E c o n o m i c s B u l l e t i n

On the theory of equalizing differences; Increasing abundances of types of workers may increase their earnings

Edward Cartwright
University of Warwick

Myrna Wooders
University of Warwick

Abstract

The theory of equalising differences recognises that wage differentials may be required to equalise the attractiveness of alternative occupations. We examine this theory using the Conley/Wooders 'crowding types' model. The crowding types model distinguishes between the tastes of a player and his crowding type, those attributes of the player that directly effect the well−being of other players in the same club − a player's skill, productivity or personality are examples. A club can be interpreted as firm in which the job attributes are the club goods; thus, the crowding types model, with its distinction between tastes and crowding types, provides a natural environment in which to study equalising differences. In contrast to results for earlier models, we demonstrate that even when small groups of players are strictly effective in a strong sense, an increase in the abundance of players of one crowding type can increase the core payoffs to players of that crowding type.

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"The whole of the advantages and disadvantages of the different employments of labour and stock must, in the same neighborhood, be either perfectly equal or continually tending to equality. If in the same neighborhood there was any employment evidently either more or less advantageous than the rest, so many people would crowd into it in one case, and so many would desert it in the other, that its advantages would soon return to the level of other employments". Adam Smith, *The Wealth of Nations.*

Equalizing differences and monotonicity of earnings.

In this passage from the Wealth of Nations we find the origins of the theory of equalizing differences, now suggested to be 'the fundamental market equilibrium construct in labour economics' (Rosen 1986,1988). The basis of the theory of equalizing differences is the recognition that wage differentials may be required to equalize the total monetary and non-monetary advantages and disadvantages among alternative employments. Thus, getting the most out of the available resources requires matching the appropriate type of worker with the appropriate type of firm: 'the labor market must solve a type of marriage problem of slotting workers into their proper "niche" within and between firms' (Rosen 1986). Current theoretical models of equalising differences neglect the importance of worker productivity (cf., Filer/Hamermesh/Rees 1996).

The Conley/Wooders *crowding types model* provides a novel way of modelling equalising differences. This model is a recent development in a literature originating in the seminal papers of Tiebout (1956) and Buchanan (1962). The model considers an economy with endogenous club formation and small effective groups in which players are described by two characteristics - a crowding type and taste type. The crowding type of a player describes those attributes of the player which directly effect the well-being of other players in the same club - a player's skill or productivity are natural examples. A club is interpreted as a group of players who engage in some collective activity. Examples include the interpretation of a club as a *local jurisdiction* providing public goods. Here we emphasize that a club can equally be interpreted as a *firm* in which the job attributes or by-products of production are seen as the club goods (cf., Bennett/Wooders 1979). With this interpretation the crowding types model, with its distinction between taste and crowding type, provides a natural environment in which to analyze equalising differences without neglecting worker productivity.

When small groups of players are strictly effective – that is, when all gains to collective activities can be realized by groups bounded in size and there are many players – it has been shown in a number of contexts that core payoffs exhibit a comparative statics property. Specifically, an increase in the abundance of players of one *type* will not increase core payoffs to players of that type. (Kovalenkov/Wooders 2000 provides recent results and references.) In this paper we ask whether an increase in the abundance of players of one *crowding type* can increase the core payoffs to players of that crowding type. For example, can an increase in the abundance of engineers *increase* the payoff to engineers who do not mind a dangerous working environment? We present an example to show that this may indeed be the case.

The Model

Following Conley/Wooders (1997) consider an economy where players are described by two uncorrelated characteristics, their taste types and crowding types. A player has one of T different taste types, denoted by $t \in 1, ..., T \equiv \mathcal{T}$ and one of C different crowding types, denoted $c \in 1, \ldots, C \equiv \mathcal{C}$.

