E C O N O M I C S   B U L L E T I N

Scaling power laws in the Sao Paulo Stock Exchange

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Abstract

The scaling of the probability distribution of the Sao Paulo Stock Exchange index is shown to be described by a Levy stable stochastic process for the modal region of the distribution. Data refer to daily records for the 30−year period 1968−1998. The truncated Levy process is characterized by a scaling index of 1.66. Scaling power laws are also shown to be present in the mean and standard deviation of the series as the time horizon is increased. A power law is also found for the autocorrelation time of the natural logs of the index series. The deviations from the line that best fits the natural logs of the series are also found to be short range autocorrelated and to follow an exponential decay.

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1. Introduction

Recently a growing interest has been devoted to the dynamical properties of highly "complex", open systems in which many subunits interact nonlinearly in the presence of feedback and stable governing rules. Such systems tend to exhibit scaling behavior governed by power laws. Scaling has been observed in a wide range of systems, from biological to economic. The attempt by physicists to map out the statistical properties of financial markets considered as complex systems has sometimes been dubbed "econophysics" (e.g. Mantegna and Stanley 2000, and Bouchard and Potters 2000). A benchmark analysis of stock markets has been an empirical assessment of the Standard & Poor 500 index (Mantegna and Stanley 1995). Here we consider data for an emerging market: the Sao Paulo Stock Exchange (Bovespa) index. In particular, we show that the scaling of the probability density function of the natural log returns of the series can be described by a Lévy stable process for the modal region of the distribution. That the stochastic properties of emerging markets should match those of advanced, well established markets (such as the S&P 500) is far from being obvious. Emerging markets are subject to severe macroeconomic instability, such as chronic inflation. Despite that, this note suggests the presence of a similar, universal process at work in stock markets, which is associated with scaling power laws.

Lo and MacKinlay (1988), for instance, rigorously demonstrate that a Gaussian random walk for US stock returns over weekly data is strongly rejected for a variety of aggregate returns indexes and size-sorted portfolios for the entire sample period (1962–1985) and for all subperiods. Here Gaussian normality is dismissed by taking into account the skewness and kurtosis of the time lags of the series. Hurst exponents (Hurst 1951, and Feder 1988) together with relative Lempel-Ziv complexity indexes (Lempel and Ziv 1976, and Kaspar and Schuster 1987) are reckoned to confirm the series is a complex, non-Gaussian random walk. Interestingly, as time horizons are increased, the evolution of mean and standard deviation is shown to be governed by non-normal scale-free power laws. This is a novel result in this type of literature. Also, the autocorrelation time of the natural logs of the Bovespa index is shown to be governed by a power law. The autocorrelation of the deviations from the line that best fits the natural logs of such a series is shown to follow an exponential decay; it is thus short range autocorrelated.

The structure of the paper is as follows. Section 2 presents the data, analysis is undertaken in Section 3, and Section 4 concludes.

2. Data

Bovespa is acronym for *Bolsa de Valores de São Paulo*, which is Portuguese for Sao Paulo Stock Exchange. Bovespa is one of the two largest stock exchange in Brazil, regulated by a securities commission; it accounts for the bulk (nearly 90 per cent) of trading in securities, including futures and options, although there are also exchanges in seven other Brazilian cities. The Bovespa index was launched in 2 January 1968 and is currently the most widely quoted share index for Brazil. Further details on the index can be found at http://www.bovespa.com.br/indexi.htm.

Three decades of daily data were taken. The sample covers the time period ranging from 2 January 1968 to 30 December 1998 and constitutes a series of 7612 datapoints. As standard, "holes" from weekends and holidays were ignored and analysis concentrated on trading days. The data set employed is the index for the closing of the market in US dollar terms, which is available online at http://www.bvpr.com.br/Ibov_us$.exe. A reason why we took the series in dollar terms rather than in the Brazilian currency is that the series expressed in local currency suffers from the distortions provoked by high inflation.

Some analysis will concentrate on four distinct subsets of data related to the performance of the Brazilian economy. These are (1) "the economic miracle" (1968–1973), which was a period of explosive Gross Domestic Product growth and low inflation; (2) the period in between the two oil shocks (1974–1979), when GDP growth slowed down; (3) "the lost decade" (1980–1989), when there was a downturn in GDP and inflation exacerbated; and (4) the nineties (1990–1998), which experienced extremely high inflation during the first part followed by a stabilization of prices since 1995. It is worth noticing that such macroeconomic instability is typical of emerging markets; it is not arguably present in well-developed markets, where upheavals are not the norm.

3. Analysis

Natural log returns

$$Z(t) \equiv \ln Y(t+\Delta t) - \ln Y(t) \quad (1)$$

were chosen as our stochastic variable, where Y is a Bovespa index closing day value in dollar terms, t is time (i.e. a trading day), and Δt is initially one trading day. The probability density function (pdf) of the natural log returns is not bell-shaped; there is a slightly longer tail to the left. A negatively skew is seen (skewness of –0.314). The pdf also show a high degree of peakedness and fat tails (kurtosis of 8.389) relative to a normal distribution (which has kurtosis of 3). (Figure 6 displays the $\Delta t = 1$ pdf together with a Lévy pdf to be presented below). In short, the pdf is almost symmetric and highly leptokurtic. Thus there is a clear departure from Gaussian normality.

