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Spatial Cournot competition and timing of endogenous wage setting

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Abstract

This paper studies the problem of location-quantity choice in a duopoly in which the wage paid by each firm is set by the corresponding monopoly union. Compared with the outcome obtained in location-price choice game, we find that the wage setting choice for both unions does not change in our model, they still choose to set wage sequentially. The equilibrium locations are not influenced by the timing of wage setting.

1. Introduction

The motivation of this paper is to propose a theoretical model of spatial Cournot competition when production costs are endogenous ¹. In our model, the time when unions set wage is not exogenously imposed but determined by the different unions once both firms are located and before the stage in which firms set their output. That is to say, the timing of wage setting takes place simultaneously or sequentially.

The analysis of spatial models with Cournot competition starts with the works of Hamilton et al. (1989) and Anderson and Neven (1991). They analyze a two-stage problem of locationthen-quantity choice in Hotelling's (1929) linear city model and find that firms agglomerate at the center of the market. Gupta et al. (1997) introduce spatial heterogeneity in the distribution of consumers into the linear city model and show that spatial Cournot competition gives rise to agglomeration under a wide variety of consumer distributions. Mayer (2000) considers a model in which production costs vary across locations and confirms that the firms may still agglomerate. Besides, great numbers of studies have been done on spatial Cournot competition in Hotelling's linear city model, such as multi-plant firms issue (Chamorro-Rivas, 2000; Pal and Sarkar, 2002; Yu and Lai, 2003; Li, 2006), production differentiation issue (Shimizu, 2002; Yu and Lai, 2003), and economic welfare analysis (Matsumura and Shimizu, 2005a; Matsumura and Shimizu, 2005b). However, the production costs of these papers are given exogenously. Wang and Chen (2008) analyze the problem of location-quantity choice with endogenous production costs in both linear and circular city model and demonstrate that the pattern of equilibrium location varies as the transport rate increases in a linear city and firms locate equidistant from each other in circular city model. However, the wage setting process in Wang and Chen (2008) is only simultaneous.

In the non-spatial context, Naylor (1998,1999) constructs a wage-then-quantity model in which the monopoly union first chooses a wage and then the corresponding firm sets its output levels simultaneously. The main result of Naylor (1998) is that a more competitive product market does not necessarily generate a more competitive labor market. Straume (2002) analyzes the scope for collusive behavior when labor markets are unionized.

In the present paper we bring together two separate literatures: spatial Cournot competition and wage setting. The basic spatial model in our paper is similar to Liang et al. (2006). However, there are some differences between their model and ours. Firstly, the demands of the two identical markets in the former are asymmetric while they are assumed symmetric for convenient computation in our paper. Secondly, the production costs are exogenous and constant in their model while they are endogenous by supposing that each firm confronts a monopoly union and wages are determined by the unions in the current paper. We attempt to address the following two questions: (1) Is the timing of wage setting process for both unions in location-quantity game the same with that in location-price game? (2) Does the timing of wage setting process affect the location choices of firms?

We find that the wage setting choices for both unions in our model are the same as the ones in Bárcena-Ruiz and Casado-Izaga (2008), which are obtained in the location-then-price game with endogenous production costs. Both unions prefer to set wages sequentially rather than simultaneously for any pair of locations chosen by firms. The timing of wage setting does not alter the location equilibrium for the wages paid by firms are independent of their locations in our model.

¹To our knowledge, it is seldom that papers in spatial competition literature consider endogenous production costs. For exceptions see Matsumura and Matsushima (2004) where they endogenize the production cost by introducing strategic cost-reducing activities into a Hotelling spatial model. Brekke and Straume(2004) establish a location-price model in which firms can choose their locations simultaneously or sequentially, but always bargain over wages simultaneously. Recently, Bárcena-Ruiz and Casado-Izaga (2008) extend Brekke and Straume's (2004) analysis by assuming that not only wages but also the timing of wage negotiations are endogenous determined.

The rest of the paper is organized as follows. Section 2 describes the basic model. The firms' location choices are analyzed under both wage setting cases in Section 3. Section 4 compares the outputs and profits of the two firms, consumer surplus, and social welfare obtained under the two types of wage setting respectively. Section 5 closes the paper with conclusions and further remarks.