The total population of an economy is described by a vector $N = (N_{11}, \ldots, N_{ct,} \ldots, N_{CT})$, where N_{ct} is the total number of players with crowding type c and taste type t. A *club* $m = (m_{11}, \ldots, m_{ct}, \ldots, m_{CT})$ describes a group of players, where m_{ct} denotes the number of players with crowding type c and taste type t in the group. When it will not cause any confusion, we shall refer to a club described by m as 'the club m' and similarly for the economy with population N. The *crowding profile* of an economy N is a vector $\overline{N} = (N_1, ..., N_C)$,listing the numbers of players of each crowding type in the economy; thus $N_c = \sum_t N_{ct}$. The set of all feasible clubs for N is denoted by $\mathcal{N}$.

Each player belongs to exactly one club. Thus, a *partition* n of the population into clubs $\{n_1, ..., n_K\}$ must satisfy $\sum_k n_k = N$. We will write $n_k \in n$ when a club n_k belongs to the partition n.

It will sometimes be useful to refer to individual players whom we denote by $i \in \{1, \ldots, I\} \equiv \mathcal{I}$, where $I = \sum_{c,t} N_{ct}$. We let $\theta : \mathcal{I} \to \mathcal{C} \times \mathcal{T}$ be a mapping describing the crowding and taste types of individual players. We will say an player i has *type* (c, t) if $\theta(i) = (c, t)$. With a slight abuse of notation, if individual i is a member of the club described by m, we shall write $i \in m$.

An economy has one private good x and club goods y. A vector $y = (y_1, y_2, ..., y_A) \in \Re^A_+$ gives club good production.[1] Each player $i \in \mathcal{I}$ of taste

[1] As Conley and Wooders (1995), club goods could be just elements in some compact topological space.

type t is endowed with $\omega_t \in \mathsf{R}_+$ of the private good, and has a quasi linear utility function

$$u_t(x, y, m) = x + h_t(y, m)$$

where $i \in m$ and y is the club good production of the club to which i belongs. The cost in terms of the private good of producing y club goods in club with membership m is given by a production function $f(y, m)$. A particular combination of preferences and endowments for players in the economy N and production possibilities available to subsets of N is referred to as the *structure* of the economy. There are two key assumptions: *taste anonymity in consumption* and *taste anonymity in production*. These capture the idea that players care only about the crowding types and not the taste types of the players that are in their club. See Conley/Wooders for formal definitions.

A *feasible state* of the economy

$$(X, Y, n) \equiv ((x_1, \ldots, x_I), (y_1, \ldots y_K), (n_1, \ldots n_K))$$

consists of a partition n of the population, an allocation of private goods to players $X = (x_1, \ldots, x_I)$ and a club goods production plan for each club, $Y = (y_1, \ldots y_K)$ such that

$$\sum_k \sum_{ct} n^k_{ct}\omega_t - \sum_i x_i - \sum_k f(y^k, n^k) \geq 0.$$

We also say that $(\overline{x}, \overline{y})$ is a *feasible allocation for a club m* if

$$\sum_{c,t} m_{ct}\omega_t - \sum_{i \in m} \overline{x}_i - f(\overline{y}, m) \geq 0.$$

A club $m \in \mathcal{N}$ producing a feasible allocation $(\overline{x}, \overline{y})$ *can improve upon* a feasible state (X, Y, n) if for all $i \in m$,

$$u_t(\overline{x}_i, \overline{y}, m) > u_t(x_i, y_k, n_k)$$

where in the original state $i \in n_k$ and $n_k \in n$. A feasible state of the economy (X, Y, n) is a *core state of the economy* or simply a *core state* if it cannot be improved upon by any club $m \in \mathcal{N}$.

An economy satisfies *strict small group effectiveness*, SSGE, if there exists a positive integer B such that:

(a). For all core states (X, Y, n) and all $n_k \in n$, it holds that $|n_k| < B$.
(b). For all $c \in \mathcal{C}$ and all $t \in \mathcal{T}$ it holds that $N_{ct} > B$.