A Hurst exponent (Hurst 1951, and Feder 1988) of 0.548 was reckoned, which suggests the series is a random walk whether the data are normally distributed or not. (This paper will show the series seems to be modeled by a Lévy random walk.) The Hurst exponent is related to how the value of a stochastic variable moves away from its initial position. A Hurst exponent of 0.5 gives an indication of a random walk even if a series is not normally distributed, whereas a value of between 0.5 and 1.0 suggests a persistent, trend-reinforcing series. An LZ (Lempel-Ziv) complexity index relative to Gaussian white noise (Lempel and Ziv 1976) of 1.010 was found using the algorithm of Kaspar and Schuster (1987), a figure which indicates maximal complexity (randomness). This is consistent with the value for the Hurst exponent. (An LZ index relative to Gaussian white noise of 0 is, by contrast, associated with perfect predictability). The series is thus highly complex.

The statistical properties of the natural log returns were also evaluated for time lags $\Delta t \geq 2$. We took Δt values of 2, 3, 5, 10, 20, 40, 60, 120, and 240, which correspond respectively to 2 and 3 trading days, a week, a fortnight, a month, two months, a quarter, a semester, and a year. As Δt is increased, the mean grows and the mean divided by standard deviation ("coefficient of variation") drops. Remarkably, these changes are governed by power laws. Figure 1 shows a straight line of slope 0.990741 in a log-log plot of mean against Δt within the time window of $1 \leq \Delta t \leq 1000$. However, the mean of $Z(t)$ can be expressed as $\omega(\Delta t)^{\beta}$ with estimated values for ω and β of 0.000550899 and 0.990741 respectively. The effect of ω on the mean of $Z(t)$ is larger the greater Δt is. As will be seen below, a Lévy functional form seems to describe well the modal region of the distributions of the Δt considered. The scaling index of the Lévy distribution is the negative inverse of the slope. Thus it is implied a scaling index of –1.0093 for the mean. Figure 2 displays the power law for the coefficient of variation of slope –0.4588 over two orders of magnitude

($1 \leq \Delta t \leq 100$), which implies a scaling index of 2.18. These scale-free power laws are consistent with non-Gaussian scaling (slope ≠ –0.5).

Figure 3 shows a semilog plot of the pdfs of $W = Z - \omega(\Delta t)^{\beta}$ observed at time intervals Δt. As Δt is increased, a spreading of the pdfs characteristic of any random walk is observed. As expected for a random process, the pdfs are roughly symmetrical (actually, they are slightly negatively skewed). The pdfs are also leptokurtic.

To characterize the functional form of the pdfs, the usual approach is to investigate the tails. Since, in our experiment, larger values of Δt also imply a reduced number of datapoints, methods that investigate the tails make the determination of the parameters characterizing a pdf difficult. To circumvent such a restriction, taking the "probability of return to the origin"

$$\mathrm{P}(Z = 0) \tag{2}$$

as a function of Δt has been a more fruitful approach (Mantegna and Stanley 1995, 2000). This method allows one to investigate the point of each probability distribution that is least affected by the noise coming from the finiteness of the experimental set of data. Here we departure from Mantegna and Stanley. Since the peak of a distribution is not exactly located at $Z = 0$ to all Δt, we take $\mathrm{P}(Z = \omega(\Delta t)^{\beta})$ to represent the "probability of return to the origin" instead. As seen, the mean grows and follows a power law as Δt is increased (ranging from 0.000229116 for $\Delta t = 1$ to 0.1955119 for $\Delta t = 1000$). Mantegna and Stanley do not let us know whether they have checked to see if the means are fixed at zero for all Δt in their study. Neglecting this, however, does not change results a great deal. But our approach is more rigorous and (in this sense) generalizes that of Mantegna and Stanley. Figure 4 presents a log-log plot of P(0) against Δt. To experimentally find P(0) we define a small threshold value c such that $\mathrm{P}(0) \approx \mathrm{P}(-c \leq W \leq c)$. A power law emerges within the time window of $1 \leq \Delta t \leq 100$. The data are fitted well by a straight line of slope –0.601207. Such a power law differs from the expected scaling behavior of a normal distribution, which has (as observed) slope of –0.5. Indeed our experimental finding seems to agree with a theoretical truncated Lévy flight (Mantegna and Stanley 1994). If the central region of the distributions in Figure 3 is well described by a Lévy stable pdf of the form

$$\mathrm{P}(W) \equiv \mathrm{L}_{\alpha}(W, \Delta t) \equiv \frac{1}{\pi}\int_{0}^{\infty} \exp(-\gamma \Delta t q^{\alpha})\cos(qW)\mathrm{d}q \tag{3}$$

of scaling index α and scale factor γ at $\Delta t = 1$, then the probability of return to the origin is given by

$$\mathrm{P}(0) = \frac{\Gamma(1/\alpha)}{\pi\alpha(\gamma\Delta t)^{1/\alpha}}, \tag{4}$$

where Γ stands for the gamma function. By plugging the slope value of –0.601207 found in Figure 4, the scaling index α =1.66332 obtains. The scale factor can also be reckoned. This is found to be γ =0.000927673.

