

**COURNOT COMPETITION**

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**Working Paper No. 06-W20**

October 2006

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September 2006

Prepared for inclusion in The New Palgrave Dictionary of Economics, 2nd edition, edited by Steven N. Durlauf and Lawrence E. Blume, Palgrave Macmillan (Basingstoke and New York), forthcoming, reproduced with permission of Palgrave MacMillan.

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### **ABSTRACT**

Cournot's 1838 model of strategic interaction between competing firms has become the primary workhorse for the analysis of imperfect competition, and shows up in a variety of fields, notably industrial organization and international trade. This entry begins with a tour of the basic Cournot model and its properties, touching on existence, uniqueness, stability, and efficiency; this discussion especially emphasizes considerations involved in using the Cournot model in multi-stage applications. A discussion of recent applications is provided as well as the URL for an extended bibliography of approximately 125 selected publications from 2001 through 2005.

Journal of Economic Literature Classification Code: D43

Keywords and phrases: Cournot models, Cournot equilibrium, oligopoly, multi-stage models of competition, best-response functions.

## 1. The Cournot Model

The classic Cournot model is static in nature, with each (single-product) firm's strategy being the quantity of output it will produce in the market for a specific homogeneous good; as Kreps observed in his essay on *Nash equilibrium* in the first edition of the New Palgrave, Cournot's model was an early progenitor of Nash's famous paper. Many recent applications have involved multi-stage games; for example, each of  $n$  firms might first simultaneously choose investment levels (say, in cost-reducing R&D) and then simultaneously choose output levels in the second stage. Often now used in such a manner, we will see that Cournot model is doing well, contributing to a range of new research, as it moves towards the two-century mark.

### *The basic one-stage model and associated concepts*

Consider an industry comprised of  $n$  firms, each firm choosing an amount of output to produce. Firm  $i$ 's *output level* is denoted as  $q_i$ ,  $i = 1, \dots, n$ ; let the vector of firm outputs be denoted  $\mathbf{q} \equiv (q_1, q_2, \dots, q_n)$ . The firms' products are assumed to be perfect substitutes (the *homogeneous-goods* case); let  $Q$  denote the aggregate industry output level (i.e.,  $Q \equiv \sum_{i=1}^n q_i$ ). We will refer to the  $(n-1)$ -vector of output levels chosen by firm  $i$ 's rivals as  $\mathbf{q}_{-i}$ ; so, let  $(\mathbf{q}_{-i}, q_i)$  also be the  $n$ -vector  $\mathbf{q}$ . Market demand for the perfect-substitutes case is a function of aggregate output and its inverse is denoted as  $p(Q)$ ; furthermore, let firm  $i$ 's cost of producing  $q_i$  be denoted as  $c_i(q_i)$ . Thus, firm  $i$ 's *profit function* is written as  $\pi^i(\mathbf{q}) \equiv p(Q)q_i - c_i(q_i)$ . All elements of the model are assumed to be commonly known by the firms, though extensions allowing incomplete information are not uncommon.

A *Cournot equilibrium* consists of a vector of output levels,  $\mathbf{q}^{CE}$ , such that no firm wishes to unilaterally change its output level when the other firms produce the output levels assigned to them in the (purported) equilibrium. Alternatively put (and reversing history), it is a Nash equilibrium of the normal-form game with quantities as strategies chosen from a compact space (for example,  $q_i$  in  $[0, Q^*]$ , for some appropriate  $Q^*$ , such as  $p(Q^*) = 0$ ) and with the  $\pi^i(\mathbf{q})$  as the payoff functions. Thus,  $\mathbf{q}^{CE}$  is a Cournot equilibrium if the following  $n$  equations are satisfied:  $\pi^i(\mathbf{q}^{CE}) \geq \pi^i(\mathbf{q}_{-i}^{CE}, q_i)$  for all values of  $q_i$ , for  $i = 1, \dots, n$ .