SSGE is a relatively strong condition; the literature shows, however, economies satisfying apparently mild conditions, such as boundedness of average payoff, can be approximated by ones satisfying SSGE (cf.,Wooders 1999 for a discussion and references).

Under SSGE, any core state must have the *equal treatment property* – in any core state, any two players of the same type must be equally well off (cf., Conley/Wooders 1997 or Wooders 1983, Theorem 3 for the general game-theoretic result). In consequence, for any core state (X, Y, n) there is a vector of payoffs $u = (u_{11}, ..., u_{ct}, ..., u_{CT}) \in \mathbb{R}^{CT}$ where u_{ct} is the utility of an player with crowding type c and taste type t. A particularly important result is that *for any core state of the economy players with the same crowding types and possibly different tastes must make the same contributions to club good provision.* This feature ensures the optimality of equilibrium with anonymous prices (that is, admission prices of players to clubs that depend only on their crowding types) and equivalence of the core and equilibrium outcomes (Conley/Wooders 1995,1997). This amounts to saying that (monetary) wages depend only on crowding types and not on taste types.

Monotonicity in Crowding Types

Taking S and G as two economies and remembering that $S_c = \sum_t S_{ct}$ and $G_c = \sum_t G_{ct}$ gives the numbers of players in economies S and G with crowding type c we now introduce *weak monotonicity in crowding types* (WMCT) as follows: Given an economic structure let S and G be two economies with that structure. The populations of S and G, represented $S = (S_{11}, \ldots, S_{ct,} \ldots, S_{CT})$ and $G = (G_{11}, \ldots, G_{ct,} \ldots, G_{CT})$, are allowed to take any form (that satisfies SSGE) for which there exists a core outcome. We let $u^s = (u^s_{11}, ..., u^s_{ct}, ..., u^s_{CT}) \in \mathbb{R}^{CT}$ and $u^g = (u^g_{11}, ..., u^g_{ct}, ..., u^g_{CT}) \in \mathbb{R}^{CT}$ represent any of these equal treatment core payoffs. We say that the payoffs to players of crowding type c and taste type t satisfy:

Weak Monotonicity in Crowding Types (WMCT) if for any populations such that, for all $c' \neq c$ and all t', it holds that $S_{c't'} = G_{c't'}$, and, for all t it holds that $S_{ct} < G_{ct}$ then

$$(u^s_{ct} - u^g_{ct}) \cdot (S_c - G_c) \leq 0.$$

Generally, we say payoffs satisfy monotonicity in crowding types if a ceteris paribus increase in the number of players with a given crowding type cannot increase the payoff to any players of that crowding type. WMCT restricts attention to scenarios in which there is a common change in the number of players with each type and the given crowding type. WMCT does not allow, for example, the number of skilled public good lovers to increase while the number of skilled public good haters falls. (For our purposes here we restrict to the situation for which we currently have the broadest positive results. In further research we consider other sorts of changes in the composition of the total player set.)

A Counter Example to WMCT

The club good is music. Taste types - players who love to hear music while they work (L), players who hate music (H) and players who are indifferent to music (I). Crowding types - workers who sing along with music (S), workers who do not sing along (N), and workers who occasionally sing (O). Players form pairs with utilities as below:

U_H(S,S)=0	U_I(S,S)=10	U_L(S,S)=20	U_H(O,O)=10	U_I(O,O)=10	U_L(O,O)=10
U_H(S,O)=5	U_I(S,O)=10	U_L(S,O)=15	U_H(O,N)=15	U_I(N,N)=10	U_L(O,N)=5
U_H(S,N)=10	U_I(S,N)=10	U_L(S,N)=10	U_H(N,N)=20	U_I(O,N)=10	U_L(N,N)=0

Figure I: Utilities of combinations of crowding types.

