Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result

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Abstract

In the presence of output subsidization, the optimal output subsidy is identical and profits, output and social welfare are also identical irrespective of whether (i)a public firm moves simultaneously with n private firms or (ii) it acts as a Stackelberg leader or (iii) all firms, public and private, behave as profit–maximizers.

1 Introduction

In recent years there has been an increased interest in the theoretical investigation of mixed oligopoly models, usually in the context of the privatization debate (see De Fraja and Delbono (1990) for an excellent review). A mixed oligopoly is defined as an oligopolistic market structure where a homogeneous or differentiated good is supplied by a relatively small number of firms and the objective of at least one of them differs from that of other firms (De Fraja and Delbono (1990), p.1), as opposed to a private oligopoly where all firms have the objective of profit maximization. Usually in a mixed oligopoly there is a public firm whose objective is to maximize a notion of social welfare, competing with a multitude of private profit-maximizing firms. With the exception of White (1996), the use of output subsidies in the presence of a mixed oligopoly has not been examined in detail.[1] White (1996), concentrating on a simultaneous-moves oligopoly, has shown that if subsidies are used and the industry is subsequently privatized, there is a reduction in social welfare. Moreover, both in the private oligopoly and in the mixed oligopoly the optimal output subsidy is identical.

In the present paper, by building upon White (1996), we provide a much stronger 'irrelevance' result with respect to firms' moves and market structure in the presence of output subsidization. In addition to a private oligopoly and a mixed oligopoly where all firms make their output decisions simultaneously, we also consider the case of the public firm acting as a Stackelberg leader. We show that the optimal output subsidy is identical and profits, output and social welfare are also identical irrespective of whether (i) the public firm moves simultaneously with the private firms or (ii) the public firm acts as a Stackelberg leader or (iii) all firms behave as profit-maximizers.

2 The model and main result

There are n identical private profit-maximizing firms and one public welfare-maximizing firm competing in a market for a homogeneous good. Demand is linear, $p = a - Q$, where Q is total output, $Q = q_0 + \sum_i^n q_i$, q_0 is the output of the public firm and $q_i, i = 1, ..., n$ is the output of the i-th private firm. All firms share the same production technology represented by a quadratic

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[1] In a different context, Pal and White (1998) consider the use of output subsidies versus import tarrifs in an international mixed oligopoly and address the welfare consequences of privatization of a public firm on the home country.

cost function[2], $C(q_j) = c + \frac{1}{2}kq_j^2$, $j = 0, 1, ..., n$ and $k > 0$. As we are not concerned with entry issues we set, with no loss of generality, $c = 0$, i.e. we assume that there are no fixed costs. The firms' strategic variable is output. We consider a setting where the government, at an initial stage, commits to an output subsidy s. Given this output subsidy, firms subsequently compete by choosing their output. We consider three regimes: (a) all $n + 1$ firms choose their output simultaneously (this is the mixed-Nash oligopoly, labeled N); (b) all $n + 1$ firms are profit-maximizers (this is a standard private oligopoly, labeled P) and (c) the public firm acts as a Stackelberg leader choosing its output before the private firms do (this is the mixed-Stackelberg oligopoly, labeled S) .

A private firm's profit is given by

$$\pi_i = q_i(a - q_0 - \sum_i^n q_i) - \frac{1}{2}kq_i^2 + sq_i, \qquad i = 1, ..., n \tag{1}$$

the public firm's profit is given by

$$\pi_0 = q0(a - q_0 - \sum_i^n q_i) - \frac{1}{2}kq_0^2 + sq_0 \tag{2}$$

and social welfare, defined as the sum of producer and consumer surplus, is

$$W = \frac{1}{2}Q^2 + \pi_0 + \sum_i^n \pi_i - s(q_0 + \sum_i^n q_i). \tag{3}$$

Note that the output subsidy exerts an indirect effect on welfare through its effect on the private firms' output.

(a) the mixed-Nash oligopoly

There are two stages: in stage 1 the government sets the output subsidy to maximize social welfare; in stage 2 all $n+1$ firms choose their output conditional on the output subsidy. The model is solved by backward induction. Thus, we solve for the second-stage equilibrium expressions first. Maximizing (1) and (3) simultaneously we obtain the Nash equilibrium outputs for a given subsidy s:

$$q_0^N = \frac{a(k + 1) - ns}{(1 + k)^2 + kn}, \qquad q_i^N = \frac{ak + (1 + k)s}{(1 + k)^2 + kn}. \tag{4}$$

Note that setting $s = 0$ in the above expressions for output gives the standard result that the public firms's output exceeds a private firm's output,

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[2]With constant marginal costs the simple solution would have the public firm pricing at marginal costs and supplying the amount not produced by the private firms. Thus the assumption of increasing marginal costs provides a justification for public intervention.

$q_0^N(s=0) > q_i^N(s=0)$ (e.g. De Fraja and Delbono, 1989). Also, the effect of a subsidy is to reduce the public firm's output and increase the output of private firms, leading to costs being equalized across firms. This is a direct result of the assumption of increasing marginal cost.