In analyzing his model applied to a duopoly (he also considered the  $n$ -firm version), Cournot provided the notion of *best-response functions*. In the duopoly case, this is a pair of functions,  $\psi^1(q_2)$  and  $\psi^2(q_1)$ , which provide the profit-maximizing choice of output for firm 1 and 2 (respectively), given conjectures about the output level chosen by the rival firm (that is, each firm's choice of its output level reflects a *best-response property*). Hence,  $\psi^i(q_j) = \arg \max_{q_i} \pi^i(q_i, q_j)$ ,  $i, j = 1, 2, i \neq j$ . That is, we want  $\psi^i(q_j)$  to be the solution to firm  $i$ 's first-order condition:  $p(\psi^i(q_j) + q_j) + p'(\psi^i(q_j) + q_j)\psi^i(q_j) - c'_i(\psi^i(q_j)) = 0$ ,  $i, j = 1, 2, i \neq j$ . We'll assume for now that the problem has a nice solution and that some sort of sufficiency condition holds (e.g., strict quasiconcavity of profits), but the discussion below on existence and uniqueness of equilibrium shows that such classical assumptions are overly-strong and are overly-restrictive for some modern applications, such as those involving multi-stage games or discontinuous cost functions. More generally,  $\psi^i(q_j)$  could be a correspondence (a point-to-set map); we generally restrict the discussion below to functions, and assume as much differentiability as needed.

If output-level choices are best responses to conjectures about each firm's rival's choice of output, and if these conjectures are correct in equilibrium, then the resulting vector of output

levels provides a Cournot equilibrium:  $q_i^{CE} = \psi^i(q_j^{CE})$  for  $i, j = 1, 2$ , and  $j \neq i$ . In other words, the equilibrium occurs where the best-response functions cross when graphed in the space of output levels. Generalizing to  $n$  firms, this condition can be written as  $q_i^{CE} = \psi^i(\mathbf{q}_{-i}^{CE})$  for  $i = 1, \dots, n$ :  $\mathbf{q}^{CE}$  is a Cournot equilibrium if it consists of mutual best-responses for all the firms.

Some variations on the basic model are worth mentioning. If the cost function for a firm has both fixed and variable components, and if the fixed component is avoidable (i.e., is zero at zero output) then the best-response function for the firm will be discontinuous at the positive output level where variable profits just cover the avoidable cost. This is important for two reasons. First, avoidable fixed costs are not unusual in many entry scenarios: think of an airline entering a market where there are already some competitors, with the avoidable cost being advertising. Second, this discontinuity could mean that the only equilibrium might involve some or all firms choosing to not enter (or to exit) the market, even if absent these avoidable costs  $\mathbf{q}^{CE}$  would be strictly positive.

Another avenue for interaction would consider imperfect factor markets, so that instead of  $c_i(q_i)$ , the cost function for firm  $i$  would be written as  $c_i(\mathbf{q}_{-i}, q_i)$ ; then strategic interaction occurs not only through revenue, but also via factor markets. Finally, if the model is one of short-run competition, then the output level of the firm may be restricted to be less than some predetermined capacity level; a simple version is that there are parameters  $k_i$ ,  $i = 1, \dots, n$ , such that a constraint on firm  $i$ 's quantity choice is  $q_i \leq k_i$ ,  $i = 1, \dots, n$ ; this induces a vertical segment (at the capacity level) in a firm's best-response function. Such capacity levels might be choices made in an earlier stage.

