an ad valorem subsidy on the usage price is a less costly instrument to increase the coverage than a per unit subsidy.

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1 Introduction

A recurrent debate in public economics is the study of commodity taxation when firms use linear prices. This topic has been the subject of many works[1]. The main result that emerges from this literature is that the type of taxation used matters as soon as there exists imperfect competition. Then, before looking at the level a commodity should be taxed, governments have to deal with the question of the balance between different types of commodity taxes (generally between *ad valorem* and unit taxation)[2]. However, the question of commodity taxation when firms use two-part tariffs (or other general tariffs) has not received much attention by public economists even if this kind of tariffs are more and more used in practice.

The main problem with the taxation of good produced by a non linear pricing monopolist is to justify public intervention. Indeed, by using discriminating prices a monopolist can extract all the consumers' surplus and then produces efficiently (first best). However, Laffont (1987) shows that if there is asymmetric information or if there exists a cost of public funds there is room for commodity taxation. Laffont then studies the sign of the tax and shows that this sign depends on the cost of public funds and on the cost of the informational rent the government has to leave to the firm. The question of the type of tax is not addressed in this paper. Cheung (1998) deals with this problem. The author compares a specific tax and an *ad valorem* one on the tariff when a monopoly uses a non linear tariff to discriminate among consumers. The author shows that the result of Skeath and Trandel (1994) could be generalized to the case of non linear pricing: the *ad valorem* tax Pareto dominates the specific one. Then, even with a non linear pricing firm, the question of the type of commodity tax to use is of significant relevance.

In this note we will take a different approach to justify public intervention. We focus on a good that is produced by local monopolies. In this kind of industry, some consumers are not served because they are 'too far' from one of the monopolies. As a consequence, the market is not fully served. A government might then want to increase the market coverage by using subsidization. This question has been the topic of discussions between the French government and GSM networks[3].

To study this question, we will use the model of linear city due to Hotelling[4]. Moreover, we will focus on particular non linear prices namely two-part tariffs. This tariff are made of a usage fee (per unit price) and of a subscription fee. The main objective of the paper is to know what kind of subsidies the government should use. After a brief description of the model (section 2) and of the equilibrium (section 3), we compare, in section 4, the use of a per unit subsidy to the use of an *ad valorem* one on the usage fee. In section 5, we turn to the comparison between the subsidization of the subscription fee and the *ad valorem* one on the per unit price. Finally, section 6 sums up the result and gives some concluding remarks.

[1] See the thorough survey of Keen (1998).

[2] See for instance Delipalla-Keen (1992), Skeath-Trandel (1994)or Suits-Musgrave (1953). Moreover, a specific is also called a unit tax.

[3] In France, 1480 cities are not connected to any GSM networks.

[4] We will suppose that the localizations of the firms are given.

2 The model

We consider competition in a linear city model between two firms. The two firms have the same cost structure. Serving a customer involves a fixed cost f. Moreover, the per unit cost c, is constant and is the same for all consumers. Total cost is then the per customer cost multiplied by the number of served customers. The firms use a particular type of non linear prices, two-part tariff. Each firm proposes one two-part tariff, which is noted, for firm i, by

$$T_i(q) = A_i + p_i q,$$

where q is the quantity consumed by a customer. Moreover, A_i, the fixed fee, can be viewed as a subscription fee for telephone services and p_i, the variable fee (or usage fee) as the price per minute of communication[5]. The profit per consumer served is noted $\pi_i(p_i, A_i)$, the total profit of firm i is then this per consumer profit times the number of subscribers (the number of consumers connected to network i) and is noted by Π_i.

Consumers are uniformly located on the segment $[0, 1]$. Moreover, firms are located[6] at the extremities of the segment, namely firm 1 is at $x_1 = 0$ and firm 2 at $x_2 = 1$. A consumer located at x, who subscribes and consumes q units of the good produced by firm i, has a utility

$$I + u(q) + \overline{u} - \delta |x - x_i|,$$

where I is the income of the consumer, $\overline{u}$ is the fixed surplus. This surplus is, for the telecommunication example, the utility obtained by being connected to one of the two networks[7]. The income and fixed surplus are the same for all consumers. The term $\delta |x - x_i|$ represents the cost, borne by the consumer, not to consume her most favorite good (or not to be connected to her most favorite network)[8]. The variable gross surplus depends on the quantity consumed and is given by

