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Pollution Taxes and Location Decision under Free Entry Oligopoly

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Abstract

This paper examines the impact of a pollution tax as a pollution control device on the output and location decisions of undifferentiated oligopolistic firms with free entry. It shows that the optimum output and location of an oligopolistic firm is independent of a change in the pollution tax if the demand function is linear. Furthermore, an increase in the pollution tax will increase (decrease) output and move the plant location toward (away from) the CBD if the demand function is concave (convex). It also shows that a higher pollution tax will increase the pollution damage if the demand function is linear and the location effect dominates the demand effect. These results are significantly different from the conventional results based on the monopolistic location model. It indicates that the demand condition plays an important role in the determination of the impact of a pollution tax on the location decision of an oligopolistic firm and the pollution damage to the CBD residents.

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1. Introduction

In his famous 1973 paper, Tintenberg (1973) showed that in a general equilibrium model firms will face different pollution taxes because the marginal contribution of waste to pollution varies spatially. Along this line, Mathur (1976), Gokturk (1979) and Forster (1988) introduced pollution taxes into the linear location model and examined the effects of a change in the pollution tax on the plant location and on abatement decisions of the firm. Recently, Hwang and Mai (2004) (henceforth HM) extended the conventional linear location model to the two-dimensional Weber triangular location model. Assuming that (1) a monopolist uses two transportable inputs located at two vertices to produce a product which is sold at CBD (i.e., Central Business District) located at the third vertex, (2) the objective of the firm is to find the profit maximizing plant location within the triangle, they obtained the following important propositions.

HM1. *The plant location of the firm is invariant with respect to a change in the pollution tax policy if the production function is constant returns to scale. Nevertheless, the plant location moves closer to (farther away from) the CBD as a result of higher pollution tax if the production function is decreasing returns to scale (increasing returns to scale). Hwang and Mai (2004, p. 61).*

HM2. *If the production function is increasing returns to scale, then a higher pollution tax will decrease the pollution damage to the CDB residents. Hwang and Mai (2004, p. 62).*

However, HM1 and HM2 emphasize the cost factors and neglect the demand factors of plant location. They only focus on the polar case of monopoly. Given that oligopolistic industries are relatively common in the economy, it is surprising that the impact of pollution taxes on the location decisions of oligopolistic firms has received little attention.

The purpose of this paper is to fill this gap. It explicitly incorporates oligopolistic market structure into the Weber triangle and examines the impact of a change in the pollution tax on the plant location and on abatement decisions of oligopolistic firms. It will be shown that HM1 and HM2 need not hold in the oligopolistic location model.

2. An Oligopolistic Location Model

Our analysis is based on the following assumptions.

(a) N firms employ two transportable inputs (M_1 and M_2) located at A and B to produce a homogenous product (q) which is sold at the output market C locating at the CBD. The location triangle in Figure 1 illustrates the location problem of oligopolistic firms. In figure 1, the distance a and b and the angle γ are known; h is the distance between the plant location (E) and the CBD (C); z_1 and z_2 are the distances of plant location (E) from A and B, respectively; θ is the angle between CA and CE.

(b) Firms make Cournot-Nash conjectures about their rivals' production and location decisions and enter the industry without any restrictions until there is no economic profit. Assume also that equilibra are symmetric. Thus, we can neglect the location dispersion of firms and focus on the impact of market demand on the location decision of a representative firm.