Finally, a number of papers develop "non-Cournot" models which generate Cournot-model results. Kreps and Scheinkman (1983) provide a two-stage model of capacity choice followed by price setting in a homogeneous-goods duopoly; the result is a unique subgame-perfect equilibrium with Cournot capacities and a market-clearing price consistent with the standard Cournot model (however, Davidson and Deneckere, 1986, show that this result is especially sensitive to the basis for rationing consumers over firms when out-of-equilibrium firm-level demand exceeds capacity). Klemperer and Meyer (1986) analyze a one-stage game wherein duopolists producing heterogeneous goods non-cooperatively choose either a price or a quantity as the firm's strategy; under either multiplicative or additive error in the demand function, if marginal costs are upward-sloping, the outcome is that predicted by the Cournot model (applied to the heterogeneous-goods case; see the discussion of this case in Section 2 below). The classic embedding of the Cournot model is that of Bowley (1924), the best-known developer of models with "conjectural variations" (CV). This is a static story wherein the first-order conditions in the analysis include firm  $i$ 's conjecture of each rival's reaction to a small change in firm  $i$ 's quantity (e.g.,  $\partial q_j / \partial q_i$  need not be zero for each  $j \neq i$ ); different values of the CV generate competitive, collusive, or Cournot outcomes (among others). Such a handy static embedding of alternative degrees of competition has been employed in a number of theoretical applications, and in a variety of empirical analyses trying to estimate market power. However, Daughety (1985) shows that a basic rationality requirement (that each firm's CV be the same as the actual slope of the best-response function) leads to the Cournot outcome, so that alternative

CV values violate this form of rational expectations. Furthermore, Korts (1999) shows that empirical analyses using the CV approach to assess market power will generally mis-measure the degree of competitiveness of the industry.

## 2. Properties of the Cournot Equilibrium

For most of this section we emphasize results for an n-firm, homogeneous-goods, complete-information model, where a firm's cost function depends only on that firm's output level. As suggested earlier, possibly one of the most important reasons for the continuing interest in the properties of the Cournot equilibrium is that Cournot competition is frequently used as a final stage in a variety of models; analysis employing such refinements as subgame perfection rely on a well-behaved subgame.

### *Existence, Uniqueness, and Stability*

Novshek (1985) provides an existence theorem that has quite practical uses (for expository purposes we consider a slightly less general version). Besides continuity and twice differentiability of the inverse demand function,  $p(Q)$ , Novshek's existence theorem requires that: 1)  $p(Q)$  crosses the quantity axis at a finite value and is strictly decreasing for quantities below that cut-point; 2) the marginal revenue for each firm is decreasing in the aggregate output of its rivals; and 3) each firm's cost function is non-decreasing and lower semi-continuous. Requirement (2) is written formally as  $p'(Q_{-i} + q_i) + p''(Q_{-i} + q_i)q_i < 0$ , where  $Q_{-i} \equiv Q - q_i$ , for all  $i$ . This is equivalent to the assumption that  $\partial^2 \pi^i(\mathbf{q}) / \partial Q_{-i} \partial q_i < 0$  for all  $i$ , that is, that  $Q_{-i}$  and  $q_i$  are *strategic substitutes*, which means that an expansion in  $Q_{-i}$  implies that the optimal  $q_i$  falls. The third requirement means that costs cannot fall as the output level is increased and that cost functions can have jumps (discontinuities), as long as the functions are continuous from the left. This was a substantial improvement over previous existence theorems and it allows for an important case: avoidable fixed costs, such as those in the airline-entry example mentioned earlier. Amir (1996) applies an ordinal version of the theory of *supermodular games* to the existence issue (see Vives, 2005, for a recent survey of supermodular games; see also Amir, 2005, for a comparison of *ordinal* and *cardinal complementarity* in this context); this change of techniques allows for weaker demand conditions (primarily that  $\log p(Q)$  is concave) but requires a slightly stronger condition on each firm's cost function (marginal costs are positive, so models wherein marginal costs might be zero - as might occur with capacity competition - are left out) in order to guarantee that a Cournot equilibrium exists. As an example of the advantages concerning demand analysis, let  $p(Q) = (Q - \bar{Q})^2$  for  $Q \leq \bar{Q}$ , and zero otherwise. Such a function satisfies (1) above, is log-concave (actually, convex), but is excluded from consideration by Novshek's second condition.

