E C O N O M I C S B U L L E T I N

Price and Quantity Competition Revisited

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Abstract

By enlarging the parameter space originally considered by Singh and Vives (1984) to allow for a wider range of cost asymmetry, Zanchettin (2006) finds that the Singh and Vives result that firms always make larger profits under quantity competition than under price competition fails to hold. This paper shows that while profit ranking between price and quantity competition can be (partially) reversed the celebrated result by Singh and Vives that firms always choose a quantity contract in a two-stage game continues to hold in the enlarged parameter space.

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1. Introduction

In their seminal paper on price and quantity competition in a differentiated duopoly, Singh and Vives (1984) present three important findings (stated here assuming that goods are substitutes): (i) both consumer surplus and total surplus are larger under price competition than under quantity competition; (ii) both firms' profits are higher under quantity competition than under price competition; and (iii) both firms choosing the quantity contract is a dominant strategy equilibrium in the two-stage game in which firms choose between a price contract and a quantity contract in the first stage and then compete accordingly in the second stage. Recently, Zanchettin (2006) relaxes the parameter restriction imposed implicitly by Singh and Vives to allow for a wider range of cost asymmetry and finds that while conclusion (i) above continues to hold conclusion (ii) above does not hold in the larger parameter space. In particular, Zanchettin (2006) finds that, with high degrees of cost asymmetry and/or low degrees of product differentiation, both the efficient firm's profits and total profits can be higher under price competition than under quantity competition.

Since conclusion (iii) above is based on the ranking of firms' profits under different modes of competition, the finding of Zanchettin (2006) of the possibility of partial reversal in profit rankings calls into question whether conclusion (iii) holds in the larger parameter space. The purpose of this paper is to investigate the two-stage game originally studied by Singh and Vives by allowing for the larger parameter space considered by Zanchettin (2006). Our main conclusion is that, in the larger parameter space, both firms choosing the quantity contract is the only Nash equilibrium in the two-stage game; it is either a dominant strategy equilibrium or a weakly dominant strategy equilibrium. Hence the possibility of reversal in profit relationships will not alter the conclusion that in the two-stage game in which firms first commit between a price contract and a quantity contract and then compete accordingly they will always choose the quantity contract.

2. Model Setup

Our model setup is the same as in Zanchettin (2006). Two goods, 1 and 2, are produced by firm 1 and firm 2, respectively. Firm i (i = 1,2) has a constant unit cost of production c_i. It is assumed that $c_1 \le c_2$ so that firm 1 is at least as efficient as firm 2. The inverse demand functions for the two goods are given by:

$$p_i = \alpha - q_i - \gamma q_j, \quad i, j = 1,2 ;\ i \neq j. \tag{1}$$

In (1), α represents consumers' reservation price for either good and γ is the substitution parameter. As in Zanchettin (2006), we focus on the case of substituting goods (i.e., $0 < \gamma < 1$). The direct demand equations are given by:

$$q_i = \frac{1}{1-\gamma^2}[(1-\gamma)\alpha - p_i + \gamma p_j], \quad i, j = 1,2 ;\ i \neq j. \tag{2}$$

Zanchettin introduced the parameter $x = (\alpha - c_2)/(\alpha - c_1)$ to measure the degree of cost asymmetry between the two firms. The range of this parameter is (0,1) and it increases as the cost gap between the two firms decreases. As pointed out by Zanchettin, the monopoly outcome in which firm 1 becomes a monopoly under either price or quantity competition prevails if $x \le \gamma/2$. Zanchettin's focus and also our focus is thus the space S_r = $\{0 < \gamma < 1;\ \gamma/2 < x \le 1\}$.[1] Compared to Singh and Vives (1984), the space S_r allows for a larger range of cost asymmetries between the two firms.