$$u(q) = \frac{q^{1-\frac{1}{\eta}}}{1 - \frac{1}{\eta}}.$$

So, when a customer faces a usage fee p the quantity consumed q is such that

$$u'(q) = p \Leftrightarrow q = p^{-\eta}. \tag{1}$$

This variable gross surplus, then yields a constant elasticity demand function. Moreover, the elasticity of demand, namely η, is assumed to be greater than 1[9]. All the customers of firm i buy the same quantity and their net surplus is

$$w_i(p_i, A_i) = v(p_i) - A_i$$

[5] A could be the right to enter in an entertainment park, and then p is the price of an attraction in the park. An other possible interpretation could be to see A as the price to enter in a night-club and p the price of a drink in this night-club.

[6] Then, in this paper we do not consider the localisation game between the two firms.

[7] This surplus might be seen as the utility to be in the park.

[8] This cost does not depend on the quantity purchased. By relaxing this assumption, we will obtain a demand that depends on the quantity purchased. This could be a possible extension of this work.

[9] These assumptions of constant elasticity of demand and that the elasticity is greater than one are made for technical convenience.

where $v(p_i)$ is the variable net surplus which is such that

$$v(p_i) = \max_q \{u(q) - p_i q\} = \frac{p_i^{-(\eta-1)}}{\eta - 1} \text{ with } v' = -q.$$

There can be three types of equilibria in this kind of model[10]:

- type one, the consumer who is indifferent between buying from the two firms strictly prefers buying to not buying, there is full coverage.
- type two, the consumer who is indifferent between buying from the two firms is also indifferent between buying and not buying, there is also full coverage.
- type three, type one, the consumer who is indifferent between buying from the two firms strictly prefers not buying to buying. Then, the sets of consumer buying from each firm is separate. There is limited participation, each firm's demand is only affected by its own tariff, firms are then local monopolies.

In this note we will only focus on type three equilibria[11]. This type of equilibria appears if the fixed utility $\overline{u}$ is small enough. Indeed, the utility of the consumer who is indifferent between buying from the two firms is such that

$$\begin{cases} w_i - \delta\alpha = w_j - \delta(1-\alpha) \\ w_i + \overline{u} - \delta\alpha < 0. \end{cases}$$

By combining this two relations, we obtain that type three equilibrium appears when $\overline{u} < \frac{\delta}{2} - \frac{w_1+w_2}{2}$. Moreover, the market share, α_i, of firm i is such that the utility of the last subscriber (the one who is the most distant from firm i) is equal to zero, $v(p_i) - A_i + \overline{u} - \delta\alpha_i = 0$ and then we have

$$\alpha_i = \frac{v(p_i) - A_i + \overline{u}}{\delta} \text{ or } \frac{w_i + \overline{u}}{\delta}. \tag{2}$$

The government's objective function is to increase the coverage of the market for many possible reasons (network externalities, universal service, local development...) for the least possible expense. The government can use three types of subsidies to enhance the coverage:

- a specific subsidy (or per unit one), noted s. For each unit of the good sold, the firm receives a fixed amount s and then s is a subsidy on the volume of sales;
- an *ad valorem* subsidy on the per unit price, noted σ[12], so σ is a subsidy on the value of sales;
- an *ad valorem* subsidy on the subscription fee, noted Σ which decreases the cost of serving a new consumer.

The public expenditure, per firm, is noted PE_i and is equal to $\alpha_i(\Sigma A_i + \sigma p_i q_i + s q_i)$. The question we address here is to determine which instrument the government has to use.

[10] See Mas Colell, Whinston and Green (1995) chapter 12 for more details on the different type of equilibrium.

[11] In a companion paper (Boldron 2001), type one equilibria are studied.

[12] The rate σ is expressed as a percentage of the consumer price (tax-inclusive). The results are not affected with a tax-exclusive rate which expresses subsidy as a proportion of the producer price.