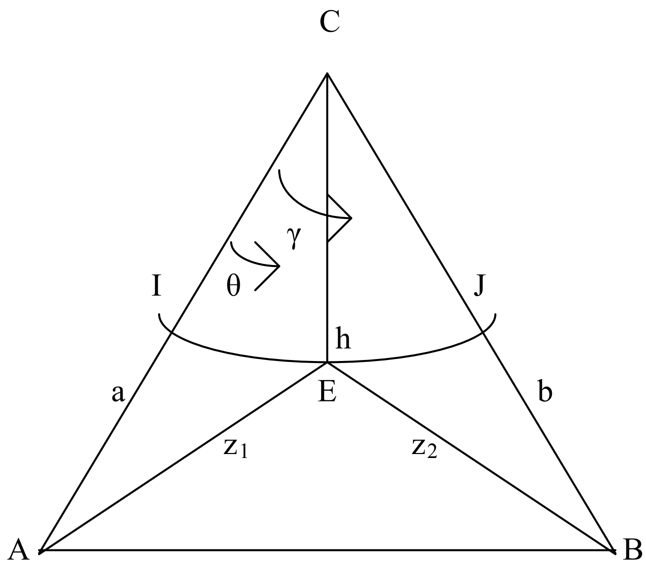

Figure 1. The Weber-Moses Triangle

(c) The production function is homothetic and can be specified as:

$$q = f(M_1, M_2) \tag{1}$$

with $f_{M1} \equiv \partial q/\partial M_1 > 0$, $f_{M2} \equiv \partial q/\partial M_2 > 0$, $f_{M1M1} \equiv \partial^2 q/\partial M_1^{\,2} < 0$, $f_{M2M2} \equiv \partial^2 q/\partial M_2^{\,2} < 0$.

(d) The industry inverse demand function for output is given by

$$P = P(Q) \tag{2}$$

where $Q = \sum q^i$ is the market quantity demanded, $P_Q \equiv \partial P/\partial Q < 0$, cf. HM (2004, p. 59). It should be noted that $\sum$ denotes $\sum_{i=1}^{N}$.

(e) The prices of inputs and output are evaluated at the plant location (E). The cost of purchasing inputs is the price of input at the source plus the freight cost, and the price of output is the market price minus the freight cost.

(f) Transportation rates are constant.

(g) The pollution tax revenue function can be specified as:

$$G(q, e) = ey(q) \tag{3}$$

where e = the pollution tax rate, y(q) = the amount of pollution generated by the production process that depends on the output level. Following HM (2004, p. 60), we assume that $y(q) = \beta q$, β is a positive constant, thus $G_q = e\beta > 0$, $G_{qq} = 0$, $G_{qe} = \beta > 0$, and $G_e = \beta q > 0$.

(h) The objective of each firm is to find the optimum location and production within the Weber triangle which maximizes the profit.

With these assumptions, the profit maximizing location problem of the representative firm is given by

$$\max \Pi = [P(Q)-th]f(M_1, M_2) - (w_1+r_1z_1)M_1 - (w_2+r_2z_2)M_2 - G(q, e) \tag{4}$$

where $z_1 = (a^2 + h^2 - 2ah\cos\theta)^{1/2}$, $z_2 = [b^2 + h^2 - 2bh\cos(\gamma-\theta)]^{1/2}$; w_1 and w_2 are the base prices of M_1 and M_2 at their sources A and B; t, r_1 and r_2 are constant transportation rates of q, M_1, M_2; z_1, z_2, and h are the distances from the plant location to the source location A, B and the market location C. It is worth mentioning that q, M_1, M_2, h and θ are choice variables and a, b, e, γ, w_1, w_2, t, r_1, r_2 are positive parameters.

Assuming that the oligopolistic firm treats q instead of M_1 and M_2 as a decision variable, we first derive the cost function by minimizing total cost subject to a given output at a given location,

$$\min L = (w_1+r_1z_1)M_1 - (w_2+r_2z_2)M_2 + \lambda[q - f(M_1, M_2)] \tag{5}$$

where λ is the Lagrange multiplier; q, h and θ are parameters. Using the standard comparative static analysis and the envelope theorem, we can show that the production function is homothetic if and only if the cost function is separable in the sense that

$$C(q; h, \theta) = c(w_1+r_1z_1, w_2+r_2z_2)H(q) \tag{6}$$

where c is a function of the delivered prices of M_1 and M_2, cf. Takayama (1993, Proposition 3.5., pp. 147-148). Hence, the average cost and marginal cost can be written as:

$$AC = C(q; h, \theta)/q = c(h, \theta)H(q)/q \tag{7}$$
$$MC = C_q = c(h, \theta)H_q \tag{8}$$

where $C_q \equiv \partial C(q; h, \theta)/\partial q$ and $H_q \equiv dH(q)/dq$.