The scaling seems to extend over the entire probability distribution as well as $W = 0$. Indeed Lévy stable symmetrical pdfs rescale under the transformations

$$W_{\mathrm{s}} \equiv \frac{W}{(\Delta t)^{1/\alpha}}, \tag{5}$$

and

$$\mathrm{L}_{\alpha}(W_{\mathrm{s}}, 1) \equiv \frac{\mathrm{L}_{\alpha}(W, \Delta t)}{(\Delta t)^{-1/\alpha}}. \tag{6}$$

The existence of the scaling law in Figure 4 thus justifies a scaled plot of the pdfs displayed in Figure 3. By using equations (5) and (6) with the scaling index α=1.66332, all the data collapse onto the $\Delta t = 1$ distribution (Figure 5). A Lévy pdf thus seems to describe well the dynamics of the probability distribution P(W) of the random process within the time window of two orders of magnitude (Figure 4).

Figure 6 shows a comparison of the experimental $\Delta t = 1$ pdf with a theoretical Lévy pdf (solid line) of α=1.66332 and γ=0.000927673 (which result from our experimental analysis) obtained from P(0) in equation (4) measured when $\Delta t = 1$. The data seem to agree well with the theoretical Lévy for the central region of the pdf. However, when $|W_s| \geq 0.10$ the data in the tails are lower than the theoretical Lévy. Variances seem to be finite. This means a truncated Lévy describes the data better than a standard Lévy, in which variances are infinite. In other words, scaling must break down for time intervals longer than two orders of magnitude (Figure 4).

We checked the kurtosis behavior for increasing time lags and found that the leptokurtosis dies out as Δt is increased. The Lévy process degenerates to a Gaussian one somewhere within the interval 20< Δt <40, when the kurtosis approaches the value of 3. Thus the self-similar Lévy scaling present in the data is not infinite.

A limitation of our findings is the assumption of i.i.d. increments underlying the theoretical truncated Lévy, which means the scaling index α and scale factor γ alike are assumed to be time independent. Figure 7 shows that α determined from the probability of return to the origin is approximately constant over time (subsets in accordance with those presented in Section 2 are drawn explicitly). However, Figure 8 shows that the same cannot be said as far as γ is concerned. Relatively, parameter γ experiences stronger fluctuations. In particular, γ is unstable for the first and third subsets of data, while it is apparently stable for the second and forth subsets. (In Figure 8, γ is determined by using the value of α presented in Figure 7 and the probability of return to the origin.) Parameter γ can be seen as a measure of volatility of this process (Mantegna and Stanley 2000). Thus volatility is time dependent and our data cannot be modeled in terms of a stochastic process with i.i.d. increments. Nevertheless, the truncated Lévy pdf still describes well the asymptotic pdfs of our data at different time horizons and their scaling properties.

Our study is in line with previous work regarding the Bovespa index (e.g. Miranda and Riera 2001). By using a shorter series (from 1986 to 2000) in local currency terms (rather than in dollar terms), a scaling index $\alpha \approx$1.6–1.7 is found in the study of Miranda and Riera, where the Lévy scaling breaks down at the time lag of 20 trading days. This is in good agreement with our results, in which α=1.66 and the Lévy scaling breaks down somewhere within the interval 20< Δt <40. Miranda and Riera's analysis departures from Mantegna and Stanley's (1995) benchmark study. Our approach is thus less idiosyncratic in that we replicate the analysis of Mantegna and Stanley. Comparisons between our own study and that of Mantegna and Stanley can be more straightforward. The scaling index of this emerging market is not quite different from the scaling index found by them for the S&P 500, in which α=1.4 (Mantegna and Stanley 1995). This is suggestive of the presence of a similar, universal process at work in stock markets.

Finally, we also studied the autocorrelation of the natural logs of the index. Figure 9 shows a straight line for the log-log plot of autocorrelation time τ against Δt for three orders of magnitude (fitting line of 0.997). So there is a power law in the autocorrelation time. Further, we took the series of deviations from the line that best fits the natural logs of the index. Figure 10 shows a straight line in a semi-log plot of autocorrelations against time within the time window of 180 trading days (fitting line to data of 0.9966). The process seems to follow an exponential decay and is thus short range autocorrelated (see e.g. Mantegna and Stanley 2000, chapter 6).

4. Concluding remarks

This paper presents an empirical study of calibrating a truncated Lévy process to model the historic log returns of the Sao Paulo Stock Exchange (Bovespa) index in US$ denomination over the 30-year period 1968–1998. We employ the tools developed in statistical physics and presented in the benchmark analysis of Mantegna and Stanley (1995) of the Standard & Poor 500 index. The scaling of the probability density function of the log returns of the series may well be modeled by a Lévy stable process for the modal region of the distribution for a holding period not exceeding 20 trading days. The scaling index of 1.66 found in this emerging market is not quite different from that of 1.4 found by Mantegna and Stanley in the S&P 500. This is suggestive of the presence of a similar, universal process at work in stock markets. The statistical properties of advanced stock markets seem to generalize to emerging markets, a result which is far from being obvious.