Gaudet and Salant (1991) provide conditions for a Cournot equilibrium to be *unique* which address an important consideration when Cournot models are used in a subgame of a larger game: their theorem allows for degeneracy (one or more firms produce zero output but have marginal cost equal to the equilibrium price); thus, such firms are just at the shut-down point in the equilibrium. In a one-stage application this could be eliminated via a small perturbation in the parameters, but in a multi-stage application, such an outcome need not be pathological, as some of the second-stage "parameters" are strategic variables in the first-stage

model (the authors provide a simple, full-information entry game to illustrate this). The sufficient conditions for uniqueness are (not surprisingly) more restrictive than those for existence (assuming Novshek's conditions hold as well): 1) each firm's cost function must be twice continuously differentiable and strictly increasing; and 2) the slope of the marginal cost function is strictly bounded above the slope of the demand function. Thus, concave costs are allowed, to some degree, but the cost function cannot be "too concave," even on subsets of its domain.

Cournot provided an explicit dynamic stability argument for his model by imagining sequential play by each agent (myopically best-responding in the current period to the existing output levels of all rivals); this is referred to as *best-reply dynamics* and when this process converges the solution is termed *stable*. Using best-reply-dynamics to rationalize a static solution has, historically, been a source of substantial criticism, but nonetheless some papers use the requirement of Cournot stability to select an equilibrium when there are multiple equilibria (dynamic stability should not be confused with equilibrium refinement criteria in game theory such as strategic stability). A sufficient condition in the duopoly case is that  $|\partial \psi^1(q_2)/\partial q_2| |\partial \psi^2(q_1)/\partial q_1| < 1$  (see Fudenberg and Tirole, 1991); ; see Seade (1980) for more general conditions (and problems) for best-reply dynamics in the n-firm case. For an approach employing an explicit evolutionary process via replicator dynamics with noise, with firms able to choose "behavioral" strategies (including, but not limited to, best-reply), see Droste, et. al. (2002).

### *Welfare*

Two types of inefficiency can occur in a Cournot equilibrium: the equilibrium price exceeds the marginal cost of production and aggregate output is inefficiently distributed over the firms. Compare the first-order conditions for firms in a duopoly, each producing under conditions of non-decreasing marginal costs (that is,  $p(Q) + p'(Q)q_i = c_i'(q_i)$ ,  $i = 1, 2$ ) with those for a central planner choosing  $q_1$  and  $q_2$  so as to maximize total surplus:  $p(Q) = c_i'(q_i)$ ,  $i = 1, 2$ . Clearly, if demand is downward-sloping at the equilibrium, aggregate output in the Cournot equilibrium will be less than what the social planner would choose. However, a second distortion can be seen in this comparison: under the social planner, each firm's marginal costs are equalized with the others'. This will only hold in a symmetric Cournot equilibrium (where  $q_1 = q_2$ ): production is, in general, inefficiently allocated across the firms.

The mal-distribution of production implies that strategic interaction readily may yield counter-intuitive welfare results. As a simple example, consider a duopoly wherein (inverse) industry demand is  $p = a - Q$  and firm  $i$ 's cost function is  $c_i(q) = C_i q$ ,  $i = 1, 2$ , with  $a > C_1 > C_2 > 0$ ; that is, the linear demand, constant-but-unequal-marginal-cost case. It is straightforward to find the equilibrium and show that it is interior and unique. Let  $W$  be the sum of producers' and consumers' surplus. Then a little work shows that  $dW/dC_1 > 0$  if  $11C_1 - 7C_2 - 4a > 0$ ; to see that these conditions are non-empty, consider the parameter specification ( $a = 20$ ,  $C_1 = 13$ ,  $C_2 = 8$ ) which satisfies all the foregoing requirements. The point of the example is that a *reduction* of firm 1's marginal cost leads to a *decrease* in equilibrium welfare. Thus, strategic interaction by the firms in the marketplace can lead to reversals of the usual welfare intuition that cost-

improving technological change is beneficial. The reason this occurs is that the cost reduction results in an increase in the high-cost firm's equilibrium output level and a (smaller) decrease in the low-cost firm's output level; this increased inefficiency in aggregate production can be sufficient to overwhelm other efficiency improvements (such as the increase in industry output). This is similarly true if in the above model firm 2 is an incumbent monopolist (using simple monopoly pricing) and firm 1 an entrant: welfare will fall due to entry.