2. The model

There are two firms, firm 1 and firm 2, who produce and sell a homogenous output to the consumers who reside in two separate identical markets(market A and market B). Firms can locate at any point along a unit length segment to set up their plants. The two markets locate at the two endpoints of the line segment. The locations of firm 1 and 2 are supposed to be x_1 and x_2 apart from the left point of the line segment, respectively. Furthermore, we assume $0 \le x_1 \le x_2 \le 1$ without of loss generality. It implies that firm 1 locates to the left of firm 2.

We suppose that all the workers employed by each firm are supplied by the corresponding monopoly labor union which has the objective of rent maximization. Furthermore, we suppose that each union negotiates with its corresponding firm over wages and has completely bargaining power. And the firm has the right to choose the employment level. The unions' competitive or reservation wage levels are assumed to be equal, and are, without loss of generality, normalized to zero. The two firms have identical technology and use one unit of labor force can produce one unit of output. To ship a unit of the product from its own location x_i to a consumer at point x, each firm i(i = 1, 2) has to pay a linear transport cost $t|x - x_i|$, where t is a positive constant. We also assume that the inverse demand functions in market A and B are linear and are respectively expressed as follows: ²

$$p_A = a - q_A,$$

$$p_B = a - q_B,$$
(1)

where q_j (j = A, B) denotes the total quantity in market j supplied by the two firms and p_j denotes delivered price in market j. Finally, $t < \frac{10}{29}a$ is assumed in order to ensure both markets will be served by the two firms.³ Given the locations and quantities of the two firms and the wages paid by both firms, the profit function of firm i (i = 1, 2) is expressed as:

$$\pi_i(x_1, x_2) = (p_A - w_i - tx_i) q_A^i + [p_B - w_i - t(1 - x_i)] q_B^i,$$
(2)

where q_j^i (j = A, B; i = 1, 2) are firm *i*'s sales in market *j*. It is obviously that the quantities of market *j* supplied by the two firms satisfy the condition $q_j = q_j^1 + q_j^2$.

The timing of the game in this paper consists of four stages. In the first stage, the two firms simultaneous select their locations. In the second stage, it is simultaneously decided whether to negotiate over wages in period 1 or in period 2 given the location choice. The decision could be taken by the unions or by the firms. If the decisions of wage take place in the same period, wage setting process is simultaneous. If not, decisions of wage are sequential. The monopoly unions choose wages of workers in stage three. The firms simultaneously choose their quantities in the fourth stage. The game equilibrium of the model is solved by backward induction from the last stage.

3. Equilibrium analysis

 $^{^{2}}$ The demand curves of both markets in the current paper are special cases of Naylor (1998). However, the results obtained in our paper also remain valid by using the demand functions in Naylor (1998).

³It implies that the quantity of each market served by each firm is positive for any pair of the two firms' locations. The inequality of $t < \frac{10}{29}a$ derives from Eqs. (9)-(10) and (14)-(15). Specifically, if wage setting process is simultaneous, from Eqs. (9)-(10), we have $a - \frac{1}{6}(2a - t) - 2t > 0$, which implies $t < \frac{4}{11}a$. Similar to simultaneous case, if wage setting process is sequential, the fact that $a - \frac{3}{16}(2a - t) - 2t > 0$ results to $t < \frac{10}{29}a$, and the fact that $a - \frac{9}{56}(2a - t) - 2t > 0$ results to $t < \frac{38}{103}a$. In order to ensure all conditions are satisfied, we have $t < \frac{10}{29}a$ for a > 0 and $\frac{10}{29}a < \frac{4}{11}a < \frac{38}{103}a$.