3 The Equilibrium

Introducing all types of subsidies, the profit of firm i is given by

$$\Pi_i = \alpha_i(p_i, A_i)\left[(1+\Sigma)A_i + (1+\sigma)p_i q(p_i) - (c-s)q(p_i) - f\right].$$

But, it is equivalent here to see competition as one in which firms choose the net surplus w_i and the usage fee rather the fixed fee and the usage fee. In this case profits are given by

$$\Pi_i = \alpha(w)\left[(1+\Sigma)(v(p)-w)A_i + (1+\sigma)pq - (c-s)qp - f\right].$$

The first order conditions[13] then give (for both firms as they are symmetric):

$$p = \frac{\eta(c-s)}{(\eta-1)\sigma + \Sigma + \eta} \tag{3}$$

$$A = \frac{1}{2(1+\Sigma)}\left[(1+\Sigma)v(p) - (1+\sigma)pq + (c-s)q + f + (1+\Sigma)\overline{u}\right] \tag{4}$$

$$\alpha = \frac{1}{2\delta(1+\Sigma)}\left[(1+\Sigma)(v(p)+\overline{u}) + (1+\sigma)pq - (c-s)q - f\right]. \tag{5}$$

We only consider situations where some consumers are not served, then it restricts the possible value of the subsidies we study. As the set of all possible values of subsidies that lead to a situation where market shares are such that $\alpha_1 + \alpha_2 < 1$ is hard to characterized, we will assume all along this paper that we compare subsidies that belong to this set. One can remark that if there is no public intervention (all the subsidies are equal to zero), the per unit price is equal to the marginal cost. Then, except when there exists a cost of public funds there is no need to set a tax on this market.

From this equilibrium values, we can deduce the effect of a variation of one of the subsidies on price:

$$\frac{\partial p}{\partial s} = \frac{-\eta}{(\eta-1)\sigma + \Sigma + \eta} < 0, \tag{6}$$

$$\frac{\partial p}{\partial \sigma} = \frac{\eta-1}{\eta} p \frac{\partial p}{\partial s} < 0, \tag{7}$$

$$\frac{\partial p}{\partial \Sigma} = \frac{1}{\eta-1}\frac{\partial p}{\partial \sigma} < 0. \tag{8}$$

The three subsidies all reduce the price and then could allow the firm to increase their market shares but also the subscription fees.

[13] We do not consider here second order conditions and we supposed that they are verified.

4 Comparison between *ad valorem* subsidy and specific subsidy

In this section, we compare this two instruments with $\Sigma = 0$ as it does not change our result but leads to a clearer exposition. From the marginal effect on price derive in the previous section (relations 6, 7 and 8), we have that

$$\frac{\partial p}{\partial s} > -1 \text{ and } \frac{\partial p}{\partial \sigma} > -p \text{ for all } (s, \sigma) \neq (0, 0)$$

$$\frac{\partial p}{\partial s} = -1 \text{ and } \frac{\partial p}{\partial \sigma} = -p \text{ for } (s, \sigma) = (0, 0)$$

Then, there is never overshifting (except when we start from the 'no intervention' point) which means here that the drop in the price is smaller than the subsidy. This indicates that firms have a strategic use of the subsidies. This observation suggests that *ad valorem* and unit taxation will have different effects on the equilibrium.

The variation of a subsidy leads to the following change in the market share:

$$\frac{\partial \alpha}{\partial s} = \frac{q}{2\delta} > 0 \text{ and } \frac{\partial \alpha}{\partial \sigma} = p\frac{\partial \alpha}{\partial s} > 0$$

This types of subsidies, even if they are not directly linked with market shares, are then useful for the government as they increase the market coverage[14]. Moreover, we can derive the effect on per firm expenditures, PE, which are equal to $\alpha(\sigma pq + sq)$:

$$\frac{\partial PE}{\partial s} = \frac{\partial \alpha}{\partial s}(\sigma pq + sq) - \alpha\frac{\partial p}{\partial s}\left[\sigma(\eta - 1) + \eta s\frac{q}{p}\right] \geqslant 0 \tag{9}$$

$$\frac{\partial PE}{\partial \sigma} = p\left[\frac{\partial PE}{\partial s} + H\right] \geqslant 0 \tag{10}$$

where $H = \frac{\alpha}{\eta}\frac{\partial p}{\partial s}\left(\sigma(\eta - 1)q + \eta s\frac{q}{p}\right)$ which is negative, nevertheless $\frac{\partial PE}{\partial \sigma}$ is positive as H is small, in absolute value, in comparison to $\frac{\partial PE}{\partial s}$. Finally we have that

$$\frac{\frac{\partial \alpha}{\partial s}}{\frac{\partial PE}{\partial s}} \leqslant \frac{\frac{\partial \alpha}{\partial \sigma}}{\frac{\partial PE}{\partial \sigma}} \tag{11}$$

The (marginal) increase of coverage per unit of public expenditure is greater with the use of the *ad valorem* subsidy than with the use of the specific one. hence, from this order relation we can write the following proposition.