In the n-firm version of the constant-marginal-cost model, changes in the distribution of production costs (holding the mean fixed) do not affect industry output; this is seen by summing over the first-order conditions, whence  $np(Q) + p'(Q)Q = \sum_{i=1}^n C_i$ . Bergstrom and Varian (1985) showed that (assuming the pre- and post-change equilibria are interior) such mean-preserving changes in the marginal costs strictly improve welfare if and only if the variance of the marginal costs strictly increases; the reason is that the aggregate cost of production has decreased if the variance increases. Salant and Shaffer (1999) extend this idea to consider the effects of changes in first-stage parameters (for example, cost-reducing R&D investments) on second-stage costs in models wherein Cournot competition is employed in the second stage. They argue that since aggregate production costs are maximized when all firms have the same costs, it is the asymmetric equilibria in such games (which are often assumed away) which may yield the most important outcomes to examine, from both a social and private perspective.

Does entry necessarily reduce the equilibrium price? A recent contribution provides a clean result if we restrict attention to the symmetric case wherein all firms have the same twice continuously differentiable and non-decreasing cost function, and demand is continuously differentiable and downward-sloping. Amir and Lambson (2000) show that the equilibrium price falls with an increase in the number of competitors if, for all  $Q$ ,  $p'(Q) < c''(q)$  for all  $q$  in  $[0, Q]$ . Thus, even with some degree of returns-to-scale (for example, as might occur with U-shaped average costs), entry will reduce price, at least with identical firms. However, Hoernig (2003) shows that, even if the equilibria are stable and there are no returns to scale, price can rise with entry if products are differentiated.

If the products of the firms are imperfect substitutes (that is, products are differentiated), then (in general) there is no aggregate demand function  $p(Q)$ ; rather firm  $i$ 's inverse demand function would be written as  $p_i(\mathbf{q})$  and profits would be written as  $\pi^i(\mathbf{q}) \equiv p_i(\mathbf{q})q_i - c_i(q_i)$ ,  $i = 1, \dots, n$ . Welfare in this model can be contrasted with a re-formulation of the model so that each firm chooses a price for its product; standard parlance is to call the price-strategy model the (*differentiated products*) *Bertrand model* (even though Bertrand's famous review of Cournot did not envision heterogeneity in products; see Friedman's 1988 translation of Bertrand's review). Without going into detail on the (*differentiated products*) Bertrand model (see the essay on *Bertrand Competition*), Singh and Vives (1984) have shown (for linear, symmetric demand and constant marginal costs in a duopoly setting) that while profits under Cournot competition exceed those under Bertrand competition, total surplus is higher under Bertrand competition than under Cournot competition. Note that this result holds in the one-stage game. However, these results may be reversed in a two-stage application. For example, Symeonidis (2003) considers R&D investment with spillovers in a two-stage game, and shows that (at least for a portion of the



parameter space) Cournot competition leads to higher welfare than Bertrand competition. The basic intuition is that if profits are higher for second-stage Cournot competition compared with second-stage Bertrand competition, and first-stage investment is inefficiently low in either case, then the increased second-stage profits may partly correct the inefficiently low first-stage investment, leading to an overall welfare gain for competition in quantities rather than prices.