3.1. Quantity competition

In the present stage, firms simultaneously set their output to maximize their own profits given locations x_1 and x_2 , wages w_1 and w_2 , and the rival's output. Calculating the first-order derivative of $\pi_i(x_1, x_2)$ with respect to q_j^i , and solving these first-order conditions, we derive the following equilibrium outcomes: ⁴

$$q_A^1 = (a - 2w_1 + w_2 - 2tx_1 + tx_2)/3,$$

$$q_A^2 = (a - 2w_2 + w_1 - 2tx_2 + tx_1)/3,$$

$$q_B^1 = [a - 2w_1 + w_2 - 2t(1 - x_1) + t(1 - x_2)]/3,$$

$$q_B^2 = [a - 2w_2 + w_1 - 2t(1 - x_2) + t(1 - x_1)]/3.$$
(3)

Eq. (3) indicates, as expected, that firm i's sales in market j increase as its location becomes closer to the market and are decreasing functions of its wage and increasing functions of the competitor's wage. We can derive the delivered prices by substituting Eq. (3) into Eq. (1):

$$p_A = (a + w_1 + w_2 + tx_1 + tx_2)/3,$$

$$p_B = [a + w_1 + w_2 + t(1 - x_1) + t(1 - x_2)]/3.$$
(4)

Therefore, the delivered price of market j increases not only with its own wage and with the wage paid by its competitor, but also with the distance between firm i's location and the market.

3.2. Wage setting

We now turn to the third stage of the game. There are two choices for both unions when setting wages. One is the simultaneous case, the other is the sequential case.

3.2.1. Simultaneous wage setting

Let w_i^C (i = 1, 2) denote the equilibrium wages in the case of simultaneous wage setting. ⁵ In this stage, each union chooses a wage taking as given the wage set by the other union and taking into account the firm's labor demand function. Hence, union *i*'s (i = 1, 2) utility can be written as:

$$U_i = w_i \left(q_A^i + q_B^i \right). \tag{5}$$

Union *i* will choose w_i^C such that

$$w_i^C = \arg \max_{w_i^c} \left\{ U_i^C = w_i^C \left(q_A^i + q_B^i \right) \right\}.$$
 (6)

Substituting (3) into (6), we obtain

$$w_i^C = \arg \max_{w_i^C} \left\{ U_i^C = \frac{1}{3} w_i^C (2a - t - 4w_i^C + 2w_k^C) \right\}, i, k = 1, 2, i \neq k,$$
(7)

where w_k^C denotes the wage chosen by union k and which union i takes as given.

From Eq. (7), calculating the first-order derivative of U_i^C with respect to w_i^C and union *i*'s wage reaction function with respect to the given wage set by union k is expressed as:

$$w_i^C = \frac{1}{8}(2a - t + 2w_k^C), \quad i, k = 1, 2, i \neq k.$$
(8)

⁴It is easy to verify that the second order conditions are satisfied.

⁵For convenient expression, we let the superscript C stands for the results in the case of simultaneous wage setting and the superscript F stands for the follower and L for the leader in the case of sequential wage setting in the rest of the paper.

Hence, the equilibrium wages set by the two unions are expressed as follows from Eq. (8):

$$w_1^C = w_2^C = \frac{1}{6} \left(2a - t \right). \tag{9}$$

Eq. (9) shows that the wages paid by each firm are equal and are independent of locations of the two firms.

From Eq. (9), we can substitute back into the labor demand functions Eq. (3) to derive the levels of the output and employment. Further substitution into Eqs. (2) and (5) then yields the equilibrium levels for profits and union utility, as shown in Eqs. (10)-(12):

$$\pi_1^C(x_1, x_2) = \frac{1}{9} \left[a - \frac{1}{6}(2a - t) - 2tx_1 + tx_2 \right]^2 + \frac{1}{9} \left[a - \frac{1}{6}(2a - t) - 2t(1 - x_1) + t(1 - x_2) \right]^2 = \left[q_A^{1C} \right]^2 + \left[q_B^{1C} \right]^2,$$
(10)

$$\pi_{2}^{C}(x_{1}, x_{2}) = \frac{1}{9} \left[a - \frac{1}{6} (2a - t) - 2tx_{2} + tx_{1} \right]^{2} + \frac{1}{9} \left[a - \frac{1}{6} (2a - t) - 2t(1 - x_{2}) + t(1 - x_{1}) \right]^{2} = \left[q_{A}^{2C} \right]^{2} + \left[q_{B}^{2C} \right]^{2}, \qquad (11)$$

$$U_1^C = U_2^C = \frac{1}{27}(2a-t)^2.$$
(12)

3.2.2. Sequential wage setting

We now turn to the case of sequential wage setting and suppose that union 1 sets wages first, without loss of generality. Thus, union 1 acts as a leader and union 2 acts as a follower.

