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International best−shot public goods and foreign aid

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Abstract

This note corrects an analytical mistake of Jayaraman and Kanbur (1999) in their analysis of a Stackelberg game of the voluntary contribution to an international best−shot public good by a donor and a recipient. It shows that, depending on players' preferences, the donor may choose not to contribute but make a positive direct income transfer to the recipient who will then contribute to the best−shot public good.

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1. Introduction

Jayaraman and Kanbur (1999) study the interaction between international public goods and direct foreign aid transfers. They consider a Stackelberg game in which there are two players, the donor country and the recipient country, each deriving utility from the consumption of a private good and an international public good. The donor is the leader who moves first and divides its endowed income into private consumption, contribution to the public good, and direct transfer to the recipient. The recipient is the follower who moves second and allocates his total income, the sum of its endowed income and the direct income transfer from the donor, between private consumption and contribution to the public good.

When the technology of public good production, which transfers individual contributions to the aggregate public good level, is of the "max" or "best-shot" type, such that the level of public good is determined by the larger contribution of the two players (e.g., high-tech research and development), Jayaraman and Kanbur state in their Proposition 3 that, "In a Stackelberg framework with a best-shot technology and identical agents, the Stackelberg equilibrium will be characterized by zero direct income transfers". It is not clear from this statement whether the two players or agents have the same income level, although it is quite evident from the context of their paper that the donor is richer than the recipient.

Unfortunately, the statement is wrong, as this note will show. Indeed, depending on players' preferences, the donor may in equilibrium make a positive direct income transfer to the recipient, even if the former is no richer than the latter. This result can be most easily understood from the following extreme case. Suppose that both the donor and the recipient derive utility from only the public good. Given the public good is of best-shot type, it is optimal for the donor, regardless of its income level, not to contribute but transfer all its income to the recipient who will then devote the sum of the transfer and it own income to the provision of the public good.

In Section 2, I analyze the Stackelberg game, as set by Jayaraman and Kanbur, of the voluntary contribution to best-shot public goods, and Section 3 summarizes the result.

2. The model

There are two players, player 1, the donor country, and player 2, the recipient country. The donor is the leader who moves first, and the recipient is the follower who moves second. Player i's, $i = 1, 2$, utility is given by

$$u_i = u(x_i, G), \tag{1}$$

where x_i is the consumption of a private good, and G is the consumption of an international public good. Both the private and public goods are normal goods. It is assumed that utility is continuous, increasing in both arguments and strictly concave. Player i's income is w_i. Jayaraman and Kanbur assume that $w_1 > w_2$, in light of the fact that the donor is usually a rich country while the recipient usually a poor country. In this note, I impose no restrictions on the relative magnitude of w_1 and w_2.

The donor moves first and divides its endowed income, w_1, into private consumption, $x_1 \geq 0$, contribution to the public good, $g_1 \geq 0$, and direct income transfers to the recipient, $t \geq 0$. So, the budget constraint for the donor is

$$x_1 + g_1 + t = w_1. \tag{2}$$

The recipient moves second and allocates its total income, the sum of its endowed income, w_2, and the transfers, t, received from the donor, into private consumption, $x_2 \geq 0$, and contribution to the public good, $g_2 \geq 0$. So, the budget constraint for the recipient is

$$x_2 + g_2 = w_2 + t. \tag{3}$$

The technology of public good production is of "max" or "best-shot" type in that the aggregate level of the public good is given by the larger of the two players' contributions. Thus,

$$G = \max\{g_1, g_2\}. \tag{4}$$

The fundamental research into the nature and evolution of infectious diseases obeys a max or best-shot technology.

Jayaraman and Kanbur (1999) state in their Proposition 3 that the Stackelberg equilibrium is characterized by $t = 0$. Unfortunately, this statement is incorrect, as I will show below.

The Stackelberg game can be solved by backward deduction. The recipient plays the best response to the donor's transfer and contribution levels, and the donor chooses transfer and contribution levels to maximize its own utility, taking into account the recipient's best response schedule. To solve for the equilibrium, the following observations are useful. First, in equilibrium, either $g_1 > 0$ and $g_2 = 0$, or $g_1 = 0$ and $g_2 > 0$. In other words, only one player makes contribution to the public good. This observation derives directly from the nature of the max technology of public good. The player who contributes less adds nothing to the level of the best-shot public good but reduces its own private consumption and hence its utility.

Second, if $w_2 \geq w_1$, then it must be the case that $g_1 = 0$ and $g_2 > 0$. The reason is simple. If $g_1 > 0$, then the recipient's best response will be one of the following, $g_2 = 0$ or $g_2 > g_1 > 0$ (recall that the recipient and donor have identical preferences and that both the private and public goods are normal goods). In the latter case, the donor's contribution is wasted. In the former, the donor can do better by not contributing, because the recipient will then contribute a larger amount than its own contribution. Therefore, when $w_2 \geq w_1$, the donor's problem is to allocate its income between private consumption and income transfers to the recipient. In the analysis below, I focus on the case in which the donor is richer than the recipient, $w_1 > w_2$.

Third, if $g_1 > 0$, then $t = 0$. If the donor chooses to contribute to the best-shot public good, it will make no direct income transfers to the recipient. Given the donor's contribution to the best-shot public good, any income transfer to the recipient from the donor reduces the donor's private consumption and hence its utility.