A contribution of this paper is to clearly indicate that successive daily log returns of the Bovespa index for sufficiently small holding periods cannot be described by a standard random walk driven by uncorrelated Gaussian distributions. A truncated Lévy random walk, rather than a Gaussian random walk, seems to be present in Wall Street as well as in other stock markets. So called technical traders can possibly take advantage of the patterns underlying the existence of self-similar non-Gaussian scaling power laws in these markets. But this can be done only temporarily because scaling must break down as time goes by.

Indeed the finite variance found in studies such as this one implies that scaling is approximate and valid only for a finite time interval. What is more, although the assumption of a constant scaling index can be considered reasonable, the truncated Lévy seems to fail to describe the time dependent volatility observed in real world data.

This study also shows results not previously found in this type of literature. As time horizons are increased, the evolution of mean and standard deviation of the time lags of the series of log returns is found to be governed by non-normal scale-free power laws. There is a power law, too, for the autocorrelation time of the logs of the Bovespa index. The deviations from the line that best fits the logs of such a series is also found to be short range autocorrelated and to follow an exponential decay.

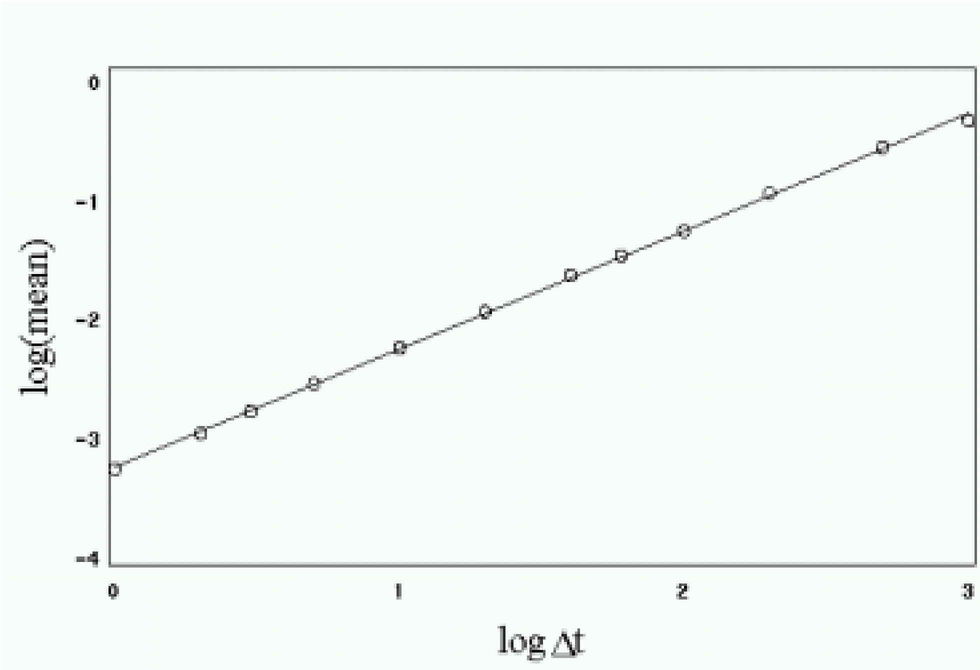

Figure 1. As the time lags are increased, a non-Gaussian power law emerges for the means within the time window of $1 \le \Delta t \le 1000$. In a log-log plot of mean against Δt a straight line of slope 0.990741 is reckoned. Given that a Lévy functional form seems to describe well the modal region of the distributions of the Δt considered, a scaling index of -1.0093 is implied.