Union 2 will choose w_2 to maximize its utility taking as given the wages w_1 set by union 1. By solving the first-order condition for union 2, we find that this union's best response function to be expressed as:

$$w_2 = (2a - t + 2w_1)/8. \tag{13}$$

As usual, Eq. (13) indicates that wages are strategic complements. That is to say, the greater the wage of the rival the greater the own wage is.

Inserting Eq. (13) into the utility function of union 1 and solving the first-order condition, the equilibrium wages w_1^L, w_2^F are respectively given as:

$$w_1^L = \frac{5}{28}(2a-t), \quad w_2^F = \frac{19}{112}(2a-t).$$
 (14)

Eq. (14) indicates that the wages set by the unions are also independent of locations of the two firms in the case of sequential wage setting. However, they are unequal.

The computational processes in this subsection are similar to the ones in the case of simultaneous wage setting. So, we can easily obtain the equilibrium levels for profits and union utility which are shown in Eq. (15)-(18):

$$\pi_1^L(x_1, x_2) = \frac{1}{9} \left[a - \frac{3}{16} (2a - t) - 2tx_1 + tx_2 \right]^2 + \frac{1}{9} \left[a - \frac{3}{16} (2a - t) - 2t(1 - x_1) + t(1 - x_2) \right]^2 = \left[q_A^{1L} \right]^2 + \left[q_B^{1L} \right]^2,$$
(15)

$$\pi_{2}^{F}(x_{1}, x_{2}) = \frac{1}{9} \left[a - \frac{9}{56} (2a - t) - 2tx_{2} + tx_{1} \right]^{2} \\ + \frac{1}{9} \left[a - \frac{9}{56} (2a - t) - 2t(1 - x_{2}) + t(1 - x_{1}) \right]^{2} \\ = \left[q_{A}^{2F} \right]^{2} + \left[q_{B}^{2F} \right]^{2},$$
(16)

$$U_1^L = \frac{25}{672}(2a-t)^2,\tag{17}$$

$$U_2^F = \frac{361}{9408} (2a-t)^2.$$
(18)

3.3. Timing of wage setting

In the present stage of the game, we will investigate the problem that whether to choose wages simultaneously or sequentially for the two unions.

We have the following propositions by comparing the equilibrium values stated in Eqs. (9)-(12) and (14)-(18).

Proposition 1 For any pair of locations x_1 and x_2 , the wages set by the monopoly unions in both wage setting cases satisfy the condition $w_1^L > w_2^F > w_1^C = w_2^C$.

This proposition shows that the wages paid by the two firms in the sequential case are higher than those in the simultaneous case in spite of the locations of firms. ⁶ We also obtain from proposition 1 that both unions will set equal levels for wages in the simultaneous case and the leader union will choose a higher wage than the follower in the sequential case.

Proposition 2 For any pair of locations x_1 and x_2 , the utility of unions in both wage setting cases satisfy the condition $U_2^F > U_1^L > U_1^C = U_2^C$.

Proposition 2 illustrates that the utility of the two unions in the sequential case is higher than that in the simultaneous case regardless of the locations of firms. We also obtain from this proposition that both unions get equal utility in the simultaneous case and the follower union gains a higher utility than the leader in the sequential case.

An intuitive explanation behind Proposition 2 is as follows. The levels for wages and employments are the two factors that determine the utility of unions. From proposition 1, we conclude that the workers employed by the two firms obtain higher wages in the sequential case than those in the simultaneous case. Moreover, the follower union can obtain more employment in the sequential case. Although the leader union supplies firm 1 with less workers in the sequential case, the higher wage implies that the greater utility obtained by the leader union under sequential wage setting. Therefore, both unions obtain greater utility in the sequential case than in the simultaneous case. Furthermore, the workers supplied by the follower union are more than those supplied by the leader union in the sequential case. The follower union obtains greater utility than the leader union although a higher wage set by the latter. Hence, both unions prefer to choose wages sequentially rather than simultaneously.