For example, if a worker who does not sing along but likes working in an environment with music forms a club with a worker who does sing along then this worker receives 10 units of utility and otherwise, zero units. All possible combinations of player pairs earn the same revenue from production, normalized to zero. The resulting worth function is:

composition	total utility	composition	total utility	composition	total utility
SL, SL	40	OL, OL	20	NL, NL	0
SI, SI	20	OI, OI	20	NI, NI	20
SH, SH	0	OH, OH	20	NH, NH	40
SH, SL	20	OH, OL	20	NH, NL	20
SH, SI	10	OH, OI	20	NH, NI	30
SI, SL	30	OI, OL	20	NI, NL	10
SL, OL	30	NL, OL	10	NL, SL	20
SI, OI	20	NI, OI	20	NI, SI	20
SH, OH	10	NH, OH	30	NH, SH	20
SH, OL	20	NH, OL	20	NH, SL	20
SL, OH	20	NL, OH	20	NL, SH	20
SH, OI	15	NH, OI	25	NH, SI	20
SI, OH	15	NI, OH	25	NI, SH	20
SI, OL	25	NI, OL	15	NI, SL	20
SL, OI	25	NL, OI	15	NL, SI	20

Figure II: The worth function

We then consider the two possible core outcomes: (where, for example, $2\times$ (NH, NH) means two jurisdictions consisting of two individuals, both of whom do not sing along (N) and hate music (H).)

1. $2\times$ (NH, NH), $4\times$ (SI, OL), $2\times$ (SL, SL), $2\times$ (OH, NI), $4\times$ (SH, NL), $2\times$ (SH, NI) and $1\times$ (OI, OI)

2. $2\times$ (NH, NH), $2\times$ (SL, SL), $2\times$ (SH, OL), $4\times$ (SI, OL), $4\times$ (OH, NI), $4\times$ (SH, NL) and $2\times$ (OI, OI)

These will give us the following scenarios:

type	SH	SI	SL	OH	OI	OL	NH	NI	NL
# of type in outcome1	6	4	4	2	2	4	4	4	4
# of type in outcome 2	6	4	4	4	4	6	4	4	4
payoff in outcome 1	10	15	20	15	10	10	20	10	10
payoff in outcome 2	7.5	12.5	20	10	10	12.5	20	15	12.5

Figure III: Core Clubs and Payoffs for Scenarios 1 and 2.

Between scenarios 1 and 2 the number of players with type OL, OH and OI increases by 2 while the number of all other players remains constant. The payoff to OL players increases by 2.5 in contradiction of WMCT.

Why is it that workers who occasionally sing along with music but like working in an environment with music are able to gain? We begin by noting that such workers prefer to work with singers and, in scenario 1, those that sing along are doing relatively well. This means that OL players have to accept a low payoff. In turn we notice that OH types prefer working with workers who do not sing, but here, it is the players not singing who are accepting lower payoffs. We now move to scenario 2. In this case, the increase in the number of OH players weakens their position and the workers who do not sing gain as a result. The 'knock on' effect of this is that workers who sing can no longer demand a high payoff . As the workers who sing lose out the workers of type OL are able to gain.[2]

The intuitive logic: 'an increase in the number of players with a particular type will weaken the bargaining position of these players' does not necessarily conform with reality. A player can gain more through the knock on effects of an increase in the number of players with his crowding type but different taste types than he loses from an increase in the number of players with his crowding type. In the example, OL workers gain more through the increase in the number of type OH workers than they lose by the increase in the number of players with their type.

Conclusion

From a general perspective one significant area of further study remains: compensating differentials on the basis of human capital. A natural way to analyze this issue is provided by the model of *genetic types* of Conley/Wooders (1995,1996) in which players are endowed with a genetic type. Players then purchase their crowding type with costs dependent on their genetic type. For example, a genetic type may be a level of intelligence and a crowding type an education level with the costs of education negatively related to intelligence. By endogenising crowding type choice this model would allow us to study the role education and training play in the theory of equalizing differences. Research in progress treats these issues.

[2] To appreciate what is going on it is useful to draw a grid of 9 dots with taste types against crowding types and then represent jurisdictions by joining the relevant dots.

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