In the space S_r, both firms produce a positive output in the Cournot equilibrium. More specifically, the Cournot equilibrium values are given by (equation (7) in Zanchettin (2006)):

$$\begin{aligned} q_1^C &= p_1^C - c_1 = \frac{(\alpha - c_1)(2-\gamma x)}{4-\gamma^2}; \qquad \pi_1^C = \left[\frac{(\alpha - c_1)(2-\gamma x)}{4-\gamma^2}\right]^2; \\ q_2^C &= p_2^C - c_2 = \frac{(\alpha - c_1)(2x-\gamma)}{4-\gamma^2}; \qquad \pi_2^C = \left[\frac{(\alpha - c_1)(2x-\gamma)}{4-\gamma^2}\right]^2. \end{aligned} \tag{3}$$

Within the space S_r, both firms produce a positive output in the Bertrand equilibrium if

$$x > x^L(\gamma) \equiv \frac{\gamma}{2-\gamma^2} \tag{4}$$

and the Bertrand equilibrium values in this case are given by (equation (8) in Zanchettin (2006)):

$$\begin{aligned} q_1^B &= \frac{p_1^B - c_1}{1-\gamma^2} = \frac{(\alpha - c_1)(2-\gamma^2-\gamma x)}{(1-\gamma^2)(4-\gamma^2)}; \qquad \pi_1^B = \frac{1}{1-\gamma^2}\left[\frac{(\alpha - c_1)(2-\gamma^2-\gamma x)}{4-\gamma^2}\right]^2; \\ q_2^B &= \frac{p_2^B - c_2}{1-\gamma^2} = \frac{(\alpha - c_1)[(2-\gamma^2)x-\gamma]}{(1-\gamma^2)(4-\gamma^2)}; \qquad \pi_2^B = \frac{1}{1-\gamma^2}\left[\frac{(\alpha - c_1)[(2-\gamma^2)x-\gamma]}{4-\gamma^2}\right]^2. \end{aligned} \tag{5}$$

Within the space S_r, the limit-pricing equilibrium in which only firm 1 produces a positive output prevails if condition (4) is not satisfied and the Bertrand equilibrium values in this case are given by (equation (10) in Zanchettin (2006)):

$$\begin{aligned} q_1^L &= \frac{1}{\gamma}(\alpha - c_1)x; \qquad p_1^L - c_1 = \frac{1}{\gamma}(\alpha - c_1)(\gamma - x); \\ \pi_1^L &= \frac{1}{\gamma^2}(\alpha - c_1)^2(\gamma - x)x; \\ p_2^L - c_2 &= q_2^L = \pi_2^L = 0. \end{aligned} \tag{6}$$

[1] The space S_r includes as a subset the space considered by Singh and Vives which is $\{0 < \gamma < 1;\ \gamma < x \le 1\}$. Here, the space S_r is slightly different from that defined in Zanchettin (2006) in that the line in which $\gamma = 1$ is excluded. This is only for convenience of discussion in the Bertrand competition case.

To complete the two-stage game, we next provide equilibrium values for the cases in which firm 1 and firm 2 choose different contracts in the first stage of the game. Consider first the (Q, P) case in which firm 1 chooses the quantity contract while firm 2 chooses the price contract. Maximizing firm 1's profit taking firm 2's price as given gives rise to firm 1's best response function in quantity as given by

$$q_1 = \frac{(1-\gamma)\alpha - c_1 + \gamma p_2}{2(1-\gamma^2)}. \tag{7}$$

Maximizing firm 2's profit taking firm 1's quantity as given gives rise to firm 2's best response function in price as given by

$$p_2 = \frac{\alpha + c_2 - \gamma q_1}{2}. \tag{8}$$

Solving the system of equations comprising (7) and (8) yields the equilibrium values for the (Q, P) case as given by

$$\begin{aligned} q_1^Q &= \frac{p_1^Q - c_1}{1-\gamma^2} = \frac{(\alpha - c_1)(2-\gamma x)}{4-3\gamma^2}; \qquad \pi_1^Q = (1-\gamma^2)\left[\frac{(\alpha - c_1)(2-\gamma x)}{4-3\gamma^2}\right]^2; \\ q_2^P &= p_2^P - c_2 = \frac{(\alpha - c_1)[(2-\gamma^2)x - \gamma]}{4-3\gamma^2}; \qquad \pi_2^P = \left[\frac{(\alpha - c_1)[(2-\gamma^2)x - \gamma]}{4-3\gamma^2}\right]^2. \end{aligned} \tag{9}$$

It is obvious to see that the solution in (9) is valid for all parameters in the space S_r.