Following Hanoch (1975), from (7) and (8), we obtain the following relation:

$$H(q)/q > (=) < H_q \tag{9}$$

if the production function exhibits increasing (constant) or decreasing returns to scale.

Substituting the cost function $C = C(q; h, \theta)$ into (4), we obtain the profit as a function of q, θ and h. The first-order condition for a maximum would be

$$\partial\Pi/\partial q = [(P + P_Q q) - th] - c(.)H_q - G_q = 0 \tag{10}$$
$$\partial\Pi/\partial\theta = - c_\theta H(q) = 0 \tag{11}$$
$$\partial\Pi/\partial h = - tq - c_h H(q) = 0 \tag{12}$$

where $G_q \equiv \partial G(.)/\partial q$, $c_\theta \equiv \partial c(.)/\partial \theta$, $c_h \equiv \partial c(.)/\partial h$. Assume that the second-order conditions are satisfied and the possibility of the corner solution is excluded; cf. Kusumoto (1986) and Mai and Hwang (1992). We can solve (10)-(12) for q, θ and h when entry is prohibited.

If free entry is allowed, each firm in the industry earns normal profit only. The following condition must be satisfied.

$$\Pi = [P(Nq) - th]q - c(.)H(q) - G(q, e) = 0 \tag{13}$$

If there is an interior solution, we can solve equations (10) – (13) for q, θ, h and N in terms of e and $v = (a, b, \gamma, w_1, w_2, r_1, r_2, t)$, where v is a vector of remaining parameters.

$$q = q(e, v), \quad \theta = \theta(e, v), \quad h = h(e, v), \quad N = N(e, v) \tag{14}$$

The expressions for the partial derivatives such as $\partial q/\partial e$, $\partial \theta/\partial e$, $\partial h/\partial e$ and $\partial N/\partial e$ can be obtained by applying the standard comparative static analyses. It is of interest to note before concluding this section that there must be increasing returns to scale for a solution as in (14). To see this, we divide both sides of equation (13) by q and utilizing (10) to obtain

$$[P(Nq) - th] = c(.)H(q)/q - G(q, e)/q \tag{15}$$

Substituting (15) into (10), we obtain

$$P_Q q = c(.)[H_q(q) - H(q)/q] + [G_q - G(q, e)/q] \tag{16}$$

Clearly, the left-hand side of (16) will be negative. Since $G_q = G(q, e)/q$ and $c(.) > 0$, for the right-hand side of (16) to be negative, the production function must exhibit increasing returns to scale, i.e., $H(q)/q > H_q(q)$. It simply implies that in equilibrium all firms produce on the downward sloping part of the average cost curve but not on minimum average cost under Cournot competition with free entry.

This completes our modeling of the basic framework for studying the effects of a pollution tax on the oligopolistic firm's production and location decisions.

3. Effects of Pollution Tax on Production and Location Decisions

We are now in a position to examine the effect of a change in the pollution tax rate on the optimum output and location. Totally differentiating equations (10)-(13) and applying Cramer's rule, we obtain the following results.

$$(\partial q/\partial e) = (-D_2/D_4)yP_{QQ}q^2 \tag{17}$$
$$(\partial \theta/\partial e) = (-1/D_4)yP_{QQ}q^2\Pi_{\theta h}c_h\{[H(q)/q] - H_q\} \tag{18}$$
$$(\partial h/\partial e) = (1/D_4)yP_{QQ}q^2\Pi_{\theta\theta}c_h\{[H(q)/q] - H_q\} \tag{19}$$
$$(\partial N/\partial e) = (y/D_4)(D_2\{[2P_Q + P_{QQ}q - c(.)H_{qq}] + (N - 1)P_{QQ}q\} - \Pi_{\theta\theta}\Pi_{qh}^2) \tag{20}$$