Equipped with the above observations, the analysis of equilibrium of the Stackelberg game is straightforward. If the donor is the contributor to the public good, it chooses g_1 to maximize $u(w_1 - g_1, g_1)$. Let the solution be $g_1^* = f(w_1)$. The assumption that both the private and public goods are normal goods implies that $0 < f'(w_1) < 1$.

If the donor does not contribute, but transfers $t \geq 0$ to the recipient, the recipient will choose g_2 to maximize $u(w_2 + t, g_2)$. The solution is $g_2^* = f(w_2 + t)$. Hence, $G = g_2^*$, and the donor's utility is $u_1 = u(w_1 - t, g_2^*)$. The donor chooses t to maximize its utility. Note that, if $du_1/dt|_{t=0} = - \partial u(w_1, f(w_2))/\partial x_1 + \partial u(w_1, f(w_2))/\partial G \, f'(w_2) > 0$, we must have $t > 0$. Furthermore, if u_1 is concave in t, then the optimal t* is given by $\partial u(w_1 - t^*, f(w_2 + t^*))/\partial x_1 = \partial u(w_1 - t^*, f(w_2 + t^*))/\partial G\, f'(w_2 + t^*)$. And the donor's utility is $u(w_1 - t^*, f(w_2 + t^*))$.

Clearly, if $u(w_1 - t^*, f(w_2 + t^*)) > u(w_1 - f(w_1), f(w_1))$, the donor will choose not to contribute but make a positive income transfer to the recipient who will then be the contributor. Note that, from the definition of t*, $u(w_1 - t^*, f(w_2 + t^*)) > u(w_1, f(w_2))$. Thus, sufficient conditions for the existence of a positive equilibrium income transfer are $u(w_1, f(w_2)) > u(w_1 - f(w_1), f(w_1))$ and $- \partial u(w_1, f(w_2))/\partial x_1 + \partial u(w_1, f(w_2))/\partial G\, f'(w_2) > 0$.

The result can be intuitively understood in a two-step thought experiment. Suppose first that income transfers from the donor to the recipient are prohibited. When a donor decides whether or not to contribute, it faces a tradeoff between private and public consumption. If it contributes, more of the public good is provided, as the donor is richer than the recipient. But the donor's private consumption is reduced by its contribution level. On the other hand, if the donor does not contribute, the recipient will contribute a smaller amount of the public good than that if the donor were to contribute. But the donor's private consumption will be its endowed income. Depending on the utility function, the donor may choose not to contribute. Moreover, when it is possible for the donor to make income transfers to the recipient, the donor in this case can never do worse, but may do better by making such transfers, if the marginal rate of substitution between public and private consumption is large enough.

I now provide an explicit example where it is optimal for the donor not to contribute but make positive income transfers to the recipient who will then be the contributor.

Example 1. Let $u_i = \gamma \ln x_i + (1 - \gamma)G$, $0 < \gamma < 1$, $i = 1, 2$. It is easy to show that, for this class of utility functions, if the donor contributes, then $x_1 = \gamma w_1$, and $G = g_1^* = (1 - \gamma)w_1$. And the donor's utility is $u_1 = \gamma \ln\gamma + (1 - \gamma)\ln(1 - \gamma) + \ln w_1$.

If the donor does not contribute but transfers $t \geq 0$ to the recipient, then $G = g_2^* = (1 - \gamma)(w_2 + t)$. And the donor's utility is $u_1(t) = \gamma\ln(w_1 - t) + (1 - \gamma)\ln[(1 - \gamma)(w_2 + t)]$. The donor chooses t to maximize $u_1(t)$. The first-order condition is $u_1'(t) = - \gamma/(w_1 - t) + (1 - \gamma)/(w_2 + t) = 0$. So, $t^* = (1 - \gamma)w_1 - \gamma w_2 > 0$ if and only if $w_1/w_2 > \gamma/(1 - \gamma)$. It is easy to verify that the second-order condition is also satisfied. The donor's utility can be shown to be $u_1(t^*) = \gamma\ln\gamma + 2(1 - \gamma)\ln(1 - \gamma) + \ln(w_1 + w_2)$. Clearly, $u_1(t^*) > u_1$ if and only if $\ln[(w_1 + w_2)/w_1] > - (1 - \gamma)\ln(1 - \gamma)$, or $w_1/w_2 < 1/(\chi - 1)$, where $\chi = 1/(1 - \gamma)^{1-\gamma} > 1$.

In conclusion, if $\gamma/(1 - \gamma) < w_1/w_2 < 1/(\chi - 1)$, then the donor will not contribute but make a positive income transfer to the recipient who will then contribute to the best-shot public good.

Note that when $\gamma \to 0$, $\chi \to 1$. Then, for all values of w_1 and w_2, it is optimal for the donor to make $t^* \to w_1$, i.e., all its endowed income to the recipient. This result is independent of the relative magnitude of w_1 and w_2. When both the donor and the recipient derive utility from only public consumption, all incomes should be devoted to the provision of public good. Given the order of move by the two players and the nature of the best-shot public good, the donor should clearly transfer all its income to the recipient.

3. Conclusion

This note points out the mistake in the analysis of the Stackelberg game by Jayaraman and Kanbur (1999) of the voluntary contribution to best-shot international public goods. It is assumed that both the donor and the recipient have identical preferences. The analysis can be easily extended to the case in which the two players have different preferences. The general conclusion is that, depending on the donor and recipient's preferences, the donor may choose not to contribute but make positive income transfers to the recipient who will then contribute to the best-shot public good.

References

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