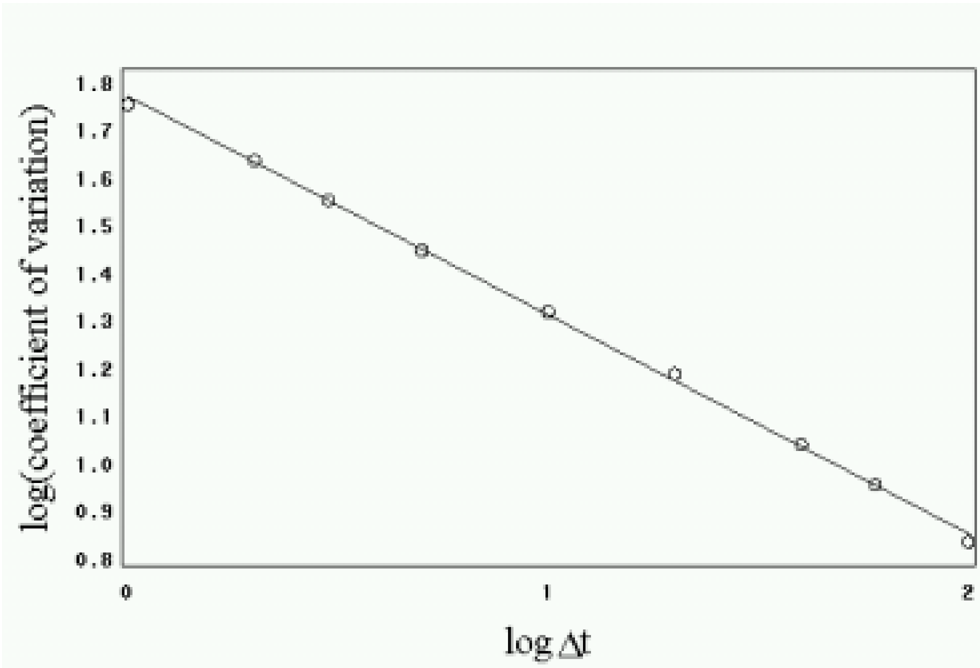

Figure 2. Power law for the coefficients of variation of slope –0.4588 over two orders of magnitude ($1 \le \Delta t \le 100$); a scaling index of 2.18 is implied.

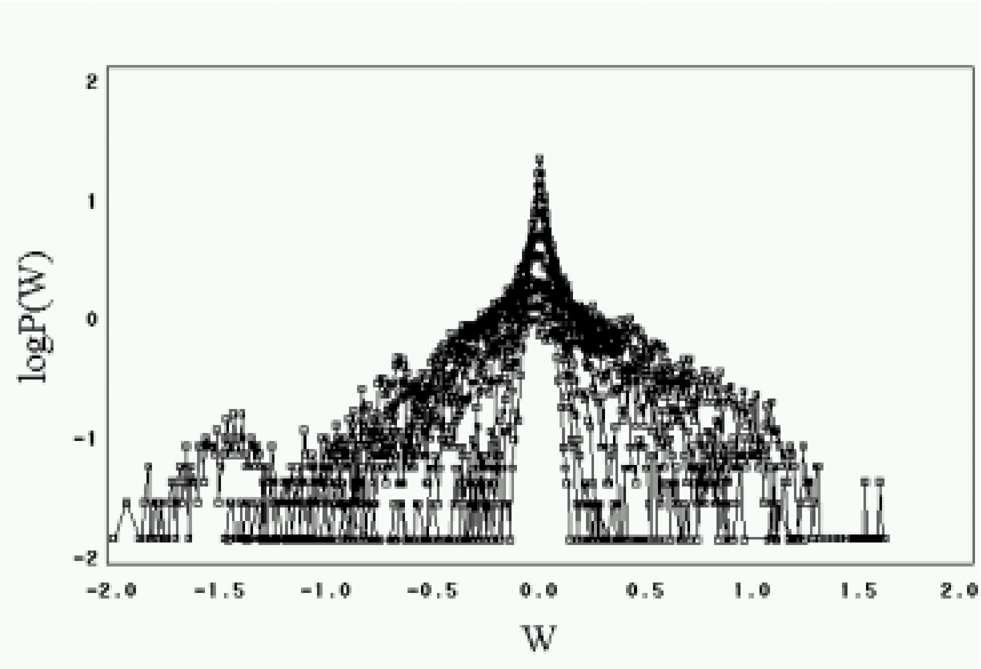

Figure 3. Probability density functions P(*W*) of the natural log returns of the Bovespa index *W*(*t*) observed at time intervals Δt, which range from 1 to 240 trading days. As Δt is increased, a spreading of the probability distribution characteristic of any random walk is observed

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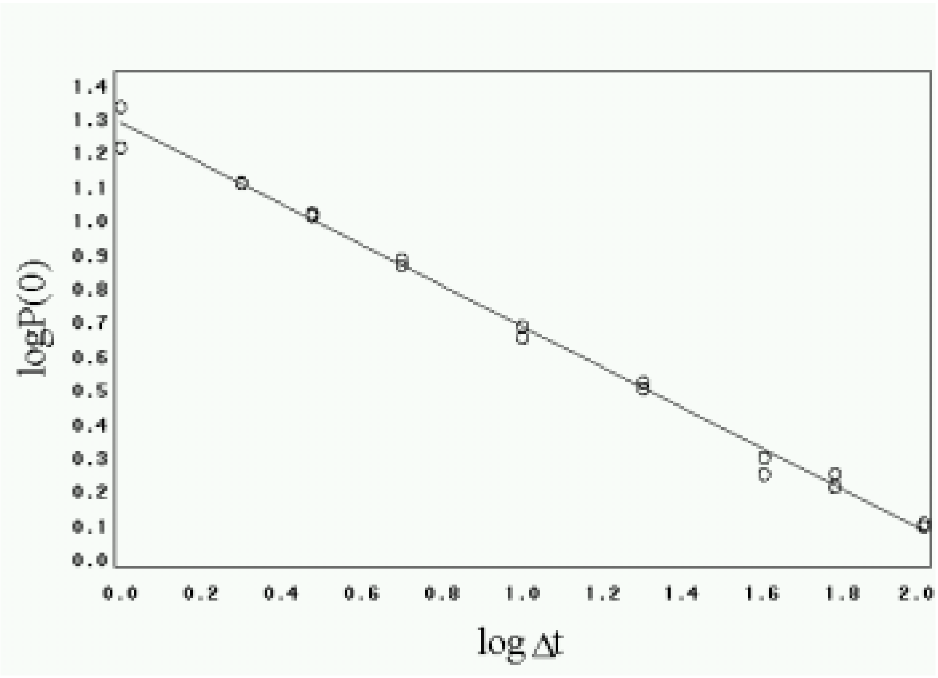

Figure 4. Log-log plot of the probability of return to the origin P(0) against the time lag Δt. A power law emerges within the time window of $1 \le \Delta t \le 100$. The data are fitted well by a straight line of slope –0.601207. This non-Gaussian scaling seems to be modeled by a Lévy process of scaling index α =1.66.