We can directly derive the following proposition from proposition 2.

Proposition 3 Sequential wage setting is better than simultaneous wage setting for both unions regardless of the locations of the two firms.

Proposition 3 indicates that if the negotiation process is taken by the two unions, the better timing of wage setting is sequential.

⁶This result is standard in the literature on wage bargaining.

Bárcena-Ruiz and Casado-Izaga (2008) also obtained the result that both unions prefer to negotiate over wages sequentially rather than simultaneously. However, our model differs from their's in the following aspect: they consider the problem of location-price choice with endogenous production costs while we consider the problem of location-quantity choice when production costs are endogenous.

Comparing the equilibrium profits of the two firms in the simultaneous and sequential cases, we straightforwardly obtain the following result.

Proposition 4 For any pair of locations x_1 and x_2 , the profits of firms in both cases are satisfied with the condition $\pi_1^C > \pi_1^L$, $\pi_2^F > \pi_2^C$.

Proposition 4 can be explained as follows. From proposition 1, we know that both the leader and the follower firms pay for higher wages under sequential wage setting than when wage setting is simultaneous. However, both the price-cost margin and the output in the two markets obtained by the leader firm in sequential wage setting case are lower than those in simultaneous wage setting case, which leads to the result $\pi_1^C > \pi_1^L$. On the other hand, both the price-cost margin and the output in the two markets obtained by the follower firm in sequential wage setting case are higher than those in simultaneous wage setting case, which leads to the result $\pi_1^C > \pi_1^L$.

If the negotiation process is determined by the two firms, proposition 4 indicates that the leader(follower) firm obtains lower(higher) profits than in the simultaneous case. Hence, each firm wants to be the follower which results that the wage setting process is simultaneous in this stage.

3.4. Location choice

In the first stage of the game, each firm chooses a location to maximize its profits given the competitor's location. We want to investigate whether the equilibrium locations of firms' are influenced by the timing of the wage setting in this section.

We have the following proposition.

Proposition 5 In the location-quantity choice game with endogenous production costs, there exists one unique equilibrium, where the two firms locate at the opposite endpoints of the line segment. Moreover, the equilibrium locations are independent of the timing of endogenous wage setting.

Proof: See the Appendix.

The intuitive explanation behind this proposition is as follows. It is the market share effect, production costs effect and the competition effect that are the three factors that determine the firms' optimal locations. However, the total quantity of the two markets sold by each firm are independent of firms' location choice in our model. Although the wages paid by the two firms are different in both simultaneous and sequential wage setting, they are also not affected by firms' location choice in the current paper. Therefore, both firms always choose their locations at the opposite endpoints markets in order to mitigate competition regardless of the timing of endogenous wage setting.

4. Comparison of the equilibria

We know from above section that the two firms will never locate in the same market whether the unions choose wages simultaneously or sequentially. Therefore, we can compare the differences in profit and outputs of the two firms, consumer surplus and social welfare of the two markets in each type of wage setting case.

We have the following propositions.

Proposition 6 Given the equilibrium locations of both firms, both firms have equal outputs and obtain equal profits in the two market if wage setting is simultaneous while the follower firm has more outputs and obtains higher profits than those gained by the leader firm if wage setting is sequential.

Proof: See the Appendix.

The consumer surplus functions of the two markets in simultaneous and sequential wage setting cases are respectively expressed as follows:

$$CS_A^C = \frac{2}{81}(2a-t)^2 = CS_B^C,$$
(19)

$$CS_A^L = \frac{5329}{225792} (2a - t)^2 = CS_B^F.$$
(20)

The social welfare is defined as the sum of the producer surplus, consumer surplus and unions' utility. Hence, the social welfare in the two markets are respectively expressed as:

$$W_A = \pi_1 + CS_A + U_1, W_B = \pi_2 + CS_B + U_2.$$
(21)

Therefore, we have the following proposition.