Proposition 1 *In order to increase the market coverage, it is less costly for the government to use an ad valorem subsidy than a per unit one.*

[14] Those marginal effects of subsidies are true for negative values (taxation). Subsidies are then the right tools to increase market shares.

By increasing the subsidy, whatever the type, the government reduces the marginal cost of firms. This creates an incentive for firms to decrease the per unit price. But, by doing so, firms could also increase their market share and/or the subscription fee. But an *ad valorem* subsidy has two other effects in comparison to the unit one. First, it reduces the per consumer fixed cost and, second, decreases the value of the subscription fees. This appears clearly by rewriting the per consumer profit as:

$$\pi_i = (1+\sigma)\left[\frac{A_i}{1+\sigma} + p_i q(p_i) - \frac{c-s}{1+\sigma} q(p_i) - \frac{f}{1+\sigma}\right]$$

Then, in a sense, with an *ad valorem* subsidy the firm has more incentives to increase its market share.

Kowalczck and Skeath (1994) shows that, in the context of international trade and with linear prices, the tariffs should be specific subsidies when the good is produced by foreign monopolies. Moreover, the results of Suits and Musgrave (1953) with a monopoly or of Delipalla and Keen (1992) in a Cournot oligopoly reverse[15] when we consider subsidization rather than taxation: for a given price the total amount of subsidies is less important with a per unit one. Then, in those cases, the government should use the specific subsidy. Here, we do not reach this conclusion but the opposite. This is an other illustration of the fact that the choice of the instrument also depends on the objective of the government[16].

5 Subsidization of the price versus subsidization of the subscription fee

The government has no incentive to use the specific subsidy compare to the *ad valorem* one on the usage fee. Then, in this section, we try to compare the subsidy on the subscription fee (we keep the per consumer subsidy equal to zero). The market share of a firm is in this case from relation (5):

$$\alpha = \frac{1}{2\delta(1+\Sigma)}\left[(1+\Sigma)(v(p)+\overline{u}) + (1+\sigma)pq - cq - f\right] \tag{12}$$

Using this relation (12) and relation (7), we can deduce the marginal effect of σ on the market share:

$$\frac{\partial \alpha}{\partial \sigma} = \frac{pq}{2\delta(1+\Sigma)} > 0.$$

Doing the same for the subsidy on the subscription fee, Σ, we obtain

$$\frac{\partial \alpha}{\partial \Sigma} = \frac{1}{pq(1+\Sigma)}\frac{\partial \alpha}{\partial \sigma}\left[f - \frac{(\sigma-\Sigma)}{(\eta-1)\sigma+\Sigma+\eta} q\right] \tag{13}$$

which could be positive or negative. This comes from the fact that one way to receive more subsidies is to increase the subscription fee, which might lead to a decrease in the market coverage. This effect does not appear for the subsidy on the usage fee. Indeed, in order to receive more subsidies, the firm has to decrease the price. Hence, it appears clearly that for all the value of (σ, Σ) such that $\frac{\partial \alpha}{\partial \Sigma} < 0$ the instrument to use is the *ad valorem* subsidy on the usage fee.

But, when $\frac{\partial \alpha}{\partial \Sigma} > 0$, it is not tractable to have a clear ranking of the two instruments. Nevertheless, we could write the following proposition.

[15] See Boldron and Hariton (2001).

[16] See Pirttilä (1997) or Keen (1998).