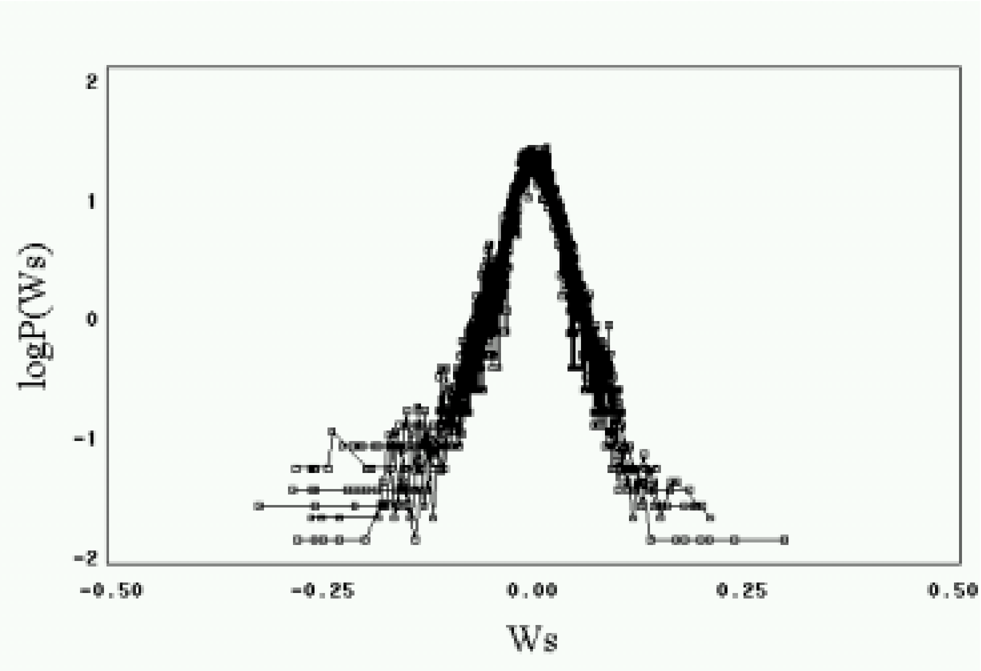

Figure 5. The same probability density functions as in Figure 3, but now plotted in scaled units P(W_s). Given the scaling index α=1.66, all the data collapse onto the $\Delta t = 1$ distribution. A Lévy pdf thus seems to describe well the dynamics of the random process within the time window of two orders of magnitude.

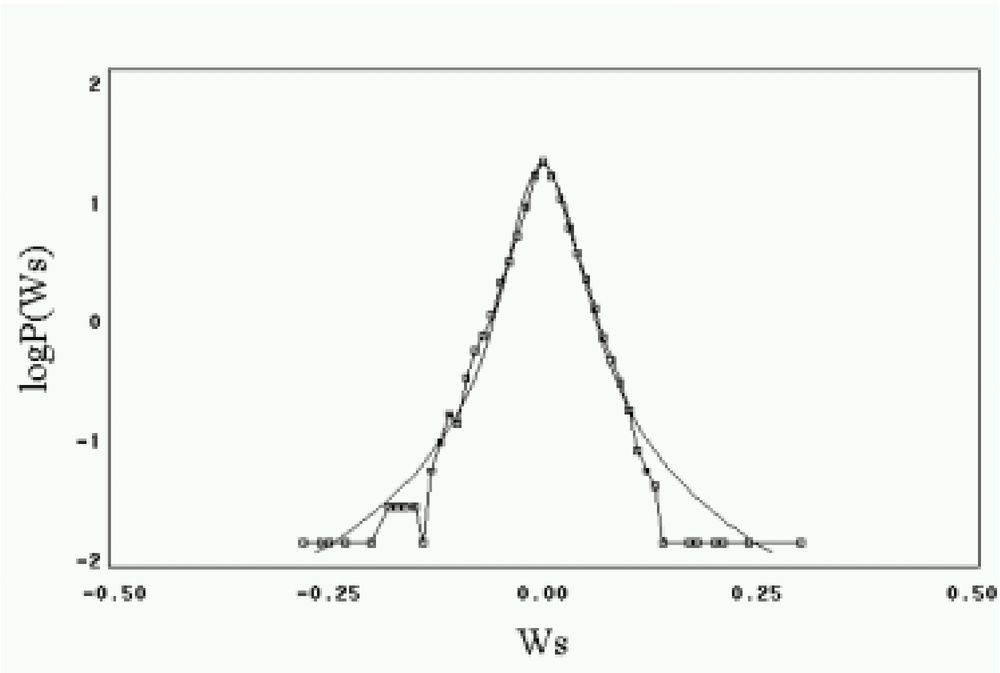

Figure 6. Comparison of the $\Delta t = 1$ distribution with a theoretical Lévy pdf (solid line) of α=1.66 and γ=0.000927673, which were obtained from our experimental analysis of P(0) when $\Delta t = 1$. The data agree well with the theoretical Lévy for the modal region of the pdf. But for $|W_s| \geq 0.10$ the data in the tails are lower than the theoretical Lévy, which means variances are finite.