Proposition 7 Given the equilibrium locations of both firms, the consumer surplus of two markets are equal respectively in both wage setting cases. Moreover, the social welfare of the two markets are equal if wage setting is simultaneous while the social welfare of the market where the follower firm locates in is higher than that of the market where the leader firm locates in.

Proof: See the Appendix.

5. Conclusions and further remarks

In this paper, we contribute to the literature of spatial Cournot competition by considering timing of endogenous wage setting in the location-quantity choice game with endogenous production costs. We show that both the monopoly unions obtain higher utility in sequential wage setting case than in simultaneous wage setting case regardless of the firms' locations. This is due to the fact that the wages paid by firms are independent of their location choices in our model. We find that both firms always locate at the endpoints of the line segment whether the wage setting is simultaneous or sequential. If wage setting process is simultaneous, both firms have equal outputs and gain equal profits, and the social welfare of the two markets is also equal. However, if wage setting process is sequential, the leader firm has less outputs and gains lower profits than those gained by the follower firm, and the social welfare of the market where the leader firm locates in is lower than that of the market where the follower firm locates in. The consumer surplus of the two markets is equal respectively in both simultaneous and sequential wage setting cases.

As a final note, there are a number of obvious directions for further work. For example, we can apply our theoretical framework to the standard models: i.e. Hotelling's linear city model and Salop's circular city model. It is worth pointing out that the wages paid by firms are also independent of their locations in Salop's circular city model from Wang and Chen (2008). Hence, we can conclude that firms locate equidistant from each other regardless of the timing of wage setting. In order to obtain general results, another way to extend our paper would be to relax the assumption that the unions have completely bargaining strength. The firms may have relative strength when they negotiate with the unions. Therefore, it is reasonable that we should use the Nash bargaining function in the wage negotiation stage of the game.

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Appendix

Proof of proposition 5

(1) Location choice with simultaneous wage setting

Calculating the first-order and the second-order derivatives of each firm's profit function $\pi_i^C(x_1, x_2)$ stated in Eq. (10) and (11) with respect to its location x_i , we obtain the following results:

$$\frac{\partial \pi_1^C(x_1, x_2)}{\partial x_1} = \frac{4t}{3} \left(-q_A^{1C} + q_B^{1C} \right), \quad \frac{\partial^2 \pi_1^C(x_1, x_2)}{\partial x_1^2} = \frac{16}{9} t^2 > 0; \\
\frac{\partial \pi_2^C(x_1, x_2)}{\partial x_2} = \frac{4t}{3} \left(-q_A^{2C} + q_B^{2C} \right), \quad \frac{\partial^2 \pi_2^C(x_1, x_2)}{\partial x_2^2} = \frac{16}{9} t^2 > 0.$$
(22)

Eq. (22) indicates that the profit function of each firm is strictly convex with respect to its location x_i . Hence, there exists a corner solution for each firm's function. So we conclude that the equilibrium locations for firms can be derived by comparing the profits at the market A and B.

Given the assumption of $0 \le x_1 \le x_2 \le 1$, there are only three possible solutions for the two firms, i.e., $(x_1, x_2) = (0, 0)$, $(x_1, x_2) = (0, 1)$, $(x_1, x_2) = (1, 1)$. However, the last candidate can be ruled out by the following condition:

$$\pi_1^C(1,1) - \pi_1^C(0,1) = -\frac{4}{9}t^2 < 0.$$
(23)

Eq. (23) illustrates that firm 1 can earn a higher profit to locate in market A than in market B, given firm 2 locate in market B.

Therefore, we can obtain the exact equilibrium locations for firms by comparing the profits of firm 2 at the two markets given firm 1 to stay in market A.

$$\pi_2^C(0,1) - \pi_2^C(0,0) = \frac{4}{9}t^2 > 0.$$
(24)

Eq. (24) shows that the best choice for firm 2 to locate in market B given firm 1 locate in market A. So, we can obtain the following result from Eq. (23) and (24).

Result 1 In the location-quantity choice game with simultaneous wage setting, the two firms always locate at the opposite endpoints of the line segment.