In the first stage the government, taking into account how firms will react to the subsidy, maximizes (3); the solution to this problem yields the optimal subsidy:

$$s^* = \frac{a}{1+k+n}. \tag{5}$$

Using (5) into (4) we obtain the subgame-perfect Nash equilibrium outcomes:

$$q_0^N(s^*) = \frac{a}{1+k+n} = q_i^N(s^*), \tag{6a}$$

$$Q^N(s^*) = \frac{a(1+n)}{1+k+n}, \tag{6b}$$

$$p^N(s^*) = \frac{ak}{1+k+n}, \tag{6c}$$

$$\pi_0^N(s^*) = \frac{a^2(2+k)}{2(1+k+n)^2} = \pi_i^N(s^*), \tag{6d}$$

$$W^N(s^*) = \frac{a^2(1+n)}{2(1+k+n)}. \tag{6e}$$

Notice that the optimal subsidy achieves the first-best as price is equal to marginal cost, $p^N(s^*) = kq$ and that output is equalized across firms.

(b) the private oligopoly

In this regime all $(n+1)$ firms maximize profit as given by (1). Following the same procedure as for case (a) we find that the second-stage equilibrium output per firm for a given subsidy is $q(s) = \frac{a+s}{2+k+n}$, indicating the standard result that output is increasing in the subsidy. Interestingly, the optimal subsidy, output, price, profit and welfare are identical with those in (a), i.e. given by expressions (5) and (6a)-(6e). This equivalence of results between the private subsidized oligopoly and the mixed subsidized oligopoly, i.e. regimes (a) and (b), was first noted by White (1996).

(c) the mixed-Stackelberg oligopoly

Here we consider a three-stage game. In stage 1 the government chooses the output subsidy; in stage 2 the public firm sets its output; and in stage 3 the private firms set their output. Starting from the last stage, we find that

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the equilibrium output per firm for given subsidy and output by the public firm is

$$q(q_0, s) = \frac{(a + s) - q_0}{1 + k + n}. \tag{7}$$

In the stage 2, the public firm sets its output for given subsidy anticipating how its choice affects the output decision of the n private firms. The solution to this problem yields

$$q_0^S(s) = \frac{a[(k+1)^2 + nk] - ns}{(k+1)^2 + nk + k(n+k+1)^2} \tag{8}$$

and using this into (7) gives the private firms' output

$$q_i^S(s) = \frac{ak(n+k+1) + [k(n+k+1) + (1+k)]s}{(k+1)^2 + nk + k(n+k+1)^2}. \tag{9}$$

Notice that in the absence of the subsidy, $q_0^S(s = 0) > q_i^S(s = 0)$, i.e. the public firm produces more than the individual private firm similarly to regime (a). In the first stage, the government anticipating how its choice of subsidy will affect firms' output choices, maximizes welfare, as in (3). The optimal subsidy in this case is

$$s^S = \frac{a}{1 + k + n} = s^*$$

exactly the same as in regimes (a) and (b). Further, output per firm, total output, price, profit and welfare are identical to those obtained in (a) and (b), i.e. given by expressions (5) and (6a)-(6e).

We thus obtain a striking irrelevance result stated in the following proposition.

Proposition 1 *When the government uses output subsidization, irrespective of whether (i) the public firm moves simultaneously with the n private firms to set output or (ii) it acts as a Stackelberg-leader or (iii) all $n + 1$ firms behave as profit-maximizers, the optimal subsidy is identical. Further, all firms produce the same level of output and make equal profits.*

At first, this irrelevance result may seem surprising. However, it may be explained by referring to the market failures that are at work in this context. First, irrespective of the market structure regime under consideration, there is under-production because of imperfect competition - this market failure

can be addressed by a positive output subsidy. In the case of a fixed number of profit-maximizing firms and oligopolistic interaction, the social optimum can be achieved by implementing the optimal subsidy (regime (b)). Furthermore, in the case of a mixed oligopoly (regimes (a) and (c)) there is a second market failure arising from firms having different objectives, i.e. the public firm is welfare-maximizing whereas the private firms are profit-maximizers; as a result, the market equilibrium distribution of production costs is not efficient. The output subsidy in this latter case restores cost efficiency by forcing all firms to produce the same amount of output at equal cost in addition to addressing under-production. Given that the social optimum is unique, it turns out that it does not matter whether the public firm acts as a Stackelberg leader or sets its output simultaneously with the private firms - the optimal subsidy is identical across regimes.

3 Conclusion

In this note we have established that in the presence of output subsidization, the optimal subsidy is unique whether a public firm moves simultaneously with n private firms to set its output or acts as a Stackelberg leader or, all firms behave as profit-maximizers. Output across firms is equalized, the distribution of production costs is efficient and social optimality, given a fixed number of firms, is achieved. Market structure, as captured by a standard Cournot-Nash oligopoly, a mixed-Nash oligopoly or a mixed-Stackelberg oligopoly, is irrelevant. This result has been obtained for linear demand and a quadratic cost function. It is our conjecture that this 'irrelevance' result is robust to alternative specifications - we leave the extension to general demand and cost functions for future research.

References

[1] De Fraja, G. and Delbono, F.,1989, Alternative strategies of a public enterprise in oligopoly, Oxford Economic Papers 41, 302-311.

[2] De Fraja, G. and Delbono, F., 1990, Game-theoretic models of mixed oligopoly, Journal of Economic Surveys 4, 1-17.

[3] Pal, D. and White, M.D., 1998, Mixed oligopoly, privatization, and strategic trade policy, Southern Economic Journal 65, 264-281.

[4] White, M.D., 1996, Mixed oligopoly, privatization and subsidization, Economics Letters 53, 189-195.