Finally, convergence of a Cournot equilibrium to a competitive equilibrium, as the number of firms grows, was considered by Cournot in one of his chapters (Chapter 8), and has been the subject of a number of papers; see Novshek and Sonnenschein (1978, 1987) for a general equilibrium treatment where appropriate replication of Cournot economies yields equilibria arbitrarily close to the Walrasian equilibrium; see Alos-Ferrer (2004) for an evolutionary model (which allows for memory) at the level of an industry.

### 3. Applications

The literature exploring and applying the Cournot model is vast; an earlier extended bibliography can be found in Daughety (1988; reprinted 2005). The more recent literature employing the Cournot model is already becoming significant in size: a survey of articles in 16 top mainline and field journals, for the time period 2001 - 2005, netted approximately 125 articles exploring or applying the Cournot model in one of its various common forms. The URL for an Excel file of (abbreviated) citations and some characteristics of each article (number of firms, number of stages, welfare considerations, informational regime, and topic classification) is: <http://www.vanderbilt.edu/Econ/faculty/Daughety/ExtendedCournotBib2001-2005.xls>

However, some excellent papers have undoubtedly been missed (not to mention papers from the 1990's) and space limitations preclude doing more than providing the briefest of tours and just a taste of the literature, so only a very few can be discussed below. This section addresses five topics which account for a significant portion of the literature, three areas that overlap other fields, and two (comparatively) new areas of research.

*Delegation.* Vickers (1985) uses an n-firm, two-stage model to examine performance measures for managers. Restricting the manager's performance measure to be a weighted average of profits and output, with the weights determined by the owner of each firm in the first stage, he shows that the weight on output is non-zero. This makes each manager more aggressive (each chooses to produce a higher output level), thereby leading to lower profits per firm. Sklivas (1987) considered the differentiated-products Bertrand version and showed that owners choose weights on revenue and profits so as to make managers more passive (they post higher prices), leading to increased profits. Miller and Pazgal (2001) have unified this literature, showing that incentive schemes based on own and rival's profits result in an equilibrium which is insensitive to whether the firm chooses price or quantity as its strategic variable.

*Information transfer.* Vives (1984), Gal-Or (1985), and Li (1985) all consider variants of "information transfer" models to examine the possibility of information sharing, wherein firms may choose to pool information on either demand or cost parameters. These models are analyzed as *Bayesian-Nash games*, so that before seeing a private signal about the parameter of

interest (for example, the demand intercept), each firm chooses whether or not to share the information with the other firms; then information is received and production (or pricing) occurs in the second stage. The nature of the good (substitutes or complements), the type of information (common or individual), and the strategy space (quantities or prices) all affect whether firms will share information. Ziv (1993) relaxes the verifiability of information and finds that firms will send misleading information if they can; he then considers mechanisms for eliciting truthful messages.

*Intellectual property.* Katz and Shapiro (1985) and Kamien and Tauman (1986) consider the licensing of innovations in an oligopoly. Katz and Shapiro employ a three-stage duopoly game in which the innovation is developed, then a single license is auctioned, and then the firms compete. Kamien and Tauman use a two-stage,  $n$ -firm game with a posted price for the innovation (a fee or a royalty), followed by competition. More recently, Fauli-Oller and Sandonis (2003) consider optimal competition policy when considering licenses as an alternative to merger. Anton and Yao (2004) allow for weak patent protection and consider how disclosure of information about an innovation (e.g., through the patent application) can be a signaling device to influence competitors, but those same competitors may be able to employ the information to successfully use (infringe on) the patent; here small innovations are patented and substantial innovations are protected through secrecy.