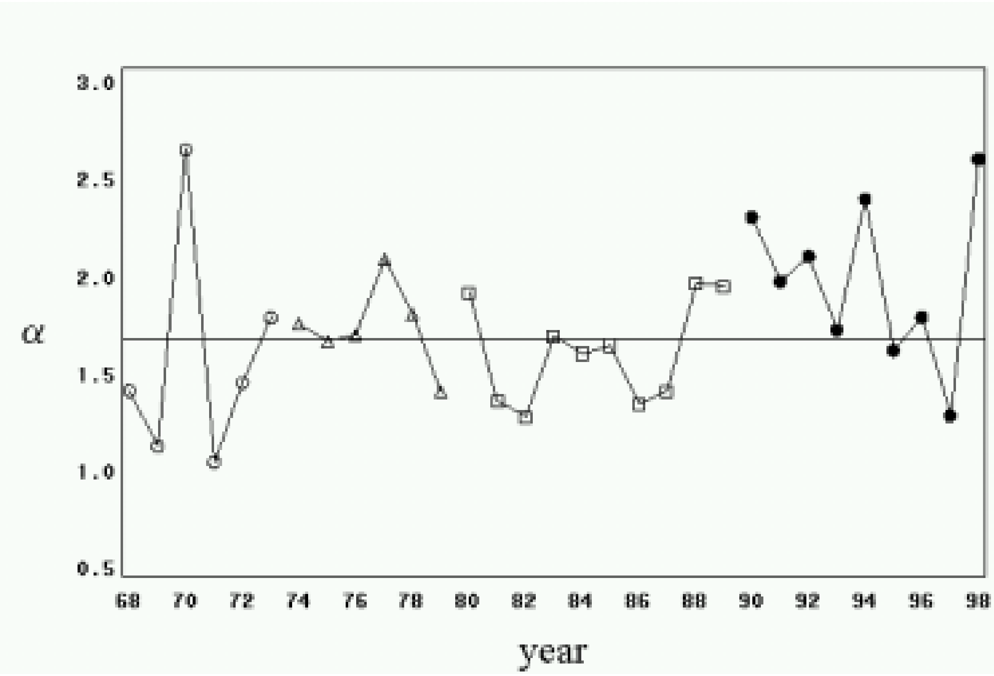

Figure 7. Relative time independence of the scaling index α determined from the probability of return to the origin. Parameter α seems to be approximately constant.

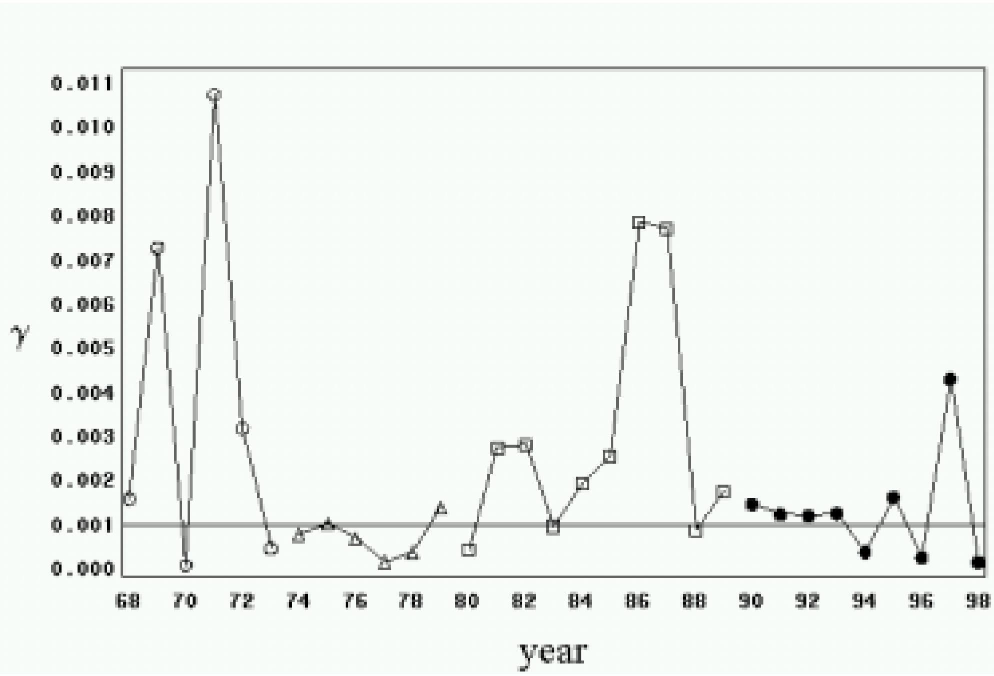

Figure 8. Time dependence of the scale factor γ determined by using the value of α presented in Figure 7 and the probability of return to the origin. Relative to α, parameter γ experiences stronger fluctuations. In particular, γ is unstable for the first and third subsets of data, while it is apparently stable for the second and forth subsets.