where $\Pi_{\theta h} = - c_{h\theta}H(q)$, $\Pi_{\theta\theta} = - c_{\theta\theta}H(q)$, $\Pi_{qq} = (N + 1)P_Q + NP_{QQ}q - cH_{qq}$, $\Pi_q = P_Q q(N - 1)$, $\Pi_{qh} = c_h\{[H(q)/q] - H_{qq}\}$, $D_2 = \Pi_{\theta\theta}\Pi_{hh} - \Pi_{\theta h}^2$ and D_4 is the relevant Hessian determinant. It should be noted that $\Pi_{\theta\theta} < 0$, $D_2 > 0$ and $D_4 > 0$ by the stability conditions and $c_h < 0$ can be seen from (12).

Assume that the market demand function is linear, i.e., $P_{QQ} = 0$. From (17) – (19), we obtain $(\partial q/\partial e) = 0$, $(\partial\theta/\partial e) = 0$ and $(\partial h/\partial e) = 0$. Thus, we can conclude that

Proposition 1. *The optimum output and location of an oligopolistic firm is independent of a change in the pollution tax rate if the demand function is linear.*

The effect of pollution tax on the optimum output and location is, perhaps, surprising. According to HM (2004), in the monopoly case, an increase in the pollution tax rate will decrease the monopolist's output level and alter its location decision if the demand function is linear. But the above result shows that HM's monopolistic result can not apply to the oligopoly case. The economic interpretation behind Proposition 1 is given as follows. A pollution tax does not change the slope of the demand curve at any output level but will increase the output price in equilibrium for firms to break even. In the case where the demand function is linear, i.e., $P_{QQ} = 0$, a higher output price will not alter the slope of demand curve and so the required tangency between demand curve and average cost curve occurs at the same output level for each firm, i.e., $(\partial q/\partial e) = 0$. Since output per firm remains unchanged, the optimum location will remain the same.

Next, we consider the case where the demand function is not linear, i.e., $P_{QQ} \neq 0$. Since the signs of P_{QQ} and $\Pi_{\theta h}$ can not a priori be determined, the signs of $(\partial q/\partial e)$, $(\partial\theta/\partial e)$ and $(\partial h/\partial e)$ in (17) - (19) are ambiguous. However, from (17) and (19), we can obtain

$$(\partial q/\partial e) < (>)\ 0, \text{ as } P_{QQ} > (<)\ 0 \tag{21}$$
$$(\partial h/\partial e) > (<)\ 0, \text{ as } P_{QQ} > (<)\ 0 \tag{22}$$

Thus, we can conclude that

Proposition 2. *The optimum output of an oligopolistic firm will increase (decrease) as the pollution tax rate increases if the demand function is concave (convex). The optimum location of an oligopolistic firm moves closer to (farther away from) the CBD as the pollution tax rate increases if the demand function is concave (convex).*

This result is different from that of HM (2004) in the monopoly case. They show that a monopoly will decrease output and move the plant location farther away from the CBD as a result of a higher pollution tax if the production function exhibits increasing returns to scale. In the oligopolistic case with increasing returns to scale, we show that a higher pollution tax will lead to a higher output level and move the plant location closer to the CBD if the demand curve is concave. The economic intuition underlying Proposition 2 is given as follow. An increase in the pollution tax rate does not change the slope of the demand curve at any output level but will increase the output price in equilibrium for firms to break even. In the case where the demand function is concave (i.e., $P_{QQ} < 0$), a higher output price decreases the absolute value of the slope of the demand curve and so the point of tangency between demand curve and average curve occurs at a larger output level for each firm. Since the production function exhibits increasing returns to scale, the quantity of inputs, M_1 and M_2, per unit of output declines, then the resources pull decreases while the market pull increases. As a result, the optimum location moves towards the CBD. In the case where the demand function convex (i.e., $P_{QQ} > 0$), the opposite applies.