*Mergers.* Salant, et. al. (1983) show that exogenously-determined mergers of a subset of firms in the constant-marginal-cost setup yields a problematical result: a sufficient condition for a merger to be unprofitable is that it involve fewer than 80 percent of the industry, hardly a resounding endorsement of using such a model to analyze mergers. This result, however, is partly driven by the assumptions of homogeneous products, constant unit costs, and industry structure. Perry and Porter (1985) show that various mergers can be profitable if firms have sufficiently-increasing marginal costs. Daughety (1990), using a two-tiered-industry,  $n$ -firm model, with  $m$  firms choosing output in the first stage (tier) and  $n-m$  firms choosing output in the second stage, shows that if  $1 < m < n$ , then when  $m$  is comparatively small ( $m < n/3$ ), mergers of two second-tier firms to make a first-tier firm, can be both profitable and social-welfare-enhancing, even though such mergers increase concentration and have no cost synergies (all firms have identical constant unit costs). Recently, Pesendorfer (2005), using a repeated game model with entry, has found that merger to monopoly may not be profitable, but merger in a non-concentrated industry can be; these differences from the previous literature partly reflect long-run versus short-run profitability computations.

*R&D.* d'Aspremont and Jacquemin (1988) considered cost-reducing R&D in the presence of spillovers, and considered both non-cooperative and cooperative R&D decision-making; there have been a number of recent papers on cost-reducing spillovers (e.g., see Zhao, 2001, for more on the negative welfare effects of cost-reducing innovation, and Symeonidis, 2003, cited in Section 2 above, as well as the work discussed below under the subject of auctions with competition). Toshimitsu (2003) considers the incentive and welfare properties of quality-based R&D subsidies for firms in a model of endogenously-determined product quality (and thus product differentiation); subsidizing high quality is welfare-enhancing (independent of whether

the Cournot or Bertrand model is employed).

Areas of ongoing effort which overlap into other fields include *experimental economics* (see the essays on this topic), *the financial structure of the firm* (e.g., Brander and Lewis, 1986, on determinate debt-equity due to imperfect competition, and see Povel and Raith, 2004, extending Brander and Lewis via endogenously-determined debt contracts); and *international trade* (e.g., Brander and Spencer, 1985, analyzing the strategic use of subsidies in international competition; Mezzetti and Dinopoulos, 1991, discussing domestic firm-union bargaining and import competition; and Spencer and Qiu, 2001, concerning relationship-specific investments and trade).

Finally, a few examples of comparatively new topics. While auctions with private information has long been an area of interest, the developing literature on *auctions with competition* has started to take seriously the combination of incomplete information and post-auction competition. For example, see Das Varma (2003) or Goeree (2003), who find that signaling by winners of an auction causes bids to be biased when post-auction interaction between the auction's winner and losers can be influenced by the size of the bid. A nice example is if firms have private information about how acquiring a cost-reducing innovation might affect the firm's production costs and bidding for a license for the innovation precedes Cournot oligopoly interaction; here signaling with a high bid suggests that the winner will have low costs and will produce a high level of output.

A second new area is *networks*; one recent example is Goyal and Moraga-Gonzalez, 2001. They model bilateral agreements to share knowledge, and allow for the possibility of partial collaboration, via considering possible networks of relationships. They examine how the nature of the firms' interaction in markets can contribute to the instability of certain types of strategic alliances, and the stability of other ones.

#### **4. In Closing: A Broader Perspective on Cournot Competition**

If alive to critique this essay, Cournot might view the interpretation of the phrase "Cournot Competition" being limited merely to the legacy of his oligopoly analysis in Chapter Seven of the *Recherches* to be an overly-restrictive interpretation of the assignment. And well he should. Hicks (1935, 1939) argues that Cournot was the first to present a modern model of monopoly as well as the precise conditions for perfect competition; furthermore, as noted earlier, Cournot's eighth chapter concerned "unlimited competition." In the 1937 Cournot Memorial session of the Econometric Society, A. J. Nichol (1938) observed that if ever there was an apt illustration of Carnegie's dictum that "It does not pay to pioneer," then Cournot's life and work would be it. Cournot's oligopoly model was essentially ignored for many years, or was relegated to dusty corners of microeconomics texts, but over the last few decades it has come to be an essential tool in many an economist's toolbox, and is likely to continue as such.

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*(for the 2001-2005 Extended Bibliography, see the URL in the essay)*

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