Consider next the (P, Q) case in which firm 1 chooses the price contract while firm 2 chooses the quantity contract in the first stage of the game. Maximizing firm 1's profit taking firm 2's quantity as given gives rise to firm 1's best response function in price as given by

$$p_1 = \frac{\alpha + c_1 - \gamma q_2}{2}. \tag{10}$$

Maximizing firm 2's profit taking firm 1's price as given gives rise to firm 2's best response function in quantity as given by

$$q_2 = \frac{(1-\gamma)\alpha - c_2 + \gamma p_1}{2(1-\gamma^2)}. \tag{11}$$

Solving the system of equations comprising (10) and (11) yields the equilibrium values for the (P, Q) case as given by

$$q_1^P = p_1^P - c_1 = \frac{(\alpha - c_1)(2-\gamma^2-\gamma x)}{4-3\gamma^2}; \qquad \pi_1^P = \left[\frac{(\alpha - c_1)(2-\gamma^2-\gamma x)}{4-3\gamma^2}\right]^2;$$
$$q_2^Q = \frac{p_2^Q - c_2}{1-\gamma^2} = \frac{(\alpha - c_1)(2x-\gamma)}{4-3\gamma^2}; \qquad \pi_2^Q = (1-\gamma^2)\left[\frac{(\alpha - c_1)(2x-\gamma)}{4-3\gamma^2}\right]^2. \tag{12}$$

From (12), both firms produce a positive quantity as long as condition (4) is satisfied. Hence, (12) gives the solution to the (Q, P) case when condition (4) holds. If condition (4) does not hold then it is straightforward to verify that the limit-pricing solution given by (6) is the corner solution for the (Q, P) case.

3. The Two-Stage Game

We now study the two-stage game in which the two firms each choose between a price contract and a quantity contract in the first stage and then in the second stage they compete according to their first-stage choice of contract. The reduced first-stage game matrix may take one of the following two forms. If condition (4) is satisfied then the game matrix is given by

		Firm 2	
		Price	Quantity
Firm 1	Price	(π_1^B, π_2^B)	(π_1^P, π_2^Q)
	Quantity	(π_1^Q, π_2^P)	(π_1^C, π_2^C)

In this game matrix, π_1^B and π_2^B are given by (5), π_1^C and π_2^C are given by (3), π_1^P and π_2^Q are given by (12), and π_1^Q and π_2^P are given by (9). If condition (4) is not satisfied then the reduced first-stage game matrix is given by

		Firm 2	
		Price	Quantity
Firm 1	Price	$(\pi_1^L, 0)$	(π_1^P, π_2^Q)
	Quantity	$(\pi_1^L, 0)$	(π_1^C, π_2^C)

In this matrix, the second column is the same as in the first matrix above and π_1^L is given by (6).

The following lemma will help us find the Nash equilibrium in each of the above two game matrices. (The proof of this lemma involves straightforward algebra and is omitted.)

Lemma 1:
(i) $\pi_i^Q > \pi_i^C > \pi_i^B > \pi_i^P$, i = 1,2;
(ii) $\pi_1^L > \pi_1^C$.

Part (ii) of Lemma 1 confirms the reversal in profit relationship between price and quantity competition. Namely, if condition (4) is not satisfied then the more efficient firm 1's profit is higher when the firms compete in prices than when they compete in quantities. However, parts (i) and (ii) of Lemma 1 together imply that each firm's profit from choosing the quantity contract is never less than that from choosing the price contract holding the other firm's choice of price or quantity contract fixed.

The following proposition follows immediately from the relationships in Lemma 1.

Proposition 1: If condition (4) holds then the two-stage game has a dominant strategy equilibrium in which both firms choose the quantity contract in the first stage. If condition (4) does not hold then the two-stage game has a weakly dominant strategy equilibrium in which both firms choose the quantity contract in the first stage.

Proposition 1 implies that the two-stage game in which firms choose between a price contract and a quantity contract in the first stage and then compete accordingly in the second stage has a unique Nash equilibrium in that both firms choose the quantity contract in the first stage of the game. The first part of this proposition is simply an extension of similar result by Singh and Vives (1984) except here the parameter space is larger than that considered by Singh and Vives. The second part of this proposition confirms that the Singh and Vives result essentially holds for the entire parameter space S_r.

We have thus shown that the possibility of (partial) reversal in profit relationships between price competition and quantity competition will not alter the conclusion that in the two-stage game they will always choose the quantity contract in equilibrium. This is unfortunate on welfare grounds since this outcome is welfare inferior to the outcome when both firms choose the price contract.

References

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