(2) Location choice with sequential wage setting

The analysis in this section is similar to the case of simultaneous wage setting. First, the first-order and the second-order derivatives of firms' profit functions stated in Eqs. (15) and (16) with respect to their locations are expressed as:

$$\frac{\partial \pi_1^L(x_1, x_2)}{\partial x_1} = \frac{4t}{3} \left(-q_A^{1L} + q_B^{1L} \right), \quad \frac{\partial^2 \pi_1^L(x_1, x_2)}{\partial x_1^2} = \frac{16}{9} t^2 > 0; \\
\frac{\partial \pi_2^F(x_1, x_2)}{\partial x_2} = \frac{4t}{3} \left(-q_A^{2F} + q_B^{2F} \right), \quad \frac{\partial^2 \pi_2^F(x_1, x_2)}{\partial x_2^2} = \frac{16}{9} t^2 > 0.$$
(25)

Hence, the profit function of each firm is strictly convex with respect to its location and there also exist three possible solutions for both the firms, i.e., $(x_1, x_2) = (0, 0), (x_1, x_2) = (0, 1), (x_1, x_2) = (1, 1).$

Secondly, we compare the profits of firm 1 at the two markets given firm 2 locate in market B.

$$\pi_1^L(1,1) - \pi_1^L(0,1) = -\frac{4}{9}t^2 < 0.$$
(26)

Thirdly, the optimal locations can be derived by comparing the profits of firm 2 at the two markets should firm 1 locate in market A:

$$\pi_2^F(0,1) - \pi_2^F(0,0) = \frac{4}{9}t^2 > 0.$$
(27)

Therefore, the following result can be straitly obtained by Eq. (26) and (27).

Result 2 In the location-quantity choice game with sequential wage setting, both firms will choose the opposite endpoints of the line segment as their optimal locations.

Result 1 and result 2 yield proposition 5.

Proof of proposition 6

Substituting $x_1 = 0, x_2 = 1$ into Eq. (3), from Eq. (9), we have

$$(q_A^{1C} + q_B^{1C}) = (q_A^{2C} + q_B^{2C}) = \frac{2}{9}(2a - t).$$
(28)

From Eq. (14), we have

$$(q_A^{1L} + q_B^{1L}) - (q_A^{2F} + q_B^{2F}) = -\frac{1}{56}(2a - t) < 0.$$
⁽²⁹⁾

Substituting the equilibrium locations $x_1 = 0, x_2 = 1$ into Eqs. (10)-(11) and (15)-(16), the total profits of both firms gained from markets A and B under simultaneous wage setting and sequential wage setting are respectively expressed as follows:

$$\pi_1^C = \frac{1}{9} \left\{ \left[a - \frac{1}{6}(2a-t) + t \right]^2 + \left[a - \frac{1}{6}(2a-t) - 2t \right]^2 \right\} = \pi_2^C.$$
(30)

$$\pi_1^L = \frac{1}{9} \left\{ \left[a - \frac{3}{16}(2a - t) + t \right]^2 + \left[a - \frac{3}{16}(2a - t) - 2t \right]^2 \right\},$$

$$\pi_2^F = \frac{1}{9} \left\{ \left[a - \frac{9}{56}(2a - t) + t \right]^2 + \left[a - \frac{9}{56}(2a - t) - 2t \right]^2 \right\}.$$
(31)

From Eq. (31), we have $\pi_1^L < \pi_2^F$ for $0 < \frac{9}{56}(2a-t) < \frac{3}{16}(2a-t)$ and $a > 0, t < \frac{10}{29}a$.

Proof of proposition 7

Using (12), (17)-(21) and (30)-(31), we can get the social welfare expressions of each market in the simultaneous game and in the sequential game straightforwardly.

$$W_A^C = \frac{1}{324} (112a^2 - 112at + 190t^2) = W_B^C,$$

$$W_A^L = \frac{1}{225792} (74516a^2 - 74516at + 131525t^2),$$

$$W_B^F = \frac{1}{28224} (9916a^2 - 9916at + 16591t^2).$$
(32)

From formula (32), we have the following result:

$$W_A^L - W_B^F = -\frac{401}{75264} (2a - t)^2 < 0.$$
(33)

The inequality of (33) derives from the assumptions of $0 < t < \frac{10}{29}a$ and a > 0.