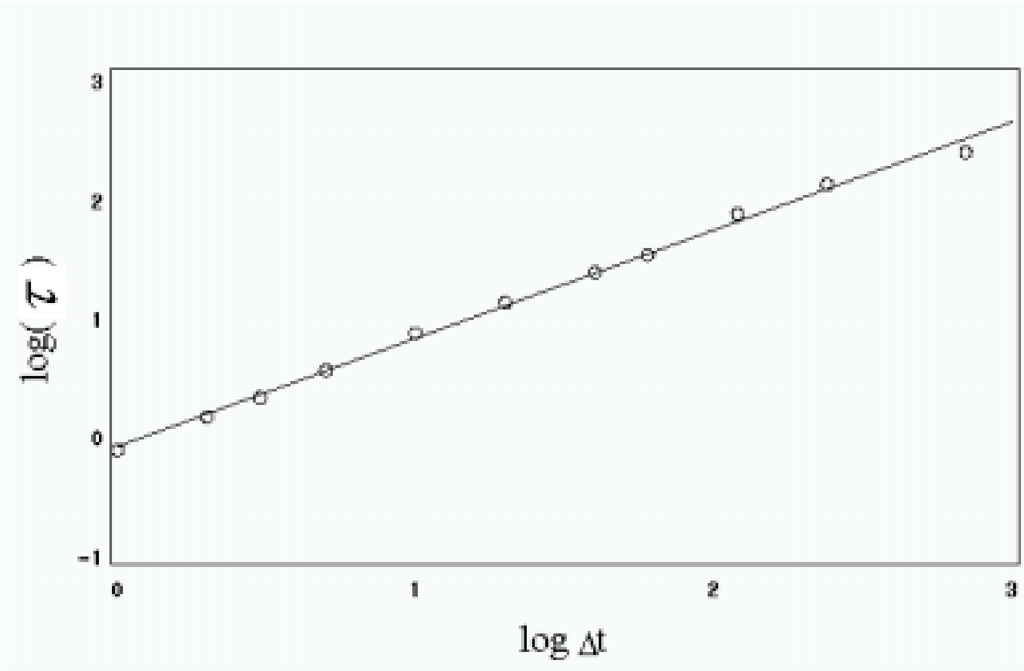

Figure 9. Log-log plot of autocorrelation time τ against Δt. A power law is present for three orders of magnitude (fitting line to data of 0.997).

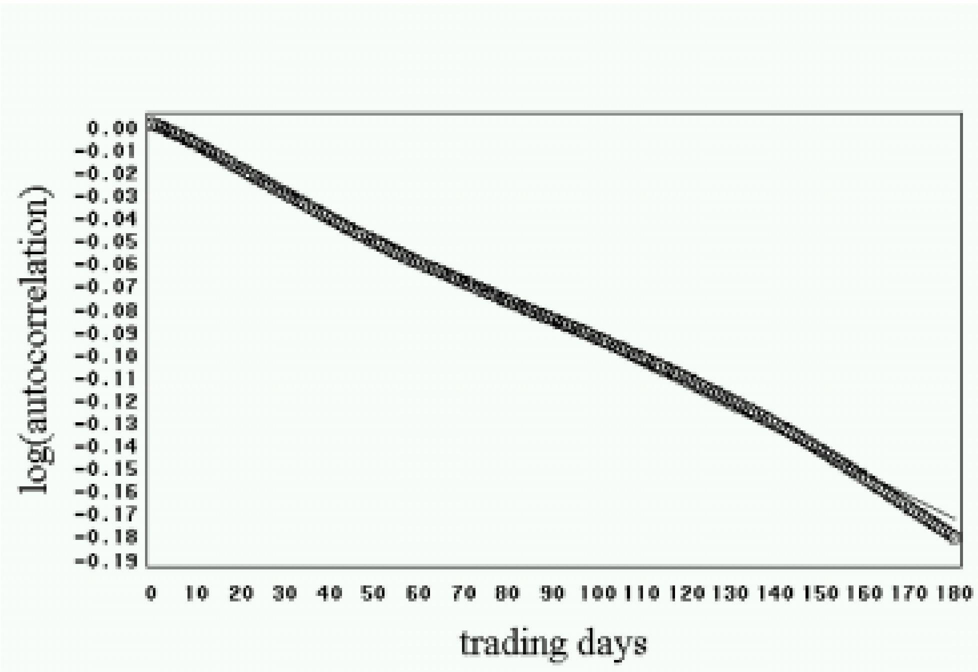

Figure 10. Log of the autocorrelations of the deviations from the line that best fits the natural logs of the Bovespa index against trading days. A straight line appears in this semi-log plot within the time window of 180 trading days (fitting line of 0.9966). The process follows an exponential decay; thus, it is short range autocorrelated.

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