4. Effect of Pollution Tax on Pollution Level

Next, we examine the impact of pollution taxes on the pollution level at the CBD. The pollution is emitted by undifferentiated oligopolistic firms located at E. The pollution level measured at the CBD is lower than at the plant location and is affected by the distance between the plant location and the CBD. Following HM (2004), the pollution level at CBD is specified as:

$$X^* = m(h)X, \quad m_h < 0, m_{hh} > 0 \tag{23}$$

where X = total pollution measured at E, X* = total pollution measured at C, m(h) = the relationship between the pollution level at CBD and the plant location. Assume that X = y(q). (23) can be rewritten as:

$$X^* = m(h)Ny(q) \tag{24}$$

Since the optimal solution of h, q and N is h = h(e, v), q = q(e, v) and N = N(e, v), we take partial derivative of X* with respect of e to obtain

$$\begin{aligned} \partial X^*/\partial e &= mNy_q(\partial q/\partial e) + Nym_h(\partial h/\partial e) + my(\partial N/\partial e) \\ &= \text{output effect} + \text{location effect} + \text{entry effect} \end{aligned} \tag{25}$$

Since $y_q > 0$ and $m_h < 0$ and the signs of $(\partial q/\partial e)$, $(\partial h/\partial e)$ and $(\partial N/\partial e)$ rely on the characteristics of the demand function and the firm's location decision, thus the sign of $(\partial X^*/\partial e)$ is ambiguous. However, use the result in (21) and (22), we can show that

$$\partial X^*/\partial e > 0, \text{ as } P_{QQ} = 0 \text{ and } - D_2[2P_Q - c(.)H_{qq}] < -\Pi_{\theta\theta}\Pi_{qh}^2 \tag{26}$$

Thus, we can conclude that

Proposition 3. *If the production function exhibits increasing returns to scale and the demand function is linear, a change in the pollution tax rate will increase the pollution damage to the CBD residents if the location effect dominates the demand effect.*

This result is quite different from HM's result in the monopoly case. In the monopolistic location model where the production function exhibits increasing returns to scale, HM shows that a higher pollution tax rate will decrease the pollution damage. In the oligopolistic location model with free entry, we show that a higher pollution tax rate will increase the pollution damage if the location effect dominates the demand effect.

5. Concluding Remarks

We have presented a simple oligopolistic location model and examined the impact of a pollution tax as a pollution control device on the production and plant location decisions of undifferentiated oligopolistic firms. Under the assumptions that (1) N oligopolistic firms are identical and symmetric; (2) they make Cournot-Nash conjectures about their rivals' production and location decisions; (3) they are price takers in the input market and the production function exhibits increasing returns to scale; (4) they produce a homogenous product, we show the impact of a pollution tax on the production and location decision of the oligopolistic firm and the pollution damage to the CBD residents crucially depends upon the characteristics of demand function. These results are different from HM's results based on the monopolistic location model. HM (2004) shows that the impact of a pollution tax on the production and location decisions of a monopolistic firm and the pollution damage to the CBD residents crucially depend upon the characteristics of production function only and the demand function plays no role.

In the case where the demand function is linear, we show that a change in the pollution tax will not change the production and location decisions of the oligopolistic firm, and may increase the pollution damage to the CBD residents. In the case where the demand function is concave (convex), we show that an increase in the pollution tax will cause each firm's output to rise (fall) and move the plant location closer to (farther away from) the CBD. We further show that an increase in the pollution tax rate may increase the pollution damage to the CBD residents even if the production function exhibits increasing returns to scale. These results are significantly different from HM's results.

Although our analysis is based on a simplified oligopolistic location model, it has shed some light on the output and location effects of a pollution tax in an oligopolistic location setting. It shows that the market demand condition plays a key role on the effects of a pollution tax on output, location decision and pollution damage. Our results are significantly different from the conventional results based on the monopoly setting. This indicates that all regulatory policies which aim at reducing the pollution damage through the pollution tax as a pollution control design should receive carefully scrutiny, and there is a need for future research in this area.

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