Proposition 2 *In order to increase the market coverage, it is less costly for the government to use the subsidy on the usage fee than the one on the subscription fee:*

- *if it starts from the 'no intervention' point,* $(\sigma, \Sigma) = (0, 0)$,
- *or if there is no fixed cost per consumer.*

The proof is detailed in the appendix. The first part of the proposition gives us the feeling that if a small subsidy is needed then it is better to use the *ad valorem* subsidy on the usage fee. The second part of the proposition is quite intuitive. Firms choose A and p, which determine α in such a way that the more the fixed cost f is small the more the market share is important. But if there is no fixed cost, the only way to increase the market share is to generate a larger consumer surplus, which means a drop in the price. Then, the better instrument is the usage fee subsidy because it is directly linked with the value of the sale[17].

6 Concluding remarks

The agreement between the French government and the GSM networks has the following features: a reduction of the VAT on all networks revenues (in particular on the fixed fees and on the per minute of communication price) and a subsidization of the relay antennas. This agreement is quite consistent with our results as the reduction of the VAT rate is an ad valorem subsidization. Moreover, by building some antennas, the government reduces indirectly a part of the consumer fixed cost.

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[17] Keen (1998) sums up this general principle by the following sentence: "The best instrument for correcting behaviour deemed inappropiate is the one that affects that behaviour the most directly and exclusively".

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Appendix

Proof of proposition 2. Consider the marginal effect on the market share of a marginal change of Σ which is from relation (13):

$$\frac{\partial \alpha}{\partial \Sigma} = \frac{1}{pq(1+\Sigma)} \frac{\partial \alpha}{\partial \sigma} \left[f - \frac{(\sigma - \Sigma)}{(\eta - 1)\sigma + \Sigma + \eta} q \right]$$

is equal to in $(\sigma, \Sigma) = (0, 0)$:

$$\frac{\partial \alpha}{\partial \Sigma} = \frac{\partial \alpha}{\partial \sigma} \frac{f}{pq}.$$

Moreover, we have the marginal effect on public expenditures, $PE_i = \alpha_i(\Sigma A + \sigma pq)$, which are:

$$\begin{aligned} \frac{\partial PE}{\partial \sigma} &= \frac{\partial \alpha}{\partial \sigma}(\Sigma A + \sigma pq) + \alpha pq - \alpha(\eta - 1) pq \frac{\partial p}{\partial \sigma} + \alpha \Sigma \frac{\partial A}{\partial \sigma} \\ \text{which gives in } (0,0) \quad &: \quad \frac{\partial PE}{\partial \sigma} = \alpha pq \end{aligned}$$

and

$$\begin{aligned} \frac{\partial PE}{\partial \Sigma} &= \frac{\partial \alpha}{\partial \Sigma}(\Sigma A + \sigma pq) + \alpha A - \alpha(\eta - 1) \sigma q \frac{\partial p}{\partial \Sigma} + \alpha \Sigma \frac{\partial A}{\partial \Sigma} \\ \text{which gives in } (0,0) \quad &: \quad \frac{\partial PE}{\partial \Sigma} = \alpha A \end{aligned}$$

Then for $(\sigma, \Sigma) = (0, 0)$ we have

$$\frac{\frac{\partial \alpha}{\partial \Sigma}}{\frac{\partial PE}{\partial \Sigma}} = \frac{f}{A} \frac{1}{2\delta\alpha} = \frac{f}{A} \frac{\frac{\partial \alpha}{\partial \sigma}}{\frac{\partial PE}{\partial \sigma}}.$$

But in $(0,0)$ the price is equal to the marginal cost (from relation (3)), then we must have $A > f$ in order to have a positive profit. Finally we get that

$$\frac{\frac{\partial \alpha}{\partial \Sigma}}{\frac{\partial PE}{\partial \Sigma}} < \frac{\frac{\partial \alpha}{\partial \sigma}}{\frac{\partial PE}{\partial \sigma}}$$

which demonstrates the first part of the proposition.

If $f = 0$ then

$$\frac{\partial \alpha}{\partial \Sigma} = -\frac{1}{pq(1+\Sigma)} \frac{\partial \alpha}{\partial \sigma} \left[\frac{(\sigma - \Sigma)}{(\eta - 1)\sigma + \Sigma + \eta} q \right]$$

then when $\sigma > \Sigma$ then it is better to use the subsidy on the usage fee. But in $(0,0)$ the government will use σ